Search for CPV in $\tau \to K\pi\nu$ at e^+e^- colliders, effect of the polarized electron beam

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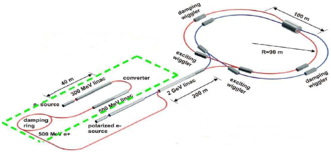
Outline:

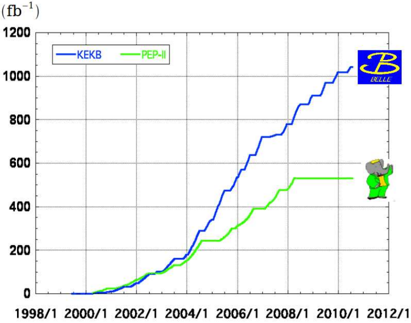


Introduction

- The world largest statistics of τ leptons collected by e^+e^- B factories (Belle and *BABAR*) opens new era in the precision tests of the Standard Model (SM).
- In the SM τ decays due to the charged weak interaction described by the exchange of $W^{\pm}$ with a pure vector coupling to only left-handed fermions. There are two main classes of τ decays:
  - Decays with leptons, like: $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$, $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma$, $\tau^- \to \ell^- \ell'^+ \ell'^- \bar{\nu}_\ell \nu_\tau$; $\ell, \ell' = e, \mu$. They provide very clean laboratory to probe electroweak couplings, which is complementary/competitive to precision studies with muon (in experiments with muon beam). Plenty of New Physics models can be tested/constrained in the precision studies of the dynamics of decays with leptons.
  - **Hadronic decays of τ offer unique tools for the precision study of low energy QCD and searches for CPV.**

Introduction: e^+e^- B factories

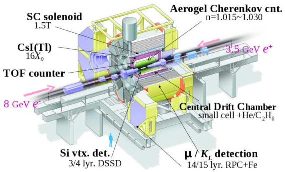
Integrated luminosity of B factories



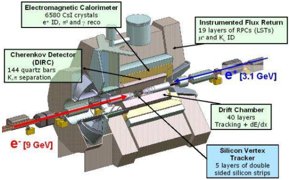
> 1 ab^{-1}
On resonance:
$\Upsilon(5S)$: 121 fb^{-1}
$\Upsilon(4S)$: 711 fb^{-1}
$\Upsilon(3S)$: 3 fb^{-1}
$\Upsilon(2S)$: 25 fb^{-1}
$\Upsilon(1S)$: 6 fb^{-1}
Off reson./scan:
$\sim$ 100 fb^{-1}

$\sim$ 550 fb^{-1}
On resonance:
$\Upsilon(4S)$: 433 fb^{-1}
$\Upsilon(3S)$: 30 fb^{-1}
$\Upsilon(2S)$: 14 fb^{-1}
Off resonance:
$\sim$ 54 fb^{-1}

Belle Detector



BABAR detector



$$\sigma(b\bar{b}) = 1.05 \text{ nb } N_{b\bar{b}} = 1.2 \times 10^9$$
$$\sigma(c\bar{c}) = 1.30 \text{ nb } N_{c\bar{c}} = 2.0 \times 10^9$$
$$\sigma(\tau\tau) = 0.92 \text{ nb } N_{\tau\tau} = 1.4 \times 10^9$$

B-factories are also charm- and τ-factories !

Introduction: hadronic τ decays

Cabibbo-allowed decays ($\mathcal{B} \sim \cos^2\theta_c$)

$\mathcal{B}(S=0) = (61.85 \pm 0.11)\%$ (PDG)

Cabibbo-suppressed decays ($\mathcal{B} \sim \sin^2\theta_c$)

$\mathcal{B}(S=-1) = (2.88 \pm 0.05)\%$ (PDG)

$$iM_{\rm fi}\left\{\begin{array}{c} S=0 \\ S=-1 \end{array}\right\} = \frac{G_F}{\sqrt{2}}\overline{u}_{\nu_\tau}\gamma^\mu(1-\gamma^5)u_\tau \cdot \left\{\begin{array}{l} \cos\theta_c \cdot \langle \text{hadrons}(q^\mu)|\hat{J}_\mu^{S=0}(q^2)|0\rangle \\ \sin\theta_c \cdot \langle \text{hadrons}(q^\mu)|\hat{J}_\mu^{S=-1}(q^2)|0\rangle \end{array}\right\},\ q^2 \le M_\tau^2$$

The main tasks

- Measurement of branching fractions with highest possible accuracy
- Measurement of low-energy hadronic spectral functions
  - Determination of the decay mechanism (what are intermediate mesons and their contributions)
  - Precise measurement of masses and widths of the intermediate mesons
- **Search for CP violation**
- Comparison with hadronic formfactors from e^+e^- experiments to check CVC theorem
- Measurement of $\Gamma_{\rm inclusive}(S=0)$ to determine α_s
- Measurement of $\Gamma_{\rm inclusive}(S=-1)$ to determine s-quark mass and V_{us}:

$$|V_{us}| = \sqrt{\frac{R_{\rm strange}}{\frac{R_{\rm non-strange}}{|V_{ud}|^2} - \delta R_{\rm theory}}}$$

- $R_{\rm strange} = \mathcal{B}_{\rm strange}/\mathcal{B}_e$
- $R_{\rm non-strange} = \mathcal{B}_{\rm non-strange}/\mathcal{B}_e$
- $\delta R_{\rm theory}$ - SU(3)-breaking contribution

Study of the $\tau^- \to K_S^0 \pi^- \nu_\tau$ decay

- **Measurement of $\mathcal{B}(\tau \to K_S^0 \pi \nu_\tau)$ branching ratio:** $\tau \to \bar{K}^0 \pi \nu_\tau$ has the largest $\mathcal{B}$ among decays with one kaon, so, it provides the dominant contribution to the s-quark mass sensitive total strange hadronic spectral function.
- **Study of the $K_S^0 \pi$ dynamics (mass spectrum):** M. Finkemeier, E. Mirkes, Z. Phys. C **72**, 619 (1996). The hadronic current in the case of two pseudoscalar hadrons with $q^\mu_{1,2}$:

$$J^\mu = F_V(q^2)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)(q_1 - q_2)_\nu + F_S(q^2) q^\mu, \; q^\mu = q_1^\mu + q_2^\mu$$

  - F_V: $K^*(892)^\pm$, $K^*(1410)^\pm$, $K^*(1680)^\pm$;
  - F_S: $K^*(800)^\pm(\kappa)$, $K^*(1430)^\pm$;
  - Precision measurement of $M(K^*(892)^\pm)$ and $\Gamma(K^*(892)^\pm)$.
- **CPV in $\tau \to K_S^0 \pi \nu_\tau$**
  - J.Kuhn, E.Mirkes, Phys. Lett. **B398**, 407 (1997).
  - Y. Grossman and Y. Nir, JHEP **1204**, 002 (2012).
  - J. P. Lees *et al.* [BABAR], Phys. Rev. D **85**, 031102 (2012).
  - M. Bischofberger *et al.* [Belle], Phys. Rev. Lett. **107**, 131801 (2011).

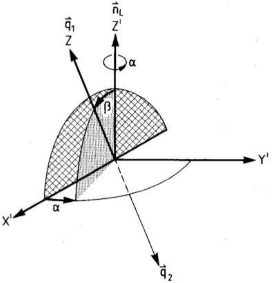
$\tau \to K\pi\nu_\tau$ hadronic spectral functions

$$d\Gamma = \frac{G_F^2}{256\pi^3 m_\tau} \sin^2\theta_c \{L_{\mu\nu}H^{\mu\nu}\} \left(1 - \frac{q^2}{m_\tau^2}\right) |\vec{q}_1| \frac{dq^2}{\sqrt{q^2}} \frac{d\alpha}{2\pi} \frac{d\cos\beta}{2} \frac{d\cos\theta}{2}$$

$$L_{\mu\nu}H^{\mu\nu} = 2m_\tau^2 \left(1 - \frac{q^2}{m_\tau^2}\right)(\overline{L}_B W_B + \overline{L}_{SA} W_{SA} + \overline{L}_{SF} W_{SF})$$

$$W_B = 4|\vec{q}_1|^2 |F_V|^2,\ W_{SA} = q^2 |F_S|^2,\ W_{SF} = 4\sqrt{q^2}|\vec{q}_1|\mathrm{Re}[F_V F_S^*]$$

$$\overline{L}_B = \frac{1}{3}\left(2 + \frac{m_\tau^2}{q^2}\right) - \frac{1}{6}\left(1 - \frac{m_\tau^2}{q^2}\right)(3\cos^2\psi - 1)(3\cos^2\beta - 1),\ \overline{L}_{SA} = \frac{m_\tau^2}{q^2},\ \overline{L}_{SF} = -\frac{m_\tau^2}{q^2}\cos\psi\cos\beta$$



$$\cos\beta = -\vec{n}_q \cdot \frac{\vec{q}_1}{|\vec{q}_1|}$$

$$\cos\theta = \frac{\left(2\frac{E_{K\pi}}{E_\tau} - 1 - \frac{q^2}{m_\tau^2}\right)}{\left(1 - \frac{q^2}{m_\tau^2}\right)\sqrt{1 - m_\tau^2/E_\tau^2}}$$

$$\cos\psi = \frac{\frac{E_{K\pi}}{E_\tau}(m_\tau^2 + q^2) - 2q^2}{(m_\tau^2 - q^2)\sqrt{(E_{K\pi}^2 - q^2)/E_\tau^2}}$$

$K\pi$ mass spectrum

$$\frac{d\Gamma}{d\sqrt{s}} \sim \frac{1}{s}\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+2\frac{s}{M_\tau^2}\right)P(s)\left\{P^2(s)|F_V|^2+\frac{3(M_K^2-M_\pi^2)^2}{4s(1+2\frac{s}{M_\tau^2})}|F_S|^2\right\}$$

$$\mathrm{F_V} = \frac{\mathrm{BW_{K^*(892)} + a(K^*(1410)) \cdot BW_{K^*(1410)} + a(K^*(1680)) \cdot BW_{K^*(1680)}}}{\mathrm{1 + a(K^*(1410)) + a(K^*(1680))}}$$

$$\mathrm{F_S = a(K_0^*(800)) \cdot BW_{K_0^*(800)} + a(K_0^*(1430)) \cdot BW_{K_0^*(1430)}}$$

$$\mathrm{BW_X = \frac{M_X^2}{M_X^2 - s - i\sqrt{s}\Gamma_X(s)}}$$

$$\Gamma_X(s) = \Gamma_X \frac{\mathrm{M_X^2}}{\mathrm{s}}\left(\frac{\mathrm{P(s)}}{\mathrm{P(M_X^2)}}\right)^{2\ell+1} \cdot \mathrm{F_R^{\ell 2}}$$

$$\mathrm{P(s) = \frac{\sqrt{(s-(M_K+M_\pi)^2)(s-(M_K-M_\pi)^2)}}{2\sqrt{s}}}$$

Spin ℓ	Blatt-Weisskopf factor $\mathrm{F_R^\ell}$
0	1
1	$\sqrt{\frac{1+R^2P^2(M_X^2)}{1+R^2P^2(s)}}$
2	$\sqrt{\frac{9+3R^2P^2(M_X^2)+R^4P^4(M_X^2)}{9+3R^2P^2(s)+R^4P^4(s)}}$

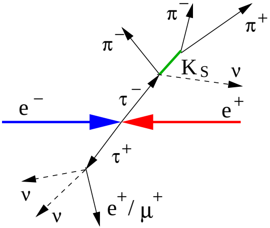
Measurement of $\mathcal{B}(\tau^- \to K_S^0\pi^-\nu_\tau)$ at Belle

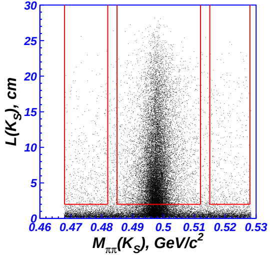
D. Epifanov *et al.* [Belle], Phys. Lett. B **654**, 65 (2007).

Statistics: $\int Ldt = 351$ fb^{-1}, $N_{\tau\tau} = 323 \times 10^6$

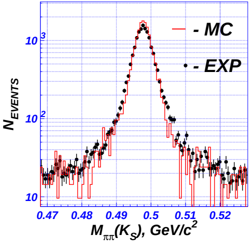
$\approx$ 53000 signal events with efficiency $\varepsilon_{\rm det} \simeq 6\%$.

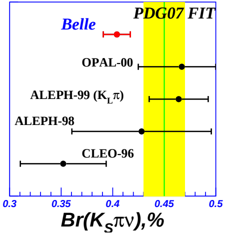
Two-lepton ($\tau \to e\nu\nu, \tau \to \mu\nu\nu$) events are used for normalization.



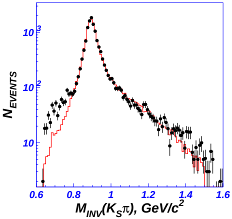


Distribution over $M_{\pi\pi}(K_S)$





Fit with K*(892) only



Mode	Contents, %
$K_S\pi\nu$	79
$K_S\pi K_L\nu$	9
$K_S\pi\pi^0\nu$	4
$K_S K\nu$	2
$3\pi\nu$	5
non-$\tau\tau$	1

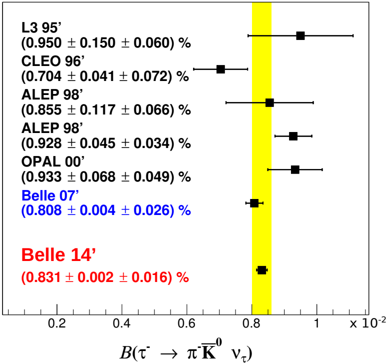
$$\mathcal{B}(\tau^- \to K_S\pi^-\nu_\tau) = (0.404 \pm 0.002(\text{stat.}) \pm 0.013(\text{syst.}))\%$$

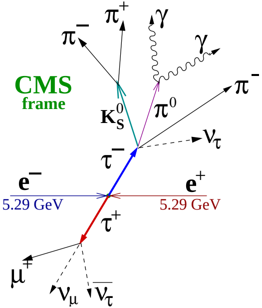
Study of $\tau^- \to K_S^0 X^- \nu_\tau$ decays at Belle

S. Ryu *et al.* [Belle], Phys. Rev. D **89**, 072009 (2014)

Data sample of $\int Ldt = 669$ fb^{-1} with $N_{\tau\tau} = 616 \times 10^6$ was used to study inclusive decay $\tau^- \to K_S^0 X^- \nu_\tau$ as well as 6 exclusive modes (1-prong tag):

$$\pi^- K_S^0 \nu_\tau \qquad K^- K_S^0 \nu_\tau \qquad \pi^- K_S^0 K_S^0 \nu_\tau$$
$$\pi^- K_S^0 \pi^0 \nu_\tau \qquad K^- K_S^0 \pi^0 \nu_\tau \qquad \pi^- K_S^0 K_S^0 \pi^0 \nu_\tau$$





≈ 144000 signal events with efficiency $\varepsilon_{\rm det} \simeq 7\%$

$\mathcal{B}(\tau^- \to K_S^0 \pi^- \nu_\tau) = (4.16 \pm 0.01 \pm 0.08) \times 10^{-3}$

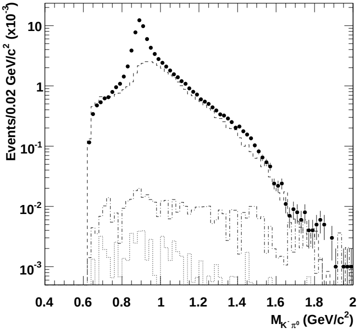
$\mathcal{B}(\tau^- \to K_S^0 X^- \nu_\tau) = (9.14 \pm 0.01 \pm 0.22) \times 10^{-3}$

Study of $\tau \to K\pi\nu$ at *BABAR*

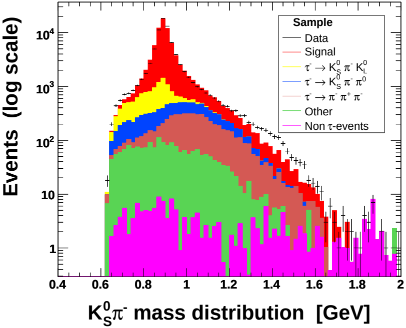
B. AUBERT *et al.* [BABAR], PHYS. REV. D **76**, 051104 (2007).

B. AUBERT *et al.* [BABAR], NUCL. PHYS. PROC. SUPPL. **189**, 193 (2009).

$\tau^- \to K^-\pi^0\nu_\tau$



$\tau^- \to K^0_S\pi^-\nu_\tau$

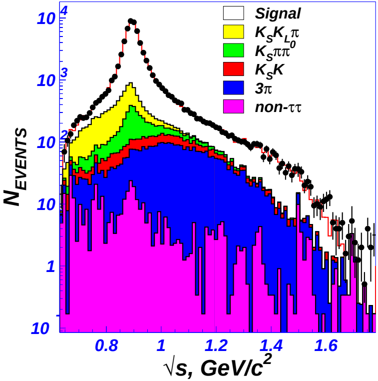


$$\mathcal{B}(\tau^- \to K^-\pi^0\nu_\tau) = (0.416 \pm 0.003(\text{stat.}) \pm 0.018(\text{syst.}))\%$$

$$\mathcal{B}(\tau^- \to K^0_S\pi^-\nu_\tau) = (0.420 \pm 0.002(\text{stat.}) \pm 0.012(\text{syst.}))\% \text{ (preliminary)}$$

$K_0^*(800) + K^*(892) + K^*(1410)$ model

The $K^*(892)$ alone is not sufficient to describe the $K_S^0\pi$ spectrum



$M_{K^*(892)} = 895.47 \pm 0.20$ MeV/c^2

$\Gamma_{K^*(892)} = 46.19 \pm 0.57$ MeV

$|a(K^*(1410))| = (75 \pm 6) \times 10^{-3}$

$\arg(a(K^*(1410))) = 1.44 \pm 0.15$

$|a(K_0^*(800))| = 1.57 \pm 0.23$

$\chi^2/\mathrm{Ndf} = 90.2/84$, $P(\chi^2) = 30\%$

We take $K_0^*(800)$ parameters:
$M_{K_0^*(800)} = (878 \pm 23 \pm 60)$ MeV/c^2, $\Gamma_{K_0^*(800)} = (499 \pm 52 \pm 71)$ MeV/c^2 from:
M. Ablikim *et al.*, [BES Collaboration], Phys. Lett. B **633**, 681 (2006).
There is large systematic uncertainty in the near $K_S^0\pi$ production threshold part of the spectrum due to the large background from the $\tau^- \to K_S^0\pi^-K_L^0\nu_\tau$ decay, whose dynamics is not precisely known. Careful study of the $\tau^- \to K_S^0\pi^-\nu_\tau$ near the $K_S^0\pi$ production threshold is needed.

CPV in hadronic τ decays at B factories

- CPV has not been observed in lepton decays
- It is strongly suppressed in the SM ($A^{\rm CP}_{\rm SM} \lesssim 10^{-12}$) and observation of large CPV in lepton sector would be clean sign of New Physics
- τ lepton provides unique possibility to search for CPV effects, as it is the only lepton decaying to hadrons, so that the associated strong phases allows us to visualize CPV in hadronic τ decays.

I. CPV in $\tau^- \to \pi^- K_S(\geq 0\pi^0)\nu_\tau$ at BaBar (Phys. Rev. D 85, 031102 (2012))

Data sample of $\int Ldt = 476$ fb^{-1} was analyzed

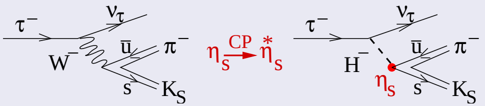
$$A_{\rm CP} = \frac{\Gamma(\tau^+\to\pi^+K^0_S(\geq 0\pi^0)\bar{\nu}_\tau) - \Gamma(\tau^-\to\pi^-K^0_S(\geq 0\pi^0)\nu_\tau)}{\Gamma(\tau^+\to\pi^+K^0_S(\geq 0\pi^0)\bar{\nu}_\tau) + \Gamma(\tau^-\to\pi^-K^0_S(\geq 0\pi^0)\nu_\tau)} = (-0.36 \pm 0.23 \pm 0.11)\%$$

2.8σ deviation from the SM expectation: $A^{K^0}_{\rm CP} = (+0.36 \pm 0.01)\%$

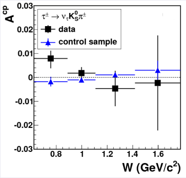
II. CPV in $\tau^- \to K^0_S\pi^-\nu_\tau$ at Belle (Phys. Rev. Lett. 107, 131801 (2011)) $\int$Ldt=699 fb^{-1}

Angular distributions were analyzed, $A_{\rm CP}(W = M_{K_S\pi})$ was measured ($d\omega = d\cos\beta d\cos\theta$):

$$A_{\rm CP}(W) = \frac{\int \cos\beta\cos\psi \left(\frac{d\Gamma_{\tau^-}}{d\omega} - \frac{d\Gamma_{\tau^+}}{d\omega}\right)d\omega}{\frac{1}{2}\int\left(\frac{d\Gamma_{\tau^-}}{d\omega} + \frac{d\Gamma_{\tau^+}}{d\omega}\right)d\omega} \simeq \langle\cos\beta\cos\psi\rangle_{\tau^-} - \langle\cos\beta\cos\psi\rangle_{\tau^+}$$



$|Im(\eta_S)| < 0.026$



CP violation in $\tau^\pm \to K_S\pi^\pm\nu_\tau$ at Belle (I)

The $K_S\pi^-$ hadronic current is parametrized by vector ($F_V(Q^2)$) and scalar ($F_S(Q^2)$) form factor:

$$J^\mu = \langle K_S(q_1)\pi^-(q_2)|\bar{s}\gamma^\mu u|0\rangle = F_V(Q^2)\left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right)(q_1 - q_2)_\nu + F_S(Q^2)Q^\mu$$

Effect of CP violating scalar boson exchange diagram can be introduced by replacing the SM scalar form factor:

$$F_S(Q^2) \to \overline{F}_S(Q^2) = F_S(Q^2) + \frac{\eta_S}{m_\tau}F_H(Q^2),\ F_H = \langle K_S(q_1)\pi^-(q_2)|\bar{s}u|0\rangle = \frac{Q^2}{m_s - m_u}F_S(Q^2)$$

$$d\Gamma_{\tau^-}(\eta_S) \xrightarrow{CP} d\Gamma_{\tau^+}(\overset{*}{\eta}_S)$$

$\tau^- \to K_S\pi^-\nu_\tau$ differential decay width:

$$\frac{d\Gamma}{dQ^2 d\cos\beta d\cos\theta} = (A(Q^2) - B(Q^2)(3\cos^2\beta - 1)(3\cos^2\psi - 1))|F_V(Q^2)|^2 + M_\tau^2|F_S|^2 +$$

$$+C(Q^2)\cos\beta\cos\psi\,\mathrm{Re}(F_V\overset{*}{\overline{F}}_S(\eta_S))$$

- β - angle between $\vec{q}_1$ and direction to CMS frame in the $K_S\pi$ rest frame
- ψ - angle between $\vec{P}_\tau$ and direction to CMS frame in the $K_S\pi$ rest frame
- θ - angle between $\vec{P}_\tau$ in CMS and momentum of $K_S\pi$ in τ rest frame (correlated with ψ)

To extract CPV term the following observable is defined in bin "i" of Q^2 ($d\omega = dQ^2 d\cos\theta d\cos\beta$):

$$A_i^{\mathrm{CP}} = \frac{\int_i \cos\beta\cos\psi\left(\frac{d\Gamma_{\tau^-}}{d\omega} - \frac{d\Gamma_{\tau^+}}{d\omega}\right)d\omega}{\frac{1}{2}\int_i\left(\frac{d\Gamma_{\tau^-}}{d\omega} + \frac{d\Gamma_{\tau^+}}{d\omega}\right)d\omega} \simeq \langle\cos\beta\cos\psi\rangle^i_{\tau^-} - \langle\cos\beta\cos\psi\rangle^i_{\tau^+}$$

CP violation in $\tau^\pm \to K_S \pi^\pm \nu_\tau$ at Belle (II)

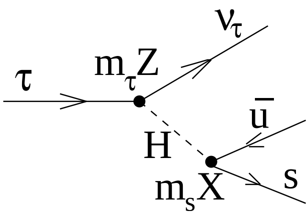
From the $A_i^{\rm CP}$ the CPV parameter $Im(\eta_S)$ can be extracted:

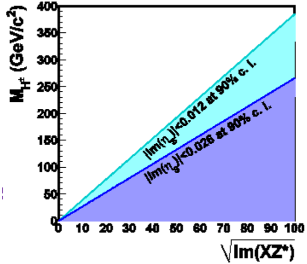
$$A_i^{\rm CP} \simeq {\rm Im}(\eta_S) \frac{N_s}{n_i} \int_i C(Q^2) \frac{{\rm Im}(F_V F_H^*)}{m_\tau} dQ^2 \equiv c_i {\rm Im}(\eta_S)$$

Use several parametrizations of F_V and F_S from our previous study of $M_{K_S\pi}$ spectrum and floating relative phase ($\phi_S = 0°...360°$):

$$|{\rm Im}(\eta_S)| < (0.012 - 0.026) \text{ at } 90\% \text{ CL}$$

Theoretical predictions for ${\rm Im}(\eta_S)$ in MHDM:





$$\eta_S \simeq \frac{m_\tau m_s}{M_{H^\pm}^2} X^* Z \quad |{\rm Im}(XZ^*)| < 0.15 \frac{M_{H^\pm}^2}{1\,{\rm GeV}^2/c^4} \quad (|{\rm Im}(\eta_S)| < 0.026)$$

CPV in $\tau^{\pm} \to K_S \pi^{\pm} \nu_\tau$ with polarized τ lepton (I)

S. Y. Choi *et al.*, PLB **437**, 191 (1998).

$$M_\sigma = \frac{G_F}{\sqrt{2}} \sin\theta_c \left[(1+\chi)\bar{u}_\nu(k,-)\gamma^\mu(1-\gamma^5)u(p,\sigma)J_\mu + \eta\bar{u}_\nu(k,-)(1+\gamma^5)u(p,\sigma)J_S \right]$$

$$J_\mu = \langle (K\pi)^- | \bar{s}\gamma_\mu u | 0 \rangle = F_V(q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right)(q_1 - q_2)^\nu + F_S(q^2) q^\mu$$

$$J_S = \langle (K\pi)^- | \bar{s} u | 0 \rangle = \frac{q^2}{m_s - m_u} F_S(q^2)$$

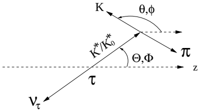
- $\sigma = \pm 1$ – helicity of τ^-;
- χ, η parametrize BSM contribution, $\xi = \frac{m^2_{K_0^*}}{(m_s - m_u)m_\tau}\left(\frac{\eta}{1+\chi}\right)$
- τ^-: $M_\pm(\chi, \xi)$, τ^+: $\overline{M}_\pm(\chi^*, \xi^*)$

If χ and η are real: $M_\pm(\Theta; q^2; \theta, \phi) = \mp \overline{M}_\mp(\Theta; q^2; \theta, -\phi)$

τ is polarized in the $(\theta_p,\ \phi_p)$ direction:

$|\theta_p,\ \phi_p\rangle = \cos\frac{\theta_p}{2}|+\rangle + \sin\frac{\theta_p}{2}|-\rangle$

$\langle \Theta,\ \Phi | \theta_p,\ \phi_p \rangle = \cos\frac{\theta_p}{2} M_+ + \sin\frac{\theta_p}{2} M_-$



$$d\Gamma = \frac{1}{2} d(\Gamma_{++} + \Gamma_{--}) + P_\tau \left(\frac{1}{2} \cos_p d(\Gamma_{++} - \Gamma_{--}) + \sin_p \cos(\phi_p - \Phi) d\mathrm{Re}\Gamma_{+-} - \sin_p \sin(\phi_p - \Phi) d\mathrm{Im}\Gamma_{+-} \right)$$

$$d\Gamma_{\sigma\sigma'} = \frac{1}{(2\pi)^3} \frac{1}{32 m_\tau} \left(1 - \frac{q^2}{m_\tau^2} \right) M_\sigma M^*_{\sigma'} P_K \, d\Phi_3 \, d\Phi, \quad d\Phi_3 = d\sqrt{q^2} \, d\cos d\phi \, d\cos$$

CPV in $\tau^\pm \to K_S \pi^\pm \nu_\tau$ with polarized τ lepton (II)

After integration on Φ (and $P_\tau = 1$):

$$\frac{d\Gamma_1}{d\Phi_3} = \frac{d(\Gamma_{++} + \Gamma_{--})}{d\Phi_3},\ \frac{d\Gamma_2}{d\Phi_3} = \frac{d(\Gamma_{++} - \Gamma_{--})}{d\Phi_3},\ \frac{d\Gamma_3}{d\Phi_3} = 2\mathrm{Re}\left(\frac{d\Gamma_{+-}}{d\Phi_3}\right),\ \frac{d\Gamma_4}{d\Phi_3} = 2\mathrm{Im}\left(\frac{d\Gamma_{+-}}{d\Phi_3}\right)$$

$$\frac{d\Gamma_i}{d\Phi_3} = \frac{1}{2}(\Sigma_i + \Delta_i),\quad \Sigma_i/\Delta_i - \text{CP even/odd part},\quad i = 1 \div 4$$

$$\Sigma_1 = \frac{d(\Gamma_1 + \overline{\Gamma}_1)}{d\Phi_3},\ \Sigma_2 = \frac{d(\Gamma_2 - \overline{\Gamma}_2)}{d\Phi_3},\ \Sigma_3 = \frac{d(\Gamma_3 - \overline{\Gamma}_3)}{d\Phi_3},\ \Sigma_4 = \frac{d(\Gamma_4 + \overline{\Gamma}_4)}{d\Phi_3},$$

$$\Delta_1 = \frac{d(\Gamma_1 - \overline{\Gamma}_1)}{d\Phi_3},\ \Delta_2 = \frac{d(\Gamma_2 + \overline{\Gamma}_2)}{d\Phi_3},\ \Delta_3 = \frac{d(\Gamma_3 + \overline{\Gamma}_3)}{d\Phi_3},\ \Delta_4 = \frac{d(\Gamma_4 - \overline{\Gamma}_4)}{d\Phi_3}$$

CP even: $\Sigma_1 \gg \Sigma_2, \Sigma_3, \Sigma_4$,
CP odd: Δ_1 – P_τ-independent part, $\Delta_{2,3,4}$ – P_τ-dependent part.
Four optimal variables to search for CPV are: $w_i^{\mathrm{opt}} = \Delta_i/\Sigma_1$.

P_τ-independent $\boldsymbol{w_1^{\mathrm{opt}}}$ was used at Belle, while 3 P_τ-dependent $\boldsymbol{w_{2\div4}^{\mathrm{opt}}}$ can be additionally measured at the Super Charm-Tau factory:

$$\boldsymbol{w_1^{\mathrm{opt}} = A_1(q^2;\Theta,\theta,\phi)\mathrm{Im}(\xi)\mathrm{Im}(F_V F_S^*)},$$

$$\boldsymbol{w_{2\div4}^{\mathrm{opt}} = A_{2\div4}(q^2;\Theta,\theta,\phi)\mathrm{Im}(\xi)\mathrm{Im}(F_V F_S^*) + B_{2\div4}(q^2;\Theta,\theta,\phi)\ \mathrm{Im}(\xi)\mathrm{Re}(F_V F_S^*)}$$

At the Super Charm-Tau factory CPV search doesn't depend on $F_V F_S^*$ phase.

CPV in $\tau^\pm \to K_S\pi^\pm\nu_\tau$ with polarized τ lepton (III)

At the center-of-mass energies close to the $\tau^+\tau^-$ production threshold the τ lepton is produced with the polarization

$|\vec{P}_\tau| = P_e \frac{2E_{\text{beam}}\sqrt{p^2_{\text{beam}}\cos^2\theta + M^2_\tau}}{E^2_{\text{beam}} + M^2_\tau + p^2_{\text{beam}}\cos^2\theta} \approx P_e$ along electron beam polarization

$((P_\tau)_Z = P_e \frac{E_{\text{beam}}\cos^2\theta + M_\tau\sin^2\theta}{\sqrt{p^2_{\text{beam}}\cos^2\theta + M^2_\tau}} \approx P_e)$.

In case of New Physics contribution, the amplitudes for the decays $\tau^- \to (K\pi)^-\nu_\tau$ and $\tau^+ \to (K\pi)^+\bar{\nu}_\tau$ are:

$$\mathcal{A} = A_1 + A_2 e^{i\phi}e^{i\delta},\ \ \overline{\mathcal{A}} = A_1 + A_2 e^{-i\phi}e^{i\delta}$$

where ϕ and δ are relative weak (CP-odd) and strong (CP-even) phases. CPV is studied comparing $|\mathcal{A}|^2$ and $|\overline{\mathcal{A}}|^2$, there are three possibilities to construct CPV asymmetry:

- decay rate asymmetry $\sim \sin\delta\sin\phi$
- weighted rate asymmetry $\sim \sin\delta\sin\phi$
- asymmetry based on $\vec{P}_\tau(\vec{p}_K \times \vec{p}_\pi)$ triple product $\sim \cos\delta\sin\phi$

At the Super Charm-Tau factory, with nonzero single τ polarization, nonzero strong-phase difference, δ, is not needed to measure CPV.

Search for CPV in $\tau^{\mp} \to (K\pi)^{\mp}\nu$ in unbinned fit

Analysis of the $(\tau^{\mp} \to (K\pi)^{\mp}\nu\ ;\ \tau^{\pm} \to \rho^{\pm}\nu)$ events, search for CPV in $\tau^- \to (K\pi)^-\nu_\tau$.

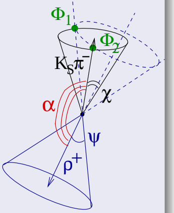
The analysis of the decay products of both taus allows one to constrain direction of $\tau^- - \tau^+$ axis. Such a constraint is efficient to suppress background from $\tau^- \to (K\pi)^- K_L^0 \nu_\tau$.

$$\frac{d\sigma(\vec{\zeta}^*, \vec{\zeta'}^*)}{d\Omega_\tau} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i^*\zeta_j'^*), \quad \frac{d\Gamma(\tau^{\pm}(\vec{\zeta'}^*) \to \rho^{\pm}\nu)}{dm_{\pi\pi}^2 d\Omega_\rho^* d\tilde{\Omega}_\pi} = A' \mp \vec{B'}\vec{\zeta'}^*$$

$$\frac{d\Gamma(\tau^{\mp}(\vec{\zeta}^*) \to (K\pi)^{\mp}\nu)}{dm_{K\pi}^2 d\Omega_{K\pi}^* d\tilde{\Omega}_\pi} = \begin{matrix} (A_0 + \eta_{CP}A_1) + (\vec{B}_0 + \eta_{CP}\vec{B}_1)\vec{\zeta}^* \\ (A_0 + \eta_{CP}^*A_1) - (\vec{B}_0 + \eta_{CP}^*\vec{B}_1)\vec{\zeta}^* \end{matrix}$$

$$\frac{d\sigma((K\pi)^{\mp}, \rho^{\pm})}{dm_{K\pi}^2 d\Omega_{K\pi}^* d\tilde{\Omega}_\pi dm_{\pi\pi}^2 d\Omega_\rho^* d\tilde{\Omega}_\pi d\Omega_\tau} = \frac{\alpha^2\beta_\tau}{64E_\tau^2}\begin{pmatrix} \mathcal{F} + \eta_{CP}\mathcal{G} \\ \mathcal{F} + \eta_{CP}^*\mathcal{G} \end{pmatrix}$$

$$\mathcal{F} = D_0A_0A' - D_{ij}B_{0i}B_j', \quad \mathcal{G} = D_0A_1A' - D_{ij}B_{1i}B_j'$$



$$\frac{d\sigma((K\pi)^{\mp}, \rho^{\pm})}{dp_{K\pi}d\Omega_{K\pi}dm_{K\pi}^2 d\tilde{\Omega}_\pi dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \sum_{\Phi_1,\Phi_2} \frac{d\sigma((K\pi)^{\mp}, \rho^{\pm})}{dm_{K\pi}^2 d\Omega_{K\pi}^* d\tilde{\Omega}_\pi dm_{\pi\pi}^2 d\Omega_\rho^* d\tilde{\Omega}_\pi d\Omega_\tau}\left|\frac{\partial(\Omega_{K\pi}^*, \Omega_\rho^*, \Omega_\tau)}{\partial(p_{K\pi}, \Omega_{K\pi}, p_\rho, \Omega_\rho)}\right|$$

$\eta_{\rm CP}$ **is extracted in the simultaneous unbinned maximum likelihood fit of the** $((K\pi)^-, \rho^+)$ **and** $((K\pi)^+, \rho^-)$ **events in the 12D phase space.**

Summary

- The world largest statistics of τ leptons collected by Belle and *BABAR* opens new era in the precision tests of the Standard Model, search for the effects of New Physics and precision studies of low energy QCD.
- Search for CP violation in $\tau^- \to \pi^- K_S (\geq \pi^0) \nu_\tau$ was done by BABAR with a 476 fb^{-1} data sample. The decay-rate asymmetry $(-0.36 \pm 0.23 \pm 0.11)\%$ is measured for the first time and differs from the SM prediction $(0.36 \pm 0.01)\%$ by 2.8σ.
- Search for CPV in $\tau^- \to K_S \pi^- \nu_\tau$ analyzing angular distributions was performed at Belle. Upper limits for CPV parameter are in range $|Im(\eta_S)| < 0.026$ at 90% CL or better, depending on the parametrizations of the $F_V(Q^2)$ and $F_S(Q^2)$.
- The unbinned analysis of the reaction $e^+e^- \to (\tau^- \to \mathrm{hadrons}^- \nu_\tau;\ \tau^+ \to \ell^+ \nu_\ell \bar{\nu}_\tau)$ or $e^+e^- \to (\tau^- \to \mathrm{hadrons}^- \nu_\tau;\ \tau^+ \to \rho^+ \bar{\nu}_\tau)$ in the full miltidimensional phase space is acute for the improved searches for the CPV in hadronic τ decays.
- Belle II experiment, which successfully started its operation, will allow us to search for CPV in τ decays on a new level of precision.
- The Super Charm-Tau factory **with polarized e^- beam**, being a source of taus with nonzero polarization, allows one to search for CPV regardless the value of the hadronic phase in hadronic tau decay.

Backup slides

$K_0^*(800) + K^*(892) + K_0^*(1430)$ model

	solution 1	solution 2
$M_{K^*(892)}$, MeV/c^2	895.42 ± 0.19	895.50 ± 0.22
$\Gamma_{K^*(892)}$, MeV	46.14 ± 0.55	46.20 ± 0.69
$\lvert a(K_0^*(1430))\rvert$	0.954 ± 0.081	1.92 ± 0.20
$arg(a(K_0^*(1430)))$	0.62 ± 0.34	4.03 ± 0.09
$a(K_0^*(800))$	1.27 ± 0.22	2.28 ± 0.47
χ^2/ndf	$86.5/84$	$95.1/84$
$P(\chi^2)$, %	41	19
$\mathcal{B}(K_0^*(1430) \to K_S\pi)$	$1/3$	$1/3$
$\mathcal{B}(\tau \to K_0^*(1430)\nu_\tau)$	$(13 \pm ^{3}_{2}) \times 10^{-5}$	$(54 \pm ^{18}_{9}) \times 10^{-5}$

M. Z. YANG, "TESTING THE STRUCTURE OF THE SCALAR MESON $K_0^*(1430)$ IN $\tau \to K_0^*(1430)\nu_\tau$ DECAY", MOD. PHYS. LETT. A **21**, 1625 (2006) [ARXIV:HEP-PH/0509102]:

$$\mathcal{B}(\tau \to K_0^*(1430)\nu_\tau) = (7.9 \pm 3.1) \times 10^{-5}$$

$K^*(892)^\pm$ mass and width (I)

The $K^*(892)^-$ width is compatible with the previous measurements within experimental errors, however the $K^*(892)^-$ mass value obtained in $\tau^- \to K_S^0\pi^-\nu_\tau$ is systematically higher than those before and is consistent with the world average value of the neutral $K^*(892)^0$ mass. None of the previous measurements in PDG, all of which were performed more than 30 years ago, present the systematic uncertainties for their measurements.

	$M(K^*(892)),\ \mathrm{MeV}/c^2$	$\Gamma(K^*(892)),\ \mathrm{MeV}$
This work	$895.47 \pm 0.20_{\rm stat} \pm 0.44_{\rm syst} \pm 0.59_{\rm mod}$	$46.2 \pm 0.6_{\rm stat} \pm 1.0_{\rm syst} \pm 0.7_{\rm mod}$
PDG-2017	891.76 ± 0.25	50.3 ± 0.8
Difference	3.71 ± 0.80	-4.1 ± 1.7

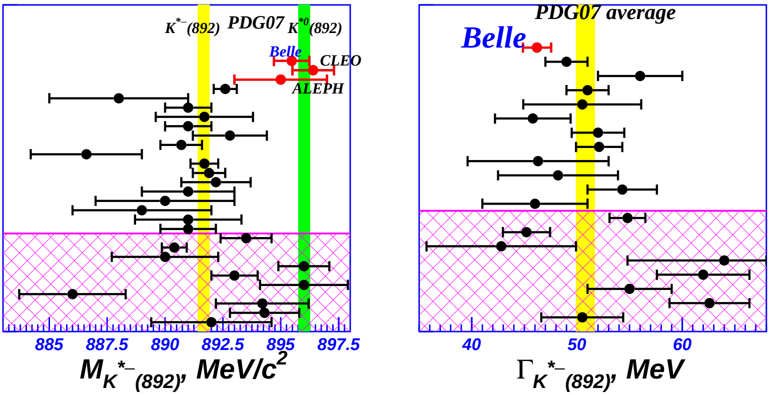
PDG average is based on the results from the fixed target experiments

894.3 ±1.5	1150	[2,3] CLARK	73	HBC	−	3.3 $K^- p \to \overline{K}^0\pi^- p$
892.0 ±2.6	341	[2] SCHWEING...68		HBC	−	5.5 $K^- p \to \overline{K}^0\pi^- p$

CHARGED ONLY, PRODUCED IN τ LEPTON DECAYS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
895.47±0.20±0.74	53k	[6] EPIFANOV	07	BELL	$\tau^- \to K_S^0\pi^-\nu_\tau$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
895.3 ±0.2		[7,8] JAMIN	08	RVUE	$\tau^- \to K_S^0\pi^-\nu_\tau$
896.4 ±0.9	11970	[9] BONVICINI	02	CLEO	$\tau^- \to K^-\pi^0\nu_\tau$
895 ±2		[10] BARATE	99R	ALEP	$\tau^- \to K^-\pi^0\nu_\tau$

$K^*(892)^\pm$ mass and width (II)



The $K^*(892)^-$ width is compatible with the previous measurements within experimental errors, however the $K^*(892)^-$ mass value obtained in $\tau^- \to K_S^0\pi^-\nu_\tau$ is systematically higher than those before and is consistent with the world average value of the neutral $K^*(892)^0$ mass. None of the previous measurements in PDG, all of which were performed more than 30 years ago, present the systematic uncertainties for their measurements.

Further studies at B factories (I)

- To elucidate the nature of the $K^*(892)^- - K^*(892)^0$ mass difference it is suggested to study: $\tau^- \to K_S^0\pi^-\nu_\tau$, $\tau^- \to K_S^0\pi^-\pi^0\nu_\tau$, $\tau^- \to K_S^0K^-\pi^0\nu_\tau$.
  - $K^*(892)^-$ mass and width can be measured in the clean experimental conditions without disturbance from the final state interactions in the $\tau^- \to K_S^0\pi^-\nu_\tau$ decay.
  - Study of the $\tau^- \to K_S^0\pi^-\pi^0\nu_\tau$ mode allows one to measure simultaneously in one mode the $K^*(892)^-(K_S^0\pi^-)$ and the $K^*(892)^0(K_S^0\pi^0)$ masses in the case of one accompanying pion. The effect of the pure hadronic interaction of the $K^*(892)^-(K^*(892)^0)$ and $\pi^0(\pi^-)$ on the $K^*(892)^-(K^*(892)^0)$ mass can be precisely measured.
  - It is possible to investigate precisely an effect of the pure hadronic interaction of the $K^*(892)^-(K^*(892)^0)$ and $K_S^0(K^-)$ on the $K^*(892)^-(K^*(892)^0)$ mass in the $\tau^- \to K_S^0K^-\pi^0\nu_\tau$ decay.
- In the analysis of the $\tau^- \to K_S^0\pi^-\nu_\tau$ decay, it is very desirable to measure separately vector (W_B), scalar (W_{SA}) form factors and the interference term (W_{SF}).
  - $K^*(892)^-$ mass and width are measured in the vector form factor (properly taking into account the effect of the interference of the $K^*(892)^-$ amplitude with the contributions from the radial exitations, $K^*(1410)^-$ and $K^*(1680)^-$).
  - The scalar form factor, W_{SA}, is important for the tests of the various fenomenological models and search for CPV.
  - The interference between vactor and scalar form factors, W_{SF}, is necessary in the search for CPV in $\tau^- \to K_S^0\pi^-\nu_\tau$ decay.
- A complete study of the hadronic τ decays into ≥ 3 hadrons can be done in the full multidimensional phase-space of the reaction
  $\boldsymbol{e^+e^- \to (\tau^- \to \mathrm{hadrons}^-\nu_\tau;\ \tau^+ \to \ell^+\nu_\ell\bar{\nu}_\tau)}$ or
  $\boldsymbol{e^+e^- \to (\tau^- \to \mathrm{hadrons}^-\nu_\tau;\ \tau^+ \to \rho^+\bar{\nu}_\tau)}$

Further studies at B factories (II)

The parametrization of the hadronic current in the $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$ decay was established by CLEO in their unbinned analysis of the $e^+e^- \to (\tau^- \to \pi^-\pi^0\pi^0\nu_\tau,\ \tau^+ \to \ell^+\nu_\ell\bar{\nu}_\tau)$ process in the full phase space: D. M. ASNER *et al.* [CLEO], PHYS. REV. D **61**, 012002 (2000).

$$J^\mu = \beta_1 j_1^\mu(\rho\pi^0)_{S-wave} + \beta_2 j_2^\mu(\rho'\pi^0)_{S-wave} + \beta_3 j_3^\mu(\rho\pi^0)_{D-wave} + \beta_4 j_4^\mu(\rho'\pi^0)_{D-wave} + \\ +\beta_5 j_5^\mu(f_2(1270)\pi)_{P-wave} + \beta_6 j_6^\mu(f_0(500)\pi)_{P-wave} + \beta_7 j_7^\mu(f_0(1370)\pi)_{P-wave}$$

- Before studying hadronic decays, leptonic decay should be analyzed (measurement of Michel parameters) to develop the fitter and polish the procedure (CLEO studied $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$ after they measured Michel parameters).
- At Belle we are now finalizing the measurement of Michel parameters.
- The developed procedure can be used to study dynamics of the $(\tau^\mp \to (K\pi)^\mp\nu\ ;\ \tau^\pm \to \rho^\pm\nu)$ and $(\tau^\mp \to (K\pi)^\mp\nu\ ;\ \tau^\pm \to \ell^\pm\nu\nu)$ processes and to search for CPV in $\tau^- \to (K\pi)^-\nu_\tau$ (also in the spin-dependent part of the differential decay width).