

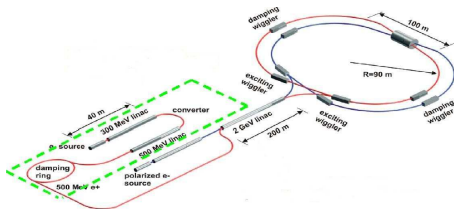
# Physics of $\tau$ lepton at the Super Charm-Tau factory

D. Epifanov (BINP)

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## Outline:

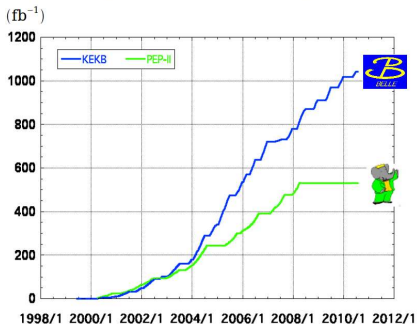
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- 3  $\tau$  decays with leptons
- 4 Test of lepton universality
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- 6 Summary



- The world largest statistics of  $\tau$  leptons collected by  $e^+e^-$   $B$  factories (Belle and *BABAR*) opens new era in the precision tests of the Standard Model (SM).
- In the SM  $\tau$  decays due to the charged weak interaction described by the exchange of  $W^\pm$  with a pure vector coupling to only left-handed fermions. There are two main classes of tau decays:
  - Decays with leptons, like:  $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau$ ,  $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau \gamma$ ,  $\tau^- \rightarrow \ell^- \ell'^+ \ell'^- \bar{\nu}_\ell \nu_\tau$ ;  $\ell, \ell' = e, \mu$ . They provide very clean laboratory to probe electroweak couplings, which is complementary/competitive to precision studies with muon (in experiments with muon beam). Plenty of New Physics models can be tested/constrained in the precision studies of the dynamics of decays with leptons.
  - Hadronic decays of  $\tau$  offer unique tools for the precision study of low energy QCD.

# Introduction: $e^+e^- B$ factories

## Integrated luminosity of B factories



**> 1 ab<sup>-1</sup>**

**On resonance:**

Y(5S): 121 fb<sup>-1</sup>

Y(4S): 711 fb<sup>-1</sup>

Y(3S): 3 fb<sup>-1</sup>

Y(2S): 25 fb<sup>-1</sup>

Y(1S): 6 fb<sup>-1</sup>

**Off reson./scan:**

~ 100 fb<sup>-1</sup>

**~ 550 fb<sup>-1</sup>**

**On resonance:**

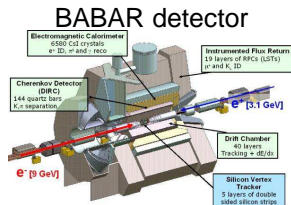
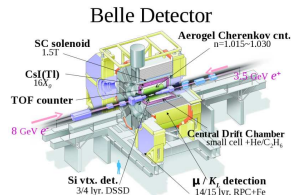
Y(4S): 433 fb<sup>-1</sup>

Y(3S): 30 fb<sup>-1</sup>

Y(2S): 14 fb<sup>-1</sup>

**Off resonance:**

~ 54 fb<sup>-1</sup>



$$\sigma(b\bar{b}) = 1.05 \text{ nb} \quad N_{b\bar{b}} = 1.2 \times 10^9$$

$$\sigma(c\bar{c}) = 1.30 \text{ nb} \quad N_{c\bar{c}} = 2.0 \times 10^9$$

$$\sigma(\tau\tau) = 0.92 \text{ nb} \quad N_{\tau\tau} = 1.4 \times 10^9$$

**B factories are also charm and  $\tau$  factories !**

**Analysis of  $\tau$  data is going on at B factories.**

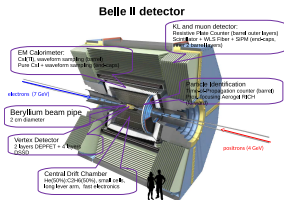
## Belle II with unpolarized beams

Planned integrated luminosity is  $50 \text{ ab}^{-1}$

$$\sigma(b\bar{b}) = 1.05 \text{ nb} \quad N_{b\bar{b}} = 53 \times 10^9$$

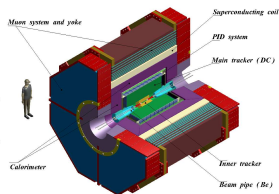
$$\sigma(c\bar{c}) = 1.30 \text{ nb} \quad N_{c\bar{c}} = 65 \times 10^9$$

$$\sigma(\tau\tau) = \mathbf{0.92 \text{ nb}} \quad \mathbf{N_{\tau\tau} = 46 \times 10^9}$$



## Super Charm-Tau factory with polarized $e^-$ beam

In five c.m.s. energy points  
( $2E = 3.554, 3.686, 3.770, 4.170, 4.650 \text{ GeV}$ )  
it is planned to accumulate  $7 \text{ ab}^{-1}$ , which  
corresponds to  $\mathbf{N_{\tau\tau} = 21 \times 10^9}$



**The polarized  $e^-$  beam results in the nonzero average polarization of single tau, which provide advantages in some particular studies with  $\tau$  lepton.**

# Michel parameters in $\tau$ decays

In the SM, charged weak interaction is described by the exchange of  $W^\pm$  with a pure vector coupling to only left-handed fermions ("V-A" Lorentz structure). Deviations from "V-A" indicate New Physics.  $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau$  ( $\ell = e, \mu$ ) decays provide clean laboratory to probe electroweak couplings.

The most general, Lorentz invariant four-lepton interaction matrix element:

$$\mathcal{M} = \frac{4G}{\sqrt{2}} \sum_{\substack{N=S,V,T \\ i,j=L,R}} g_{ij}^N \left[ \bar{u}_i(\ell^-) \Gamma^N \nu_n(\bar{\nu}_\ell) \right] \left[ \bar{u}_m(\nu_\tau) \Gamma_N u_j(\tau^-) \right],$$

$$\Gamma^S = 1, \quad \Gamma^V = \gamma^\mu, \quad \Gamma^T = \frac{i}{2\sqrt{2}} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$$

Ten couplings  $g_{ij}^N$ , in the SM the only non-zero constant is  $g_{LL}^V = 1$

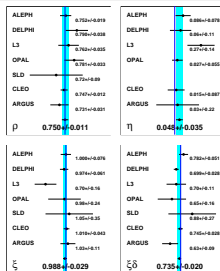
Four bilinear combinations of  $g_{ij}^N$ , which are called as Michel parameters (MP):  $\rho$ ,  $\eta$ ,  $\xi$  and  $\delta$  appear in the energy spectrum of the outgoing lepton:

$$\frac{d\Gamma(\tau^\mp)}{d\Omega dx} = \frac{4G_F^2 M_\tau E_{\max}^4}{(2\pi)^4} \sqrt{x^2 - x_0^2} \left( x(1-x) + \frac{2}{9} \rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x) \right. \\ \left. \mp \frac{1}{3} P_\tau \cos\theta_\ell \xi \sqrt{x^2 - x_0^2} \left[ 1 - x + \frac{2}{3} \delta(4x - 4 + \sqrt{1 - x_0^2}) \right] \right), \quad x = \frac{E_\ell}{E_{\max}}, \quad x_0 = \frac{m_\ell}{E_{\max}}$$

In the SM:  $\rho = \frac{3}{4}$ ,  $\eta = 0$ ,  $\xi = 1$ ,  $\delta = \frac{3}{4}$

# Michel parameters of $\tau$ , current status

Michel par.	Measured value	Experiment	SM value
$\rho$ (e or $\mu$ )	$0.747 \pm 0.010 \pm 0.006$	CLEO-97	3/4
	<b>1.2%</b>		
$\eta$ (e or $\mu$ )	$0.012 \pm 0.026 \pm 0.004$	ALEPH-01	0
	<b>2.6%</b>		
$\xi$ (e or $\mu$ )	$1.007 \pm 0.040 \pm 0.015$	CLEO-97	1
	<b>4.3%</b>		
$\xi\delta$ (e or $\mu$ )	$0.745 \pm 0.026 \pm 0.009$	CLEO-97	3/4
	<b>2.8%</b>		
$\xi_h$ (all hadr.)	$0.992 \pm 0.007 \pm 0.008$	ALEPH-01	1
	<b>1.1%</b>		



## Current systematic uncertainties at Belle (study is going on)

Source	$\Delta(\rho)$ , %	$\Delta(\eta)$ , %	$\Delta(\xi\rho\xi)$ , %	$\Delta(\xi\rho\xi\delta)$ , %
Physical corrections				
ISR+ $\mathcal{O}(\alpha^3)$	0.10	0.30	0.20	0.15
$\tau \rightarrow \ell\nu\nu\gamma$	0.03	0.10	0.09	0.08
$\tau \rightarrow \rho\nu\gamma$	0.06	0.16	0.11	0.02
Background	0.20	0.60	0.20	0.20
Apparatus corrections				
Resolution $\oplus$ brems.	0.10	0.33	0.11	0.19
$\sigma(E_{\text{beam}})$	0.07	0.25	0.03	0.15
Normalization				
$\Delta\mathcal{N}$	0.11	0.50	0.17	0.13
<b>without EXP/MC corr.</b>	<b>0.3</b>	<b>1.0</b>	<b>0.4</b>	<b>0.4</b>

At Belle we are working on the various EXP/MC efficiency corrections which produce the systematic uncertainties in MP of about few percent.

# Effect of the $e^-$ beam polarization

At the Super Charm-Tau factory with polarized electron beam the average polarization of single  $\tau$  is nonzero, hence the differential decay probability will contain both,  $\tau$  spin-dependent and spin-independent parts.

$$\frac{d\sigma(\vec{\zeta}^-, \vec{\zeta}^+)}{d\Omega_\tau} = \frac{\alpha^2}{64E_\tau^2} \beta_\tau (D_0 + D_{ij} \zeta_i^- \zeta_j^+ + \mathcal{P}_e (F_i^- \zeta_i^- + F_j^+ \zeta_j^+))$$

$$D_0 = 1 + \cos^2 \theta + \frac{1}{\gamma_\tau^2} \sin^2 \theta, \quad \mathcal{P}_e = \frac{N_e(+)-N_e(-)}{N_e(+)+N_e(-)}$$

$$D_{ij} = \begin{pmatrix} (1 + \frac{1}{\gamma_\tau^2}) \sin^2 \theta & 0 & \frac{1}{\gamma_\tau} \sin 2\theta \\ 0 & -\beta_\tau^2 \sin^2 \theta & 0 \\ \frac{1}{\gamma_\tau} \sin 2\theta & 0 & 1 + \cos^2 \theta - \frac{1}{\gamma_\tau^2} \sin^2 \theta \end{pmatrix}$$

Single  $\tau$  studies at the Super Charm-Tau factory:

$$\frac{d\sigma(\vec{\zeta}^-)}{d\Omega_\tau} = \frac{\alpha^2}{32E_\tau^2} \beta_\tau (D_0 + \mathcal{P}_e F_i^- \zeta_i^-)$$

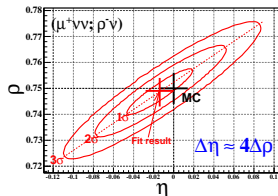
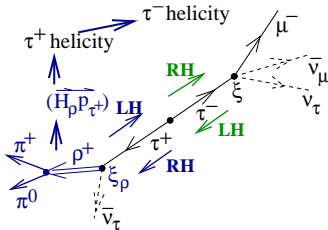
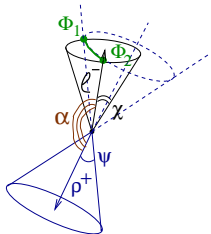
As a result, there are two methods to measure MP:

- **(I) Unbinned fit of the  $(\ell, \rho)$  events in 9D phase space (spin-spin correlations + polarized  $e^-$  beam)**
- **(II) Unbinned fit of the  $(\ell, \text{all})$  events in 3D lepton phase space (only polarized  $e^-$  beam)**

# Method at $e^+e^-$ factory with unpolarized beams

Effect of  $\tau$  spin-spin correlation is used to measure  $\xi$  and  $\delta$  MP.

Events of the  $(\tau^\mp \rightarrow \ell^\mp \nu \nu; \tau^\pm \rightarrow \rho^\pm \nu)$  topology are used to measure:  $\rho$ ,  $\eta$ ,  $\xi_\rho \xi$  and  $\xi_\rho \xi \delta$ , while  $(\tau^\mp \rightarrow \rho^\mp \nu; \tau^\pm \rightarrow \rho^\pm \nu)$  events are used to extract  $\xi_\rho^2$ .



$$\frac{d\sigma(\ell^\mp \nu \nu, \rho^\pm \nu)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = A_0 + \rho A_1 + \eta A_2 + \xi_\rho \xi A_3 + \xi_\rho \xi \delta A_4 = \sum_{i=0}^4 A_i \Theta_i$$

$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp \nu \nu, \rho^\pm \nu)}{d\rho_\ell d\Omega_\ell d\rho_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp \nu \nu, \rho^\pm \nu)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} \left| \frac{\partial(E_\ell^*, \Omega_\ell^*, \Omega_\rho^*, \Omega_\tau)}{\partial(\rho_\ell, \Omega_\ell, \rho_\rho, \Omega_\rho, \Phi_\tau)} \right| d\Phi_\tau$$

$$L = \prod_{k=1}^N \mathcal{P}^{(k)}, \quad \mathcal{P}^{(k)} = \mathcal{F}(\vec{z}^{(k)}) / \mathcal{N}(\vec{\Theta}), \quad \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z}) d\vec{z}, \quad \vec{\Theta} = (1, \rho, \eta, \xi_\rho \xi_\ell, \xi_\rho \xi_\ell \delta_\ell)$$

$$\mathcal{P}^{\text{total}} = (1 - \sum_{i=1}^4 \lambda_i) \mathcal{P}_{\text{signal}}^{\ell-\rho} + \lambda_1 \mathcal{P}_{\text{bg}}^{\ell-3\pi} + \lambda_2 \mathcal{P}_{\text{bg}}^{\pi-\rho} + \lambda_3 \mathcal{P}_{\text{bg}}^{\rho-\rho} + \lambda_4 \mathcal{P}_{\text{bg}}^{\text{other}} (\text{MC})$$

**MP are extracted in the unbinned maximum likelihood fit of  $(\ell \nu \nu; \rho \nu)$  events in the 9D phase space  $\vec{z} = (\rho_\ell, \cos \theta_\ell, \phi_\ell, \rho_\rho, \cos \theta_\rho, \phi_\rho, m_{\pi\pi}^2, \cos \tilde{\theta}_\pi, \tilde{\phi}_\pi)$  in CMS.**



# Analysis of ( $\ell$ , all) events in 3D

$$\frac{d\sigma(\vec{\zeta})}{d\Omega_\tau} = \frac{\alpha^2}{32E_\tau^2} \beta_\tau (D_0 + \mathcal{P}_e F_i \zeta_i)$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*) \rightarrow \ell^\mp \nu \nu)}{dx^* d\Omega_\ell^*} = \kappa_\ell (A(x^*) \mp \xi_\ell \vec{n}_\ell^* \vec{\zeta}^* B(x^*)), \quad x^* = E_\ell^* / E_{\ell \max}^*$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*), \quad B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\sigma(\ell^\mp)}{dE_\ell^* d\Omega_\ell^* d\Omega_\tau} = \kappa_\ell \frac{\alpha^2 \beta_\tau}{32E_\tau^2} (D_0 A(E_\ell^*) \mp \mathcal{P}_e \xi_\ell F_i n_{\ell i}^* B(E_\ell^*))$$

$$\frac{d\sigma(\ell^\mp)}{dp_\ell d\Omega_\ell} = \int_{\Omega_\tau \text{-sector}} \frac{d\sigma(\ell^\mp)}{dE_\ell^* d\Omega_\ell^* d\Omega_\tau} \left| \frac{\partial(E_\ell^*, \Omega_\ell^*)}{\partial(p_\ell, \Omega_\ell)} \right| d\Omega_\tau$$

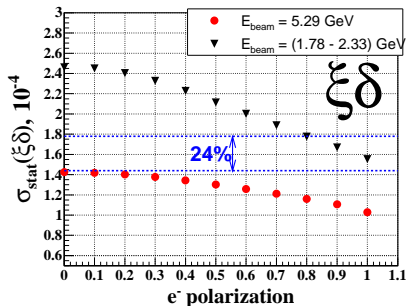
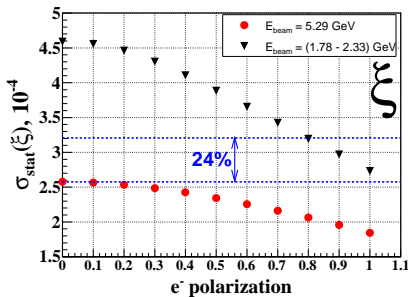
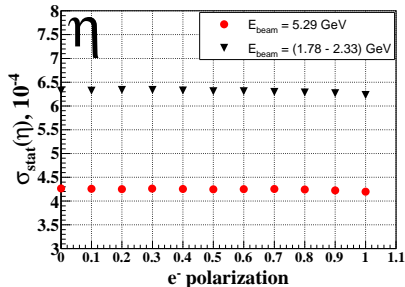
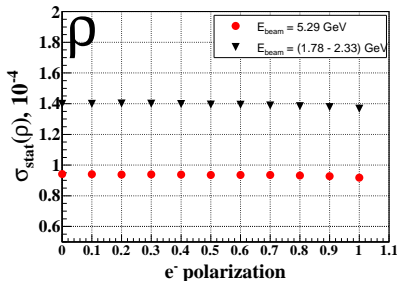
$\Omega_\tau$ -sector is determined by the kinematical constraint  $m_{\nu\nu} > 0$

- All Michel parameters ( $\rho, \eta, \mathcal{P}_e \xi, \mathcal{P}_e \xi \delta$ ) are measured in the unbinned maximum likelihood fit of ( $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau; \tau^+ \rightarrow \text{all}$ ) events in the 3D phase space.
- The reduced 3D phase space allows one to tabulate various EXP/MC corrections to the detection efficiency more precisely.
- **The crucial point in this method is to have high-efficiency 1-track trigger.**

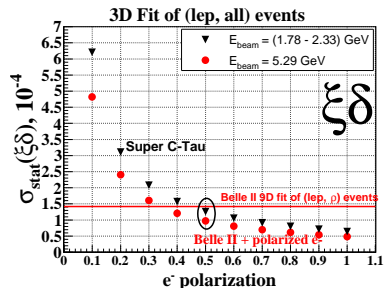
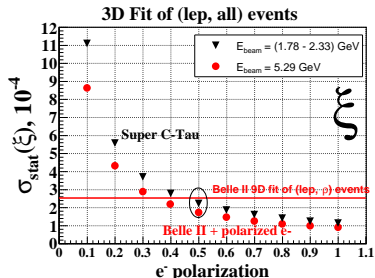
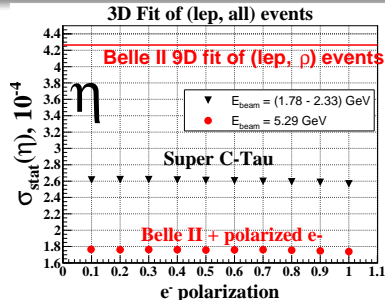
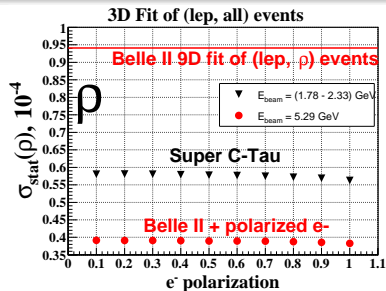
# Toy MC studies of the effect of polarized $e^-$ beam

- 66 10M  $(\mu, \rho)$  samples, at 6 center-of-mass (c.m.s.) energies (according to Table 1.1 in Super Charm-Tau factory CDR part I) :  $2E = 3.554$  GeV ( $\tau^+\tau^-$  production threshold),  $2E = 3.686$  GeV ( $\psi(2S)$ ),  $2E = 3.770$  GeV ( $\psi(3770)$ ),  $2E = 4.170$  GeV ( $\psi(4160)$ ),  $2E = 4.650$  GeV (maximum of the  $\sigma(e^+e^- \rightarrow \Lambda_c^+\Lambda_c^-)$ ),  $2E = 10.58$  GeV (Belle II), for 11 values of  $e^-$  beam polarization: 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, were generated for the calculation of the normalizations. 66 statistically independent 1M samples at the same energies and polarizations were generated for the fit.
- To evaluate MP sensitivities (rescaling the sensitivities obtained in the fits of 1M samples) we took the detection efficiency of  $(\mu, \rho)$  events to be 20% (to be compared with 12% efficiency obtained at Belle, where the  $\pi^0$  rec. efficiency is only 40%). The detection efficiency of  $(\mu, \text{all})$  events was taken to be 30%.
- To measure  $\rho$ ,  $\xi$  and  $\xi\delta$  MP, samples with  $\ell = e, \mu$  were taken into account, while  $\eta$  MP is measured in samples with  $\ell = \mu$  only.

# Fit of $(\ell, \rho)$ in 9D at Belle II/Super C-Tau



# Fit of ( $\ell$ , all) in 3D at Belle II/Super C-Tau



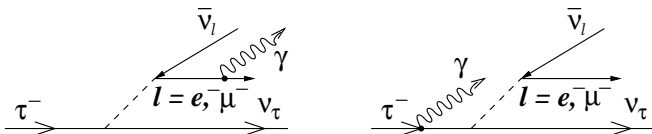
The sensitivity to the  $\xi$  and  $\xi\delta$  parameters at the Super Charm-Tau factory becomes better than that at Belle II (with unpolarized  $e^-$  beam) for the  $e^-$  beam polarizations larger than 0.5.

# Summary on Michel par. at Belle II/Super C-Tau

- Two methods were studied, (I) 9D fit of the  $(\ell, \rho)$  events, (II) 3D fit of the  $(\ell, \text{all})$  events.
- **In the method (I)**, the sensitivities to  $\rho$  and  $\eta$  parameters for the expected Belle II (with unpolarized  $e^-$  beam) and Super Charm-Tau factory statistics differ by only a factor of 1.5, Belle II has the best sensitivities. The sensitivities to the  $\xi$  and  $\xi\delta$  MP differ by only 25% (with unpolarized  $e^-$  beam for Belle II and  $e^-$  beam polarization of 0.8 for Super Charm-Tau factory), with Belle II best sensitivities.
- **In the method (II)**, the sensitivities to  $\rho$  and  $\eta$  parameters for the expected Belle II (with unpolarized  $e^-$  beam) and Super Charm-Tau factory statistics differ by only a factor of 1.5, Super Charm-Tau factory has the best sensitivities. The sensitivities to the  $\xi$  and  $\xi\delta$  MP become equal with unpolarized  $e^-$  beam for Belle II and  $e^-$  beam polarization of 0.5 for Super Charm-Tau factory. For the higher  $e^-$  beam polarization the sensitivities to  $\xi$  and  $\xi\delta$  MP improve as  $1/\mathcal{P}_e$ , and Super Charm-Tau factory wins Belle II. For the high  $e^-$  beam polarizations there is some notable room to decrease luminosity while keeping priority in the sensitivities to  $\xi$  and  $\xi\delta$  MP at Super Charm-Tau factory. The reduced 3D phase space in method (II) allows one to tabulate various EXP/MC corrections to the detection efficiency more precisely.
- It is seen that the expected MP statistical uncertainties are of the order of  $10^{-4}$ , to reach similar level systematic uncertainty, the NNLO corrections to the  $e^+e^- \rightarrow \tau^+\tau^-$  cross section are mandatory.

# Michel parameters in $\tau \rightarrow \ell\nu\nu\gamma$ at Belle (I)

C. Fronsdal and H. Uberall, Phys. Rev. **113** (1959) 654. ( $m_\ell = 0$ )  
 A. B. Arbuzov and T. V. Kopylova, JHEP **1609** (2016) 109. ( $m_\ell \neq 0$ )



Photon carries information about spin state of outgoing lepton, as a result two additional parameters,  $\bar{\eta}$  and  $\xi\kappa$ , can be extracted.

**These parameters were measured in  $\tau$  decays at Belle for the first time.**

$$\frac{d\Gamma(\tau^\mp \rightarrow \ell^\mp \nu_\ell \nu_\tau \gamma)}{dx dy d\Omega_\ell d\Omega_\gamma} = \Gamma_0 \frac{\alpha}{64\pi^3} \frac{\beta_\ell}{y} \left[ F(x, y, d) \pm P_\tau (\beta_\ell \cos \theta_\ell G(x, y, d) + \cos \theta_\gamma H(x, y, d)) \right],$$

$$\Gamma_0 = G_F^2 m_\tau^5 / 192\pi^3, \quad \beta_\ell = \sqrt{1 - m_\ell^2/E_\ell^2}, \quad x = 2E_\ell/m_\tau, \quad y = 2E_\gamma/m_\tau, \quad d = 1 - \beta_\ell \cos \theta_{\ell\gamma}$$

$$F = F_0 + \bar{\eta}F_1, \quad G = G_0 + \xi\kappa G_1, \quad H = H_0 + \xi\kappa H_1, \quad \frac{d\sigma(\ell^\mp \nu_\ell \nu_\tau, \rho^\pm \nu)}{dE_\ell^* d\Omega_\ell^* dE_\gamma^* d\Omega_\gamma^* d\Omega_\rho^* dm_\pi^2 d\Omega_\pi d\Omega_\tau} = A_0 + \bar{\eta}A_1 + \xi\kappa A_2$$

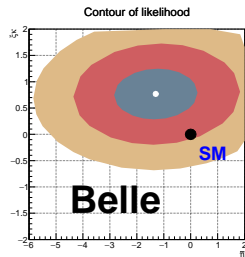
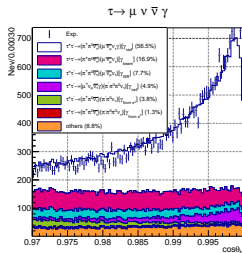
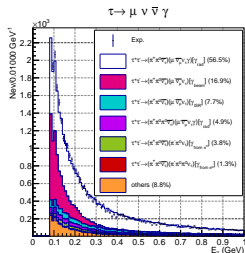
$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp \nu_\ell \nu_\tau, \rho^\pm \nu)}{d\rho_\ell d\Omega_\ell d\rho_\gamma d\Omega_\gamma d\rho_\rho d\Omega_\rho dm_\pi^2 d\Omega_\pi} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp \nu_\ell \nu_\tau, \rho^\pm \nu)}{dE_\ell^* d\Omega_\ell^* dE_\gamma^* d\Omega_\gamma^* d\Omega_\rho^* dm_\pi^2 d\Omega_\pi d\Omega_\tau} |\text{JACOBIAN}| d\Phi_\tau$$

$$L = \prod_{k=1}^N \mathcal{P}^{(k)}, \quad \mathcal{P}^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})} = \frac{\mathcal{F}_0 + \mathcal{F}_1 \bar{\eta} + \mathcal{F}_2 \xi\delta}{\mathcal{N}_0 + \mathcal{N}_1 \bar{\eta} + \mathcal{N}_2 \xi\delta}, \quad \mathcal{N}_k = \int \mathcal{F}_k(\vec{z}) d\vec{z}, \quad (k = 0, 1, 2)$$

$\bar{\eta}$  and  $\xi\delta$  are extracted in the unbinned maximum likelihood fit of  $(\ell\nu\nu\gamma; \rho\nu)$  events in the 12D phase space in CMS.

# Michel parameters in $\tau \rightarrow \ell \nu \nu \gamma$ at Belle (II)

$N_{\tau\tau} = 646 \times 10^6$ , selected: 71171 ( $\mu\nu\nu\gamma$ ;  $\rho\nu$ ) and 776834 ( $e\nu\nu\gamma$ ;  $\rho\nu$ ) events



Source	$\sigma_{\bar{\eta}}^e$	$\sigma_{\xi\kappa}^e$	$\sigma_{\bar{\eta}}^\mu$	$\sigma_{\xi\kappa}^\mu$
Normalization	4.3	0.94	0.15	0.04
Background PDF	2.5	0.24	0.67	0.22
Branching ratios	3.8	0.05	0.25	0.01
Cluster merge in ECL	2.2	0.46	0.02	0.06
Detector resolution	0.74	0.20	0.22	0.02
Data/MC eff. corr.	1.9	0.14	0.04	0.04
Total	7.0	1.1	0.76	0.24

## Belle result

$$\bar{\eta} = -1.3 \pm 1.5 \pm 0.8$$

$$\xi\kappa = 0.5 \pm 0.4 \pm 0.2$$

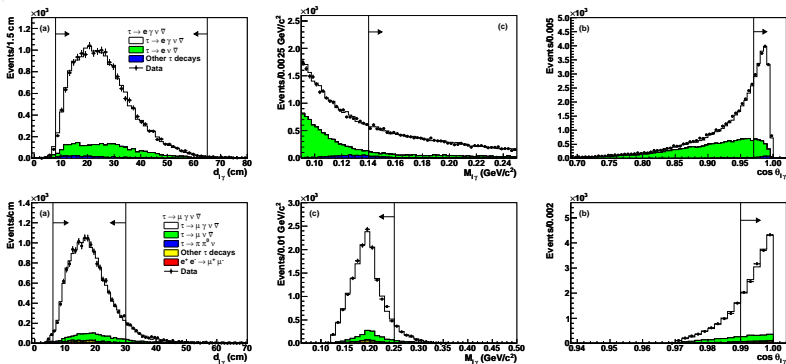
N. Shimizu *et al.* [Belle Collab.], PTEP 2018 (2018) no.2, 023C01.

# Measurement of $\mathcal{B}(\tau \rightarrow \ell\nu\nu\gamma)$ at BABAR (I)

$$\int L dt = 431 \text{ fb}^{-1}$$

## Selections:

- 2-track events with zero net charge and 1 photon with  $E_\gamma > 50 \text{ MeV}$ ;
- $0.9 < \text{thrust} < 0.995$ , signal hemisphere:  $\ell + \gamma$ , tag hemisphere: track+neutrals;
- reject  $\ell^\mp - \ell^\pm$  events,  $E_{\text{tot}} < 9 \text{ GeV}$ , distance between track and photon clusters  $d_{\ell\gamma} < 100 \text{ cm}$ .



$$\begin{array}{l}
 e\nu\nu\gamma \quad 0.22 \leq E_\gamma \leq 2.0 \text{ GeV}, M_{e\gamma} \geq 0.14 \text{ GeV}/c^2, \cos \theta_{e\gamma} \geq 0.97, 8 \leq d_{e\gamma} \leq 65 \text{ cm} \\
 \mu\nu\nu\gamma \quad 0.10 \leq E_\gamma \leq 2.5 \text{ GeV}, M_{\mu\gamma} \leq 0.25 \text{ GeV}/c^2, \cos \theta_{\mu\gamma} \geq 0.99, 6 \leq d_{\mu\gamma} \leq 30 \text{ cm}
 \end{array}$$

$$N_{\text{sel}}(\mu\nu\nu\gamma) = 15688 \pm 125 \quad N_{\text{sel}}(e\nu\nu\gamma) = 18149 \pm 135$$



# Measurement of $\mathcal{B}(\tau \rightarrow \ell\nu\nu\gamma)$ at BABAR (II)

$$\mathcal{B} = \frac{N_{\text{sel}}(1 - f_{\text{bg}})}{2\sigma_{\tau\tau}\mathcal{L}\epsilon}$$

	$\mu\nu\nu\gamma$	$e\nu\nu\gamma$
$\epsilon$ (%)	$0.480 \pm 0.010$	$0.105 \pm 0.003$
$f_{\text{bg}}$	$0.102 \pm 0.002$	$0.156 \pm 0.003$

	$\tau \rightarrow \mu\nu\nu\gamma$	$\tau \rightarrow e\nu\nu\gamma$
Photon efficiency	1.8	1.8
Particle identification	1.5	1.5
Background evaluation	0.9	0.7
BF	0.7	0.7
Luminosity and cross section	0.6	0.6
MC statistics	0.5	0.6
Selection criteria	0.5	0.5
Trigger selection	0.5	0.6
Track reconstruction	0.3	0.3
Total	2.8	2.8

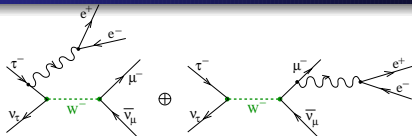
$$\mathcal{B}(\tau \rightarrow \mu\nu\nu\gamma)[E_\gamma^* > 10 \text{ MeV}] = (3.69 \pm 0.03 \pm 0.10) \times 10^{-3}$$

$$\mathcal{B}(\tau \rightarrow e\nu\nu\gamma)[E_\gamma^* > 10 \text{ MeV}] = (1.847 \pm 0.015 \pm 0.052) \times 10^{-2}$$

Measured branching ratios agree with the LO predictions ( $\mathcal{B}(\mu\nu\nu\gamma) = 3.663 \times 10^{-3}$ ,  $\mathcal{B}(e\nu\nu\gamma) = 1.834 \times 10^{-2}$ ), however the LO+NLO prediction for the  $\tau \rightarrow e\nu\nu\gamma$  ( $\mathcal{B}(e\nu\nu\gamma) = 1.645 \times 10^{-2}$ ) differs from the experimental result by  $3.5\sigma$ . It is important to embed NLO corrections to the MC generator (TAUOLA) of the radiative leptonic decay. Also background from the doubly-radiative leptonic decays should be properly studied and subtracted.

**M. Fael, L. Mercolli and M. Passera, JHEP 1507 (2015) 153.**

# Tau decays into 5 leptons



D. A. Dicus and R. Vega, Phys. Lett. B **338** (1994) 341.

M. S. Alam *et al.* [CLEO Collaboration], Phys. Rev. Lett. **76** (1996) 2637.

## A. Flores-Tlalpa, G. Lopez Castro and P. Roig, JHEP **1604** (2016) 185.

Mode	$\mathcal{B}_{\text{theory}}$	$\mathcal{B}_{\text{CLEO}}$
$e^{\mp}e^{+}e^{-}2\nu$	$(4.21 \pm 0.01) \times 10^{-5}$	$(2.7^{+1.6}_{-1.2}) \times 10^{-5}$
$\mu^{\mp}e^{+}e^{-}2\nu$	$(1.984 \pm 0.004) \times 10^{-5}$	$< 3.2 \times 10^{-5}$ (90% CL)
$e^{\mp}\mu^{+}\mu^{-}2\nu$	$(1.247 \pm 0.001) \times 10^{-7}$	
$\mu^{\mp}\mu^{+}\mu^{-}2\nu$	$(1.183 \pm 0.001) \times 10^{-7}$	

A. Kersch, N. Kraus and R. Engfer [SINDRUM], Nucl. Phys. A **485** (1988) 606.

$$\frac{d\Gamma(\tau)}{dPS} = Q_{LL}d_1 + Q_{LR}d_2 + Q_{RL}d_3 + Q_{RR}d_4 + B_{RL}d_5 + B_{LR}d_6$$

Up to now  $Q_{LL}$ ,  $Q_{LR}$ ,  $Q_{RL}$ ,  $Q_{RR}$ ,  $B_{RL}$ ,  $B_{LR}$  were measured only in muon decays ( $\mu^{-} \rightarrow e^{-}e^{-}e^{+}\nu_{\mu}\bar{\nu}_e$ ) with the accuracy of about 10 ÷ 20%.

Michel parameters can be measured in two ways: in the study of the dynamics and from the measurement of the branching fraction:

$$\mathcal{B}_{\text{exp}}/\mathcal{B}_{\text{SM}} = Q_{LL} + \alpha_{LR}Q_{LR} + \alpha_{RL}Q_{RL} + \alpha_{RR}Q_{RR} + \beta_{RL}B_{RL} + \beta_{LR}B_{LR}$$

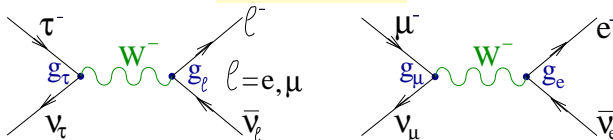
**Analysis of 5-lepton  $\tau$  decays is being finalized at Belle.**

# Tau decays with leptons at Belle II/Super C-Tau

- Good potential to study precisely doubly radiative decay  $\tau^- \rightarrow \ell^- \nu \nu \gamma \gamma$  at Belle II/Super Charm-Tau factory.
- Properly understand/investigate radiative corrections in  $\tau$  decays with leptons, update TAUOLA generator.
- Good potential to discover  $\tau^- \rightarrow e^- \mu^+ \mu^- 2\nu$  and  $\tau^- \rightarrow \mu^- \mu^+ \mu^- 2\nu$  at Belle II/Super Charm-Tau factory.
- With  $\tau \rightarrow \ell \nu \nu \gamma$  and  $\tau \rightarrow \ell \ell'^+ \ell'^- \nu \nu$  measure full set of Michel parameters ( $\xi', \xi'', \eta'', \alpha'/A, \beta'/A$  in addition to  $\rho, \eta, \xi, \delta$ ) at Belle II/Super Charm-Tau factory.
- Search for T symmetry violation in  $\tau \rightarrow \ell \ell'^+ \ell'^- \nu \nu$  (through T-odd correlation terms).

# Lepton universality in the SM

$$g_e = g_\mu = g_\tau$$



$$\Gamma(L^- \rightarrow \ell^- \bar{\nu}_\ell \nu_L(\gamma)) = \frac{\mathcal{B}(L^- \rightarrow \ell^- \bar{\nu}_\ell \nu_L(\gamma))}{\tau_L} = \frac{g_L^2 g_\ell^2}{32M_W^4} \frac{m_L^5}{192\pi^3} F_{\text{corr}}(m_L, m_\ell)$$

$$F_{\text{corr}}(m_L, m_\ell) = f(x) \left( 1 + \frac{3}{5} \frac{m_L^2}{M_W^2} \right) \left( 1 + \frac{\alpha(m_L)}{2\pi} \left( \frac{25}{4} - \pi^2 \right) \right)$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x, \quad x = m_\ell/m_L$$

$$\mathcal{B}(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu(\gamma)) = 1$$

$$\frac{g_\tau}{g_e} = \sqrt{\frac{\mathcal{B}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau(\gamma))}{\mathcal{B}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau(\gamma))} \frac{\tau_\mu}{\tau_\tau} \frac{m_\mu^5}{m_\tau^5} \frac{F_{\text{corr}}(m_\mu, m_e)}{F_{\text{corr}}(m_\tau, m_\mu)}}, \quad \frac{g_\tau}{g_e} = 1.0029 \pm 0.0015 \text{ (HFAG2017)}$$

$$\frac{g_\tau}{g_\mu} = \sqrt{\frac{\mathcal{B}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau(\gamma))}{\mathcal{B}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau(\gamma))} \frac{\tau_\mu}{\tau_\tau} \frac{m_\mu^5}{m_\tau^5} \frac{F_{\text{corr}}(m_\mu, m_e)}{F_{\text{corr}}(m_\tau, m_e)}}, \quad \frac{g_\tau}{g_\mu} = 1.0010 \pm 0.0015 \text{ (HFAG2017)}$$

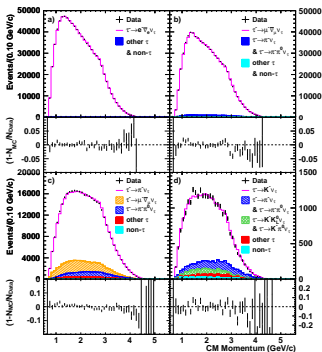
$$\frac{g_\mu}{g_e} = \sqrt{\frac{\mathcal{B}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau(\gamma))}{\mathcal{B}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau(\gamma))} \frac{F_{\text{corr}}(m_\tau, m_e)}{F_{\text{corr}}(m_\tau, m_\mu)}}, \quad \frac{g_\mu}{g_e} = 1.0019 \pm 0.0014 \text{ (HFAG2017)}$$

# Test of lepton universality at *BABAR*

$$\int L dt = 467 \text{ fb}^{-1}$$

## Selections:

- 4-track events with zero net charge;
- $0.1\sqrt{s} < E_{\text{miss}}^{\text{CMS}} < 0.7\sqrt{s}$ ,  $|\cos(\theta_{\text{miss}}^{\text{CMS}})| < 0.7$
- thrust  $> 0.9$ , signal hemisphere:  $\ell/h$  ( $\ell = e, \mu$ ;  $h = \pi, K$ ), tag hemisphere:  $\tau \rightarrow \pi\pi\pi\nu$ ;
- signal hemisphere:  $E_{\text{extra}\gamma}^{\text{LAB}} < \{1.0, 0.5, 0.2, 0.2\} \text{ GeV}$  for  $\{e, \mu, \pi, K\}$ , respectively



	$\mu$	$\pi$	$K$
$N^D$	731102	369091	25123
Purity	97.3%	78.7%	76.6%
Total Efficiency	0.485%	0.324%	0.330%
Particle ID Efficiency	74.5%	74.6%	84.6%
Systematic uncertainties:			
Particle ID	0.32	0.51	0.94
Detector response	0.08	0.64	0.54
Backgrounds	0.08	0.44	0.85
Trigger	0.10	0.10	0.10
$\pi^- \pi^- \pi^+$ modelling	0.01	0.07	0.27
Radiation	0.04	0.10	0.04
$\mathcal{B}(\tau^- \rightarrow \pi^- \pi^- \pi^+ \nu_\tau)$	0.05	0.15	0.40
$\mathcal{L}\sigma_{\tau\tau}$	0.02	0.39	0.20
<b>Total [%]</b>	<b>0.36</b>	<b>1.0</b>	<b>1.5</b>

$$\tau \rightarrow e\nu\nu: N_{\text{sel}} = 884426, \varepsilon = (0.589 \pm 0.010)\%, \text{ purity is } (99.69 \pm 0.06)\%$$

# Test of lepton universality

$$R_\mu = \frac{\mathcal{B}(\tau \rightarrow \mu\nu\nu)}{\mathcal{B}(\tau \rightarrow e\nu\nu)} = 0.9796 \pm 0.0016 \pm 0.0036$$

$$R_\pi = \frac{\mathcal{B}(\tau \rightarrow \pi\nu)}{\mathcal{B}(\tau \rightarrow e\nu\nu)} = 0.5945 \pm 0.0014 \pm 0.0061$$

$$R_K = \frac{\mathcal{B}(\tau \rightarrow K\nu)}{\mathcal{B}(\tau \rightarrow e\nu\nu)} = 0.03882 \pm 0.00032 \pm 0.00057$$

$$\left(\frac{g_\mu}{g_e}\right)_\tau = \sqrt{R_\mu \frac{F_{\text{corr}}(m_\tau, m_e)}{F_{\text{corr}}(m_\tau, m_\mu)}} = 1.0036 \pm 0.0020$$

$$(g_\tau/g_\mu)_h^2 = \frac{\mathcal{B}(\tau \rightarrow h\nu_\tau)}{\mathcal{B}(h \rightarrow \mu\nu_\mu)} \frac{2m_h m_\mu^2 \tau_h}{(1 + \delta_h) m_\tau^3 \tau_\tau} \left( \frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2} \right)^2$$

$$\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9856 \pm 0.0057, \quad \left(\frac{g_\tau}{g_\mu}\right)_K = 0.9827 \pm 0.0086$$

$$\left(\frac{g_\tau}{g_\mu}\right)_h = 0.9850 \pm 0.0054 \quad (2.8\sigma \text{ away from SM})$$

$$\left(\frac{g_\tau}{g_\mu}\right)_{\tau+\pi+K} = 1.0000 \pm 0.0014 \quad (\text{HFAG2017})$$

At the Super Charm-Tau factory at the  $\tau^+\tau^-$  production threshold ( $\tau$  is at rest) the pion from  $\tau \rightarrow \pi\nu$  and kaon from  $\tau \rightarrow K\nu$  can be easily separated via their momentum difference (of about 63 MeV). The clean sample of  $\tau \rightarrow K\nu$  is also used to measure precisely  $f_K V_{us}$ .

# Hadronic $\tau$ decays

Cabibbo-allowed decays ( $\mathcal{B} \sim \cos^2 \theta_c$ )

$$\mathcal{B}(S = 0) = (61.85 \pm 0.11)\% \text{ (PDG)}$$

Cabibbo-suppressed decays ( $\mathcal{B} \sim \sin^2 \theta_c$ )

$$\mathcal{B}(S = -1) = (2.88 \pm 0.05)\% \text{ (PDG)}$$

$$iM_{fi} \left\{ \begin{array}{l} S = 0 \\ S = -1 \end{array} \right\} = \frac{G_F}{\sqrt{2}} \bar{u}_{\nu\tau} \gamma^\mu (1 - \gamma^5) u_\tau \cdot \left\{ \begin{array}{l} \cos \theta_c \cdot \langle \text{hadrons}(q^\mu) | \hat{J}_{\mu}^{S=0}(q^2) | 0 \rangle \\ \sin \theta_c \cdot \langle \text{hadrons}(q^\mu) | \hat{J}_{\mu}^{S=-1}(q^2) | 0 \rangle \end{array} \right\}, \quad q^2 \leq M_\tau^2$$

## The main tasks

- Measurement of branching fractions with highest possible accuracy
- Measurement of low-energy hadronic spectral functions
  - Determination of the decay mechanism (what are intermediate mesons and their contributions)
  - Precise measurement of masses and widths of the intermediate mesons
- Search for CP violation
- Comparison with hadronic formfactors from  $e^+e^-$  experiments to check CVC theorem
- Measurement of  $\Gamma_{\text{inclusive}}(S = 0)$  to determine  $\alpha_S$
- Measurement of  $\Gamma_{\text{inclusive}}(S = -1)$  to determine s-quark mass and  $V_{us}$ :

$$|V_{us}| = \sqrt{\frac{R_{\text{strange}}}{\frac{R_{\text{non-strange}}}{|V_{ud}|^2} - \delta R_{\text{theory}}}}$$

- $R_{\text{strange}} = \mathcal{B}_{\text{strange}} / \mathcal{B}_e$
- $R_{\text{non-strange}} = \mathcal{B}_{\text{non-strange}} / \mathcal{B}_e$
- $\delta R_{\text{theory}}$  - SU(3)-breaking contribution

# CPV in hadronic $\tau$ decays at $B$ factories

- CPV has not been observed in lepton decays
- It is strongly suppressed in the SM ( $A_{SM}^{CP} \lesssim 10^{-12}$ ) and observation of large CPV in lepton sector would be clean sign of New Physics
- $\tau$  lepton provides unique possibility to search for CPV effects, as it is the only lepton decaying to hadrons, so that the associated strong phases allows us to visualize CPV in hadronic  $\tau$  decays.

## I. CPV in $\tau^- \rightarrow \pi^- K_S^0(\geq 0\pi^0)\nu_\tau$ at BaBar (Phys. Rev. D 85, 031102 (2012))

Data sample of  $\int Ldt = 476 \text{ fb}^{-1}$  was analyzed

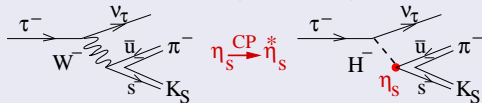
$$A_{CP} = \frac{\Gamma(\tau^+ \rightarrow \pi^+ K_S^0(\geq 0\pi^0)\bar{\nu}_\tau) - \Gamma(\tau^- \rightarrow \pi^- K_S^0(\geq 0\pi^0)\nu_\tau)}{\Gamma(\tau^+ \rightarrow \pi^+ K_S^0(\geq 0\pi^0)\bar{\nu}_\tau) + \Gamma(\tau^- \rightarrow \pi^- K_S^0(\geq 0\pi^0)\nu_\tau)} = (-0.36 \pm 0.23 \pm 0.11)\%$$

**2.8 $\sigma$  deviation** from the SM expectation:  $A_{CP}^{K^0} = (+0.36 \pm 0.01)\%$

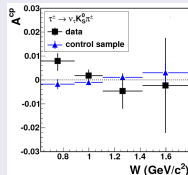
## II. CPV in $\tau^- \rightarrow K_S^0 \pi^- \nu_\tau$ at Belle (Phys. Rev. Lett. 107, 131801 (2011)) $\int Ldt=699 \text{ fb}^{-1}$

Angular distributions were analyzed,  $A_{CP}(W = M_{K_S \pi})$  was measured ( $d\omega = d \cos \beta d \cos \theta$ ):

$$A_{CP}(W) = \frac{\int \cos \beta \cos \psi \left( \frac{d\Gamma_{\tau^-}^-}{d\omega} - \frac{d\Gamma_{\tau^+}^+}{d\omega} \right) d\omega}{\frac{1}{2} \int \left( \frac{d\Gamma_{\tau^-}^-}{d\omega} + \frac{d\Gamma_{\tau^+}^+}{d\omega} \right) d\omega} \simeq \langle \cos \beta \cos \psi \rangle_{\tau^-} - \langle \cos \beta \cos \psi \rangle_{\tau^+}$$



$$|Im(\eta_S)| < 0.026$$





# CPV in $\tau^\pm \rightarrow K_S \pi^\pm \nu_\tau$ with polarized $\tau$ lepton (I)

S. Y. CHOI *et al.*, PLB **437**, 191 (1998).

$$M_\sigma = \frac{G_F}{\sqrt{2}} \sin \theta_c \left[ (1 + \chi) \bar{u}_\nu(k, -) \gamma^\mu (1 - \gamma^5) u(p, \sigma) J_\mu + \eta \bar{u}_\nu(k, -) (1 + \gamma^5) u(p, \sigma) J_S \right]$$

$$J_\mu = \langle (K\pi)^- | \bar{s} \gamma_\mu u | 0 \rangle = F_V(q^2) \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) (q_1 - q_2)^\nu + F_S(q^2) q^\mu$$

$$J_S = \langle (K\pi)^- | \bar{s} u | 0 \rangle = \frac{q^2}{m_s - m_u} F_S(q^2)$$

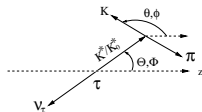
- $\sigma = \pm 1$  – helicity of  $\tau^-$ ;
- $\chi, \eta$  parametrize BSM contribution,
- $\tau^-: M_\pm(\chi, \xi), \tau^+: \bar{M}_\pm(\chi^*, \xi^*)$

$$\xi = \frac{m_{K_S^*}^2}{(m_s - m_u) m_\tau} \left( \frac{\eta}{1 + \chi} \right)$$

If  $\chi$  and  $\eta$  are real:  $M_\pm(\Theta; q^2; \theta, \phi) = \mp \bar{M}_\mp(\Theta; q^2; \theta, -\phi)$   
 $\tau$  is polarized in the  $(\theta_p, \phi_p)$  direction:

$$|\theta_p, \phi_p\rangle = \cos \frac{\theta_p}{2} |+\rangle + \sin \frac{\theta_p}{2} |-\rangle$$

$$\langle \Theta, \Phi | \theta_p, \phi_p \rangle = \cos \frac{\theta_p}{2} M_+ + \sin \frac{\theta_p}{2} M_-$$



$$d\Gamma = \frac{1}{2} d(\Gamma_{+++} + \Gamma_{---}) + P_\tau \left( \frac{1}{2} \cos \rho d(\Gamma_{+++} - \Gamma_{---}) + \sin \rho \cos(\phi_p - \Phi) d\text{Re}\Gamma_{+-} - \sin \rho \sin(\phi_p - \Phi) d\text{Im}\Gamma_{+-} \right)$$

$$d\Gamma_{\sigma\sigma'} = \frac{1}{(2\pi)^3} \frac{1}{32m_\tau} \left( 1 - \frac{q^2}{m_\tau^2} \right) M_\sigma M_{\sigma'}^* P_K d\Phi_3 d\Phi, \quad d\Phi_3 = d\sqrt{q^2} d\cos\theta d\phi d\cos\theta'$$

# CPV in $\tau^\pm \rightarrow K_S \pi^\pm \nu_\tau$ with polarized $\tau$ lepton (II)

After integration on  $\Phi$  (and  $P_\tau = 1$ ):

$$\frac{d\Gamma_1}{d\Phi_3} = \frac{d(\Gamma_{++} + \Gamma_{--})}{d\Phi_3}, \quad \frac{d\Gamma_2}{d\Phi_3} = \frac{d(\Gamma_{++} - \Gamma_{--})}{d\Phi_3}, \quad \frac{d\Gamma_3}{d\Phi_3} = 2\text{Re}\left(\frac{d\Gamma_{+-}}{d\Phi_3}\right), \quad \frac{d\Gamma_4}{d\Phi_3} = 2\text{Im}\left(\frac{d\Gamma_{+-}}{d\Phi_3}\right)$$

$$\frac{d\Gamma_i}{d\Phi_3} = \frac{1}{2}(\Sigma_i + \Delta_i), \quad \Sigma_i/\Delta_i - \text{CP even/odd part}, \quad i = 1 \div 4$$

$$\Sigma_1 = \frac{d(\Gamma_1 + \bar{\Gamma}_1)}{d\Phi_3}, \quad \Sigma_2 = \frac{d(\Gamma_2 - \bar{\Gamma}_2)}{d\Phi_3}, \quad \Sigma_3 = \frac{d(\Gamma_3 - \bar{\Gamma}_3)}{d\Phi_3}, \quad \Sigma_4 = \frac{d(\Gamma_4 + \bar{\Gamma}_4)}{d\Phi_3},$$

$$\Delta_1 = \frac{d(\Gamma_1 - \bar{\Gamma}_1)}{d\Phi_3}, \quad \Delta_2 = \frac{d(\Gamma_2 + \bar{\Gamma}_2)}{d\Phi_3}, \quad \Delta_3 = \frac{d(\Gamma_3 + \bar{\Gamma}_3)}{d\Phi_3}, \quad \Delta_4 = \frac{d(\Gamma_4 - \bar{\Gamma}_4)}{d\Phi_3}$$

CP even:  $\Sigma_1 \gg \Sigma_2, \Sigma_3, \Sigma_4$ ,

CP odd:  $\Delta_1 - P_\tau$ -independent part,  $\Delta_{2,3,4} - P_\tau$ -dependent part.

Four optimal variables to search for CPV are:  $w_i^{\text{opt}} = \Delta_i/\Sigma_1$ .

$P_\tau$ -independent  $w_1^{\text{opt}}$  was used at Belle, while 3  $P_\tau$ -dependent  $w_{2\div 4}^{\text{opt}}$  can be additionally measured at the Super Charm-Tau factory:

$$w_1^{\text{opt}} = A_1(q^2; \Theta, \theta, \phi) \text{Im}(\xi) \text{Im}(F_V F_S^*),$$

$$w_{2\div 4}^{\text{opt}} = A_{2\div 4}(q^2; \Theta, \theta, \phi) \text{Im}(\xi) \text{Im}(F_V F_S^*) + B_{2\div 4}(q^2; \Theta, \theta, \phi) \text{Im}(\xi) \text{Re}(F_V F_S^*)$$

**At the Super Charm-Tau factory CPV search doesn't depend on  $F_V F_S^*$  phase.**

# CPV in $\tau^\pm \rightarrow K_S \pi^\pm \nu_\tau$ with polarized $\tau$ lepton (III)

At the center-of-mass energies close to the  $\tau^+ \tau^-$  production threshold the  $\tau$  lepton is produced with the polarization

$$|\vec{P}_\tau| = P_e \frac{2E_{\text{beam}} \sqrt{p_{\text{beam}}^2 \cos^2 \theta + M_\tau^2}}{E_{\text{beam}}^2 + M_\tau^2 + p_{\text{beam}}^2 \cos^2 \theta} \approx P_e \text{ along electron beam polarization}$$
$$((P_\tau)_Z = P_e \frac{E_{\text{beam}} \cos^2 \theta + M_\tau \sin^2 \theta}{\sqrt{p_{\text{beam}}^2 \cos^2 \theta + M_\tau^2}} \approx P_e).$$

In case of New Physics contribution, the amplitudes for the decays  $\tau^- \rightarrow (K\pi)^- \nu_\tau$  and  $\tau^+ \rightarrow (K\pi)^+ \bar{\nu}_\tau$  are:

$$\mathcal{A} = A_1 + A_2 e^{i\phi} e^{i\delta}, \quad \bar{\mathcal{A}} = A_1 + A_2 e^{-i\phi} e^{i\delta}$$

where  $\phi$  and  $\delta$  are relative weak (CP-odd) and strong (CP-even) phases. CPV is studied comparing  $|\mathcal{A}|^2$  and  $|\bar{\mathcal{A}}|^2$ , there are three possibilities to construct CPV asymmetry:

- decay rate asymmetry  $\sim \sin \delta \sin \phi$
- weighted rate asymmetry  $\sim \sin \delta \sin \phi$
- asymmetry based on  $\vec{P}_\tau (\vec{p}_K \times \vec{p}_\pi)$  triple product  $\sim \cos \delta \sin \phi$

**At the Super Charm-Tau factory, with nonzero single  $\tau$  polarization, nonzero strong-phase difference,  $\delta$ , is not needed to measure CPV.**

# Search for CPV in $\tau^\mp \rightarrow (K\pi)^\mp \nu$ in unbinned fit

Analysis of the  $(\tau^\mp \rightarrow (K\pi)^\mp \nu ; \tau^\pm \rightarrow \rho^\pm \nu)$  events, search for CPV in  $\tau^- \rightarrow (K\pi)^- \nu_\tau$ .

**The analysis of the decay products of both taus allows one to constrain direction of  $\tau^- - \tau^+$  axis. Such a constraint is efficient to suppress background from  $\tau^- \rightarrow (K\pi)^- K_L^0 \nu_\tau$ .**

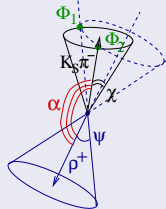
$$\frac{d\sigma(\vec{\zeta}^*, \vec{\zeta}'^*)}{d\Omega_\tau} = \frac{\alpha^2}{64E_\tau^2} \beta_\tau (D_0 + D_{ij} \zeta_i^* \zeta_j'^*), \quad \frac{d\Gamma(\tau^\pm(\vec{\zeta}'^*) \rightarrow \rho^\pm \nu)}{dm_\pi^2 d\Omega_\rho^* d\tilde{\Omega}_\pi} = A' \mp \vec{B}' \vec{\zeta}'^*$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*) \rightarrow (K\pi)^\mp \nu)}{dm_{K\pi}^2 d\Omega_{K\pi}^* d\tilde{\Omega}_\pi} = (A_0 + \eta_{CP} A_1) + (\vec{B}_0 + \eta_{CP} \vec{B}_1) \vec{\zeta}^*$$

$$(A_0 + \eta_{CP}^* A_1) - (\vec{B}_0 + \eta_{CP}^* \vec{B}_1) \vec{\zeta}^*$$

$$\frac{d\sigma((K\pi)^\mp, \rho^\pm)}{dm_{K\pi}^2 d\Omega_{K\pi}^* d\tilde{\Omega}_\pi dm_{\pi\pi}^2 d\Omega_{\pi\pi}^* d\tilde{\Omega}_\pi d\Omega_\tau} = \frac{\alpha^2 \beta_\tau}{64E_\tau^2} \begin{pmatrix} \mathcal{F} + \eta_{CP} \mathcal{G} \\ \mathcal{F} + \eta_{CP}^* \mathcal{G} \end{pmatrix}$$

$$\mathcal{F} = D_0 A_0 A' - D_{ij} B_{0i} B'_j, \quad \mathcal{G} = D_0 A_1 A' - D_{ij} B_{1i} B'_j$$



$$\frac{d\sigma((K\pi)^\mp, \rho^\pm)}{dp_{K\pi} d\Omega_{K\pi} dm_{K\pi}^2 d\tilde{\Omega}_\pi dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \sum_{\Phi_1, \Phi_2} \frac{d\sigma((K\pi)^\mp, \rho^\pm)}{dm_{K\pi}^2 d\Omega_{K\pi}^* d\tilde{\Omega}_\pi dm_{\pi\pi}^2 d\Omega_{\pi\pi}^* d\tilde{\Omega}_\pi d\Omega_\tau} \left| \frac{\partial(\Omega_{K\pi}^*, \Omega_\rho^*, \Omega_\tau)}{\partial(p_{K\pi}, \Omega_{K\pi}, p_\rho, \Omega_\rho)} \right|$$

$\eta_{CP}$  is extracted in the simultaneous unbinned maximum likelihood fit of the  $((K\pi)^-, \rho^+)$  and  $((K\pi)^+, \rho^-)$  events in the 12D phase space.

# Summary

- The world largest statistics of  $\tau$  leptons collected by Belle and *BABAR* opens new era in the precision tests of the Standard Model, search for the effects of New Physics and precision studies of low energy QCD.
- Nonzero average polarization of single  $\tau$  at the Super Charm-Tau factory provides the possibility to measure all Michel parameters without tagging the opposite tau. In this case, for the electron beam polarization  $\mathcal{P}_e > 0.5$ , the statistical uncertainties of Michel parameters are smaller than the values, which can be reached at Belle II (with unpolarized beams). Better systematic uncertainty can be reached due to the smaller impact of the ISR as well as smaller number of the phase space dimensions.
- Study of  $\tau \rightarrow \ell\nu\nu\gamma$  and  $\tau \rightarrow \ell\ell'^+\ell'^-\nu\nu$  decays allows one to measure full set of Michel parameters ( $\xi', \xi'', \eta'', \alpha'/A, \beta'/A$  in addition to  $\rho, \eta, \xi, \delta$ ) at Belle II/Super Charm-Tau factory. Precision study of radiative and doubly radiative leptonic  $\tau$  decays is important to understand higher order corrections in  $\tau$  decays with leptons for the better test of lepton universality. Good potential to discover rare decays with the  $\mathcal{B} \lesssim 10^{-7}$ .
- The Super Charm-Tau factory with polarized electron beam, being a source of taus with nonzero polarization, allows one to search for CPV regardless the value of the hadronic phase in hadronic  $\tau$  decay.
- The unbinned analysis of the reaction  
 $e^+e^- \rightarrow (\tau^- \rightarrow \text{hadrons}^- \nu_\tau; \tau^+ \rightarrow \ell^+ \nu_\ell \bar{\nu}_\tau)$  or  
 $e^+e^- \rightarrow (\tau^- \rightarrow \text{hadrons}^- \nu_\tau; \tau^+ \rightarrow \rho^+ \bar{\nu}_\tau)$  in the full multidimensional phase space is acute for the improved searches for the CPV in hadronic  $\tau$  decays.