Study of Michel parameters in τ decays at Belle

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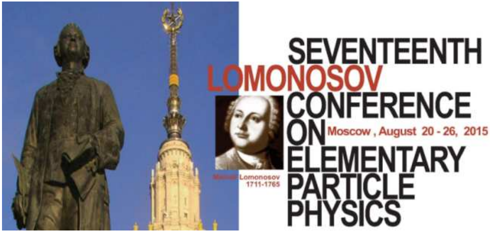
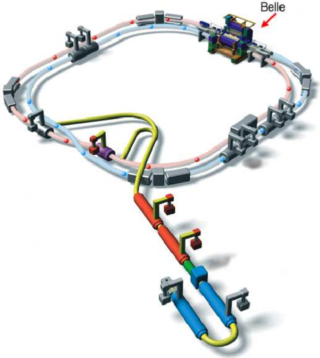
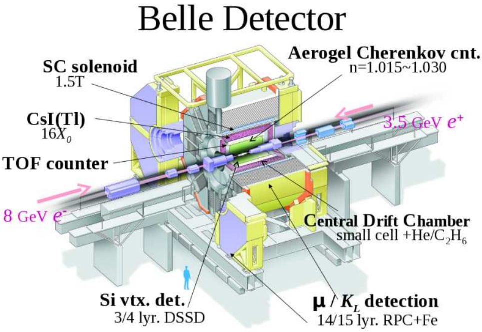


Introduction: Belle experiment





Process	σ, nb
$e^+e^- \to e^+e^-(\gamma)$ $15^o \le \theta \le 165^o$	123.5
$e^+e^- \to \mu^+\mu^-(\gamma)$	1.005
$e^+e^- \to q\bar{q}$ $(q = u, d, s, c)$	3.39
$e^+e^- \to b\bar{b}$	1.05
$e^+e^- \to e^+e^- f\bar{f}$ $(f = u, d, s, c, e, \mu, \tau)$	72.6
$e^+e^- \to \tau^+\tau^-(\gamma)$	0.919

- $E_{e^-} = 8$ **GeV,** $E_{e^+} = 3.5$ **GeV**
- **Peak luminosity:** $L = 2.11 \times 10^{34}$ $\mathbf{cm^{-2}s^{-1}}$
- **Integrated luminosity:** $\int Ldt \simeq 1$ $\mathbf{ab^{-1}}$**,** $N_{\tau\tau} \simeq 10^9$
- B-factory is also τ-factory

Introduction

In the SM charged weak interaction is described by the exchange of $W^{\pm}$ with a pure vector coupling to only left-handed fermions ("V-A" Lorentz structure). Deviations from "V-A" indicate New Physics. $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ ($\ell = e, \mu$) decays provide clean laboratory to probe electroweak couplings.
The most general, Lorentz invariant four-lepton interaction matrix element:

$$\mathcal{M} = \frac{4G}{\sqrt{2}} \sum_{\substack{N=S,V,T \\ i,j=L,R}} g_{ij}^N \left[\bar{u}_i(l^-)\Gamma^N v_n(\bar{\nu}_l)\right]\left[\bar{u}_m(\nu_\tau)\Gamma_N u_j(\tau^-)\right],$$

$$\Gamma^S = 1,\ \Gamma^V = \gamma^\mu,\ \Gamma^T = \frac{i}{2\sqrt{2}}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

Ten couplings g_{ij}^N, in the SM the only non-zero constant is $g_{LL}^V = 1$

Four bilinear combinations of g_{ij}^N, which are called as Michel parameters (MP): ρ, η, ξ and δ appear in the energy spectrum of the outgoing lepton:

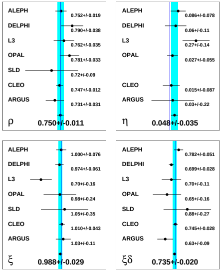
$$\frac{d\Gamma(\tau^\mp)}{d\Omega dx} = \frac{4G_F^2 M_\tau E_{\max}^4}{(2\pi)^4}\sqrt{x^2 - x_0^2}\left(x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)\right.$$

$$\left.\mp\frac{1}{3}P_\tau \cos\theta_\ell \xi\sqrt{x^2 - x_0^2}\left[1 - x + \frac{2}{3}\delta\left(4x - 4 + \sqrt{1 - x_0^2}\right)\right]\right),\ x = \frac{E_\ell}{E_{\max}},\ x_0 = \frac{m_\ell}{E_{\max}}$$

In the SM: $\rho = \frac{3}{4}$, $\eta = 0$, $\xi = 1$, $\delta = \frac{3}{4}$

Status of Michel parameters in τ decays

Michel par.	Measured value	Experiment	SM value
ρ (e or μ)	$0.747 \pm 0.010 \pm 0.006$ 1.2%	CLEO-97	3/4
η (e or μ)	$0.012 \pm 0.026 \pm 0.004$ 2.6%	ALEPH-01	0
ξ (e or μ)	$1.007 \pm 0.040 \pm 0.015$ 4.3%	CLEO-97	1
$\xi\delta$ (e or μ)	$0.745 \pm 0.026 \pm 0.009$ 2.8%	CLEO-97	3/4
ξ_{h} (all hadr.)	$0.992 \pm 0.007 \pm 0.008$ 1.1%	ALEPH-01	1



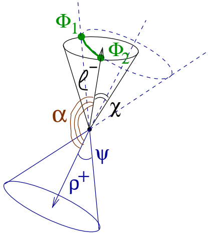
With $\times$300 Belle statistics we can improve MP uncertainties by one order of magnitude

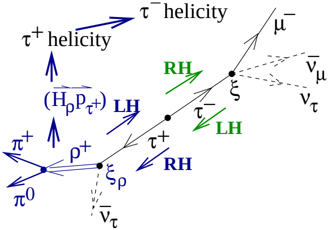
In BSM models the couplings to τ are expected to be enhanced in comparison with μ.
Also contribution from New Physics in τ decays can be amplified by $(\frac{m_\tau}{m_\mu})^n$.

- **Type II 2HDM:** $\eta_\mu(\tau) = \frac{m_\mu M_\tau}{2}\left(\frac{\tan^2\beta}{M^2_{H^\pm}}\right)^2$; $\frac{\eta_\mu(\tau)}{\eta_e(\mu)} = \frac{M_\tau}{m_e} \approx 3500$
- **Tensor interaction:** $\mathcal{L} = \frac{g}{2\sqrt{2}} W^\mu \left\{ \bar{\nu}\gamma_\mu(1-\gamma^5)\tau + \frac{\kappa^W_\tau}{2m_\tau}\partial^\nu\left(\bar{\nu}\sigma_{\mu\ nu}(1-\gamma^5)\tau\right)\right\}$,
  $-0.096 < \kappa^W_\tau < 0.037$: DELPHI Abreu EPJ C16 (2000) 229.
- **Unparticles:** Moyotl PRD 84 (2011) 073010, Choudhury PLB 658 (2008) 148.
- **Lorentz and CPTV:** Hollenberg PLB **701** (2011) 89
- **Heavy Majorana neutrino:** M. Doi *et al.*, Prog. Theor. Phys. **118** (2007) 1069.

Method, study of $\ell - \rho$ and $\rho - \rho$ events

Effect of τ spin-spin correlation is used to measure ξ and δ MP.
Events of ($\tau^\mp \to \ell^\mp \nu\nu$; $\tau^\pm \to \rho^\pm \nu$) topology are used to measure: ρ, η, $\xi_\rho \xi$ and $\xi_\rho \xi \delta$,
while ($\tau^\mp \to \rho^\mp \nu$; $\tau^\pm \to \rho^\pm \nu$) events are used to extract ξ_ρ^2.





$$\frac{d\sigma(\ell^\mp, \rho^\pm)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = A_0 + \rho A_1 + \eta A_2 + \xi_\rho \xi A_3 + \xi_\rho \xi \delta A_4 = \sum_{i=0}^{4} A_i \Theta_i$$

$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp, \rho^\pm)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp, \rho^\pm)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} \left| \frac{\partial(E_\ell^*, \Omega_\ell^*, \Omega_\rho^*, \Omega_\tau)}{\partial(p_\ell, \Omega_\ell, p_\rho, \Omega_\rho, \Phi_\tau)} \right| d\Phi_\tau$$

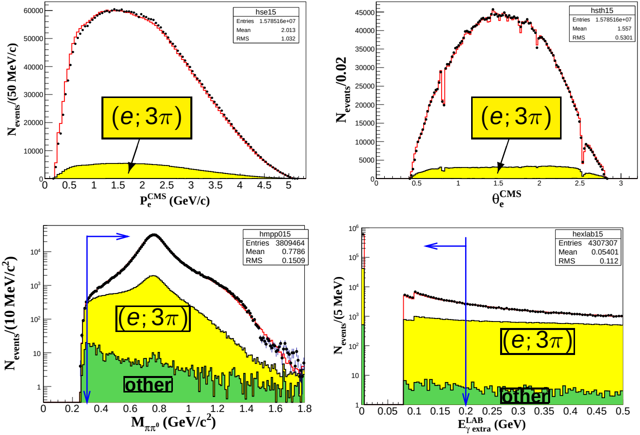
$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)}, \; \mathcal{P}^{(k)} = \mathcal{F}(\vec{z}^{(k)})/\mathcal{N}(\vec{\Theta}), \; \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z}) d\vec{z}, \; \vec{\Theta} = (1, \rho, \eta, \xi_\rho \xi_\ell, \xi_\rho \xi_\ell \delta_\ell)$$

MP are extracted in the unbinned maximum likelihood fit of (ℓ, ρ) events in the 9D phase space $\vec{z} = (p_\ell, \cos\theta_\ell, \phi_\ell, p_\rho, \cos\theta_\rho, \phi_\rho, m_{\pi\pi}^2, \cos\tilde{\theta}_\pi, \tilde{\phi}_\pi)$ in CMS.

Selection criteria

- After the standard preselections we take events with two oppositely charged tracks, one of them is identified as lepton (eID, $\mu ID > 0.9$) and the other one as pion ($PID(\pi/K) > 0.4$).
- π^0 candidate is reconstructed from the pair of gammas ($E_{\gamma}^{\rm LAB} > 80$ MeV) satisfying 115 MeV/$c^2 < M_{\gamma\gamma} <$ 150 MeV/c^2, $P_{\pi^0}^{\rm CMS} > 0.3$ GeV/c.
- $\cos(\vec{P}_{\rm lep}, \vec{P}_{\pi}) < 0$, $\cos(\vec{P}_{\rm lep}, \vec{P}_{\pi^0}) < 0$, 0.3 GeV/$c^2 < M_{\pi\pi^0} <$ 1.8 GeV/c^2.
- $E_{{\rm rest}\gamma}^{\rm LAB} < 0.2$ GeV

Detection efficiency $\varepsilon_{\rm det} \simeq 12\%$



Corrections, detector effects, background

Physical corrections:

- All $\mathcal{O}(\alpha^3)$ QED and electroweak higher order corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$ are included
- Radiative leptonic decays $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau\gamma$
- Radiative decay $\tau^- \to \pi^-\pi^0\nu_\tau\gamma$

Detector effects:

- Track momentum resolution
- γ energy and angular resolution
- Effect of external bremsstrahlung for $e - \rho$ events
- Beam energy spread
- EXP/MC efficiency corrections (trigger, track rec., π^0 rec., ℓID, πID)

Background:

The main background comes from $\ell - \pi\pi^0\pi^0$($\sim$10%) and $\pi - \pi\pi^0(\pi \to \mu)$($\sim$1.5%) events, it is included in PDF analytically.
The remaining background($\sim$2.0%) is taken into account using MC-based approach.
Background from the non-$\tau\tau$ events is $\lesssim$ 0.1%.

Description of background

Total PDF

$$\mathcal{P}(x) = \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\left((1-\sum_i \lambda_i)\frac{S(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} S(x)dx} + \lambda_{3\pi}\frac{\tilde{B}_{3\pi}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} \tilde{B}_{3\pi}(x)dx} + \lambda_{\pi}\frac{\tilde{B}_{\pi}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} \tilde{B}_{\pi}(x)dx}\right) + \lambda_{other}\mathcal{P}^{MC}_{other}(x)$$

$$\tilde{B}_{3\pi}(x) = \int 2(1-\varepsilon_{\pi^0}(y))\varepsilon_{add}(y)B_{3\pi}(x,y)dy,\ \tilde{B}_{\pi}(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)}B_\pi(x)$$

$$\overline{\varepsilon(x)} = \epsilon^{trg}_{corr}(x)\epsilon^{\ell ID}_{corr}(x)\varepsilon(x)$$

- $x = (p_\ell,\ \Omega_\ell,\ p_\rho,\ \Omega_\rho,\ m^2_{\pi\pi},\ \tilde{\Omega}_\pi);\ y = (p_{\pi^0},\ \Omega_{\pi^0})$;
- $S(x)$ - theoretical density of signal ($\ell^\mp,\ \pi^\pm\pi^0$) events;
- $B_{3\pi}(x,y)$ - theoretical density of background ($\ell^\mp,\ \pi^\pm 2\pi^0$) events;
- $B_\pi(x)$ - theoretical density of background ($\pi^\mp,\ \pi^\pm\pi^0$) events;
- $\varepsilon(x)$ - detection efficiency for signal events (**common multiplier**);
- $\mathcal{P}^{MC}_{other}(x)$ - PDF for the remaining background (MC-based procedure);
- λ_i - i-th background fraction (from MC)
- $\varepsilon_{\pi^0}(y)$ - π^0 detection efficiency (tabulated from MC);
- $\varepsilon_{\rm add}(y) = \varepsilon^{3\pi}_{\rm add}(y)/\varepsilon^{\rm sig}_{\rm add}$ - ratio of the $E^{LAB}_{\gamma rest}$ cut efficiencies (tabulated from MC);
- $\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)/\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)$ is tabulated from MC;
- $\epsilon^{trg}_{corr}(x),\ \epsilon^{\ell ID}_{corr}(x)$ - EXP/MC efficiency corrections.

Validation of the fitter with MC

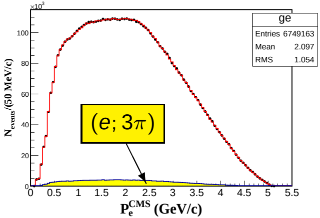
For each configuration 5M MC sample is fitted. The other, statistically independent, 5M MC sample was used to calculate normalization.

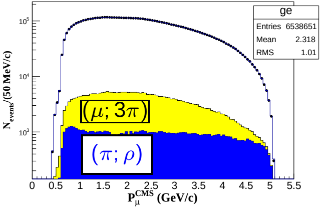
$(e^+; \pi^-\pi^0)$

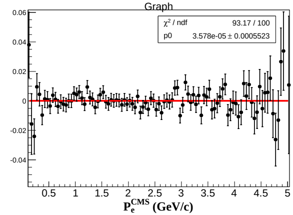
ρ	=	0.7517	$\pm$	0.0010
η	=	0	–	fixed
ξ	=	1.0092	$\pm$	0.0043
$\xi\delta$	=	0.7538	$\pm$	0.0027

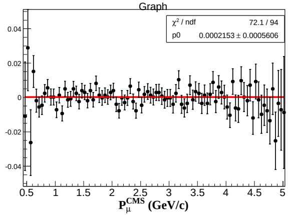
$(\mu^+; \pi^-\pi^0)$

ρ	=	0.7494	$\pm$	0.0027
η	=	0.0052	$\pm$	0.0101
ξ	=	0.9995	$\pm$	0.0050
$\xi\delta$	=	0.7519	$\pm$	0.0033









Description of the remaining background

D. Schmidt et al., Nucl. Instr. and Meth. A328 (1993) 547.

PDF for the remaining background

$$\mathcal{P}_{other}(x_i) = \frac{N^{sel}_{other}(x_i)/V_i}{N^{sel\ TOT}_{other}},\quad \mathcal{P}_{sig}(x_i) = \frac{N^{sel}_{sig}(x_i)/V_i}{N^{sel\ TOT}_{sig}},\quad S(x_i) = \frac{N^{gen}_{sig}(x_i)/V_i}{N^{gen\ TOT}_{sig}}$$

$$\frac{\bar{\varepsilon}}{\varepsilon(x_i)} = \frac{N^{sel\ TOT}_{sig}}{N^{sel}_{sig}(x_i)}\frac{N^{gen}_{sig}(x_i)}{N^{gen\ TOT}_{sig}},\quad \frac{\bar{\varepsilon}}{\varepsilon(x_i)}\mathcal{P}_{other}(x_i) = \frac{1-\sum\lambda_i}{\lambda_{other}}\frac{N^{sel}_{other}(x_i)}{N^{sel}_{sig}(x_i)}S(x_i)$$

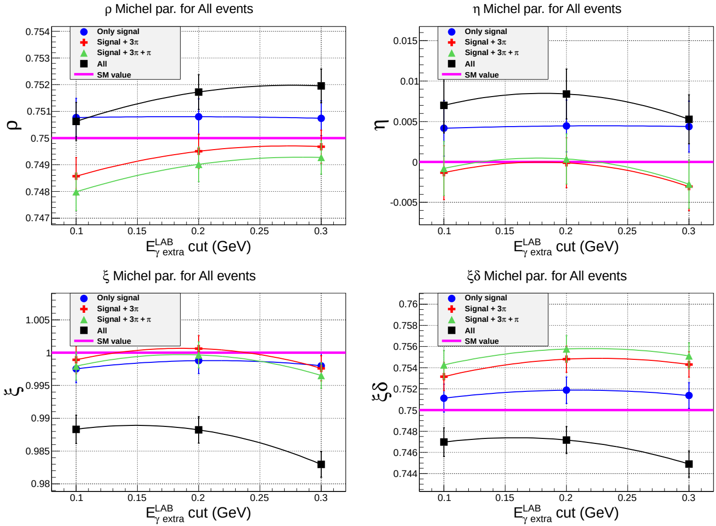
$$\mathcal{P}(x) = \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\left((1-\sum_i\lambda_i)\frac{S(x)}{\int\frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}S(x)dx} + \lambda_{3\pi}\frac{\tilde{B}_{3\pi}(x)}{\int\frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\tilde{B}_{3\pi}(x)dx} + \lambda_{\pi}\frac{\tilde{B}_{\pi}(x)}{\int\frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\tilde{B}_{\pi}(x)dx} + (1-\sum\lambda_i)\frac{N^{sel}_{other}(x_i)}{N^{sel}_{sig}(x_i)}S_{SM}(x_i)\right)$$

MC statistics we have is not enough to tabulate PDF for the remaining background in 9D with sufficient accuracy. Therefore we try to determine PDF in the reduced phase space in such a way that the main correlations are still taken into account.

The best scheme (with the lowest systematic bias of MP) we found so far is: $\mathcal{P}_{4D}(P_\ell,\ P_\rho,\ \omega,\ \cos\psi_{\ell\rho})$

$(P_\ell,\ P_\rho,\ \omega,\ \cos\psi_{\ell\rho})$-scheme, MC fit

Simultaneous fit of MC $(e^+;\rho^-)$, $(e^-;\rho^+)$, $(\mu^+;\rho^-)$, $(\mu^-;\rho^+)$ events

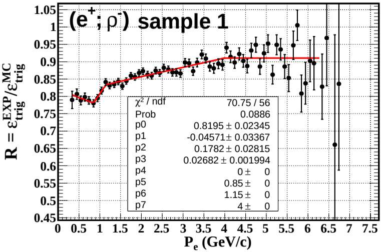


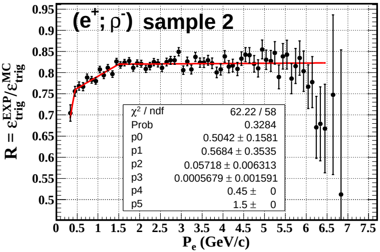
Still, systematic bias of about 1% is seen for $\xi_\rho\xi$ parameter.

We are working on to improve the description of the remaining background.

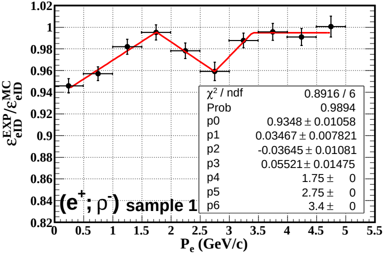
EXP/MC efficiency corrections ($(e^+; \rho^-)$ events)

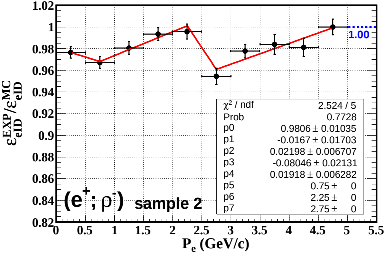
Two independent subtriggers (neutral (ECL) and charged (CDC⊕TOF⊕KLM)) are used to evaluate EXP/MC trigger efficiency correction.



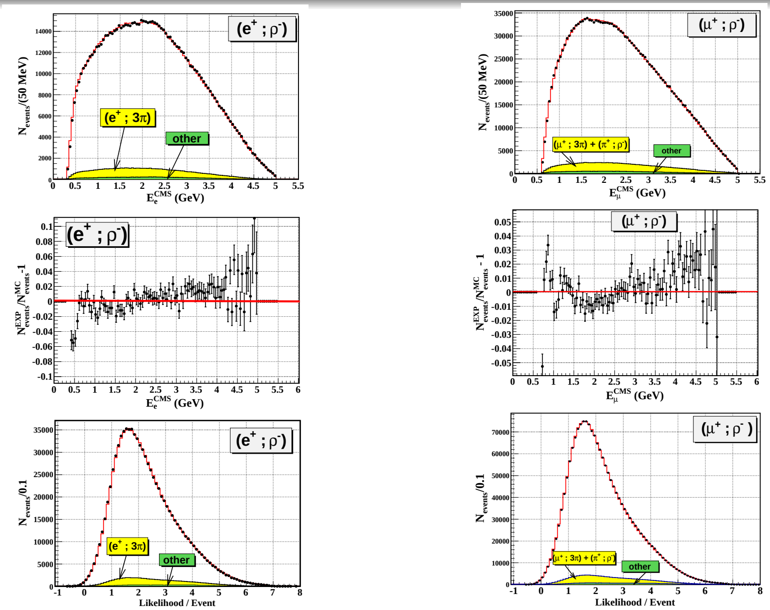


The lepton detection efficiency is corrected using the $e^+e^- \to e^+e^-\ell^+\ell^-,\ \ell = e, \mu$ two-photon data sample.





Fit of the experimental data, $(\ell^+; \rho^-)$

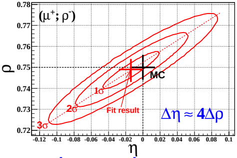


Experimental data sample of $485~\mathrm{fb}^{-1}$ **(446** $\times 10^6~\tau^+\tau^-$**) was analyzed**

Systematic uncertainties

Source	$\sigma(\rho)$, %	$\sigma(\eta)$, %	$\sigma(\xi_\rho \xi)$, %	$\sigma(\xi_\rho \xi \delta)$, %
Physical corrections				
ISR+$\mathcal{O}(\alpha^3)$	0.10	0.30	0.20	0.15
$\tau \to \ell \nu \nu \gamma$	0.03	0.10	0.09	0.08
$\tau \to \rho \nu \gamma$	0.06	0.16	0.11	0.02
Dom. background	0.20	0.60	0.20	0.20
Apparatus corrections				
Resolution $\oplus$ brems.	0.10	0.33	0.11	0.19
$\sigma(E_{\mathrm{beam}})$	0.07	0.25	0.03	0.15
Normalization				
$\Delta\mathcal{N}$	0.21	0.60	0.38	0.26
Total	0.33	1.01	0.51	0.45

We observe a systematic bias of about 1% in the $\xi_\rho \xi$ MP, which originates from the incomplete description of the remaining background.



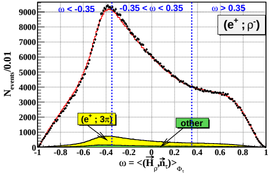
The dominant systematic uncertainties coming from the various EXP/MC efficiency corrections are under investigation.

Summary

- The procedure to measure 4 Michel parameters (MP) (ρ, η, ξ, $\xi\delta$) in leptonic τ decays at B factory has been developed and tested. It is based on the analysis of the $(\ell^{\mp},\ \rho^{\pm})$, $\ell = e,\ \mu$ and $(\rho^{\mp},\ \rho^{\pm})$ events and utilizes spin-spin correlation of tau leptons.
- We confirmed that with Belle data the statistical accuracy of MP is by one order of magnitude better than in the previous best measurements (CLEO, ALEPH).
- Various EXP/MC efficiency corrections provide the dominant contribution to the systemtic uncertainties of MP. The trigger and lepton identification EXP/MC efficiency corrections have been already taken into account.
- The main background components ($\ell - \pi\pi^0\pi^0$, $\pi - \pi\pi^0(\pi \to \mu)$) are described analytically in the fitter, the related systematic uncertainty is $(0.2 \div 0.6)\%$.
- The remaining background (with the fraction of $\sim$2.0%), coming from the other τ decays, is described with help of the MC-based method. Currently we have about 1% systematic bias in $\xi_\rho\xi$ MP due to incomplete description of this background.
- The analysis is going on.

Backup slides

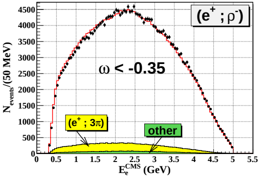
Helicity sensitive variable ω

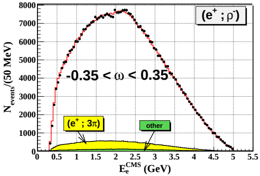


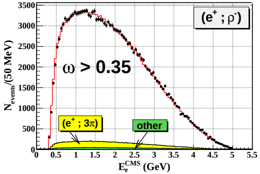
M. Davier *et. al* Phys. Lett. B **306** (1993) 411.

Helicity sensitive variable ω is introduced as:

$$\omega = \frac{1}{\Phi_2 - \Phi_1} \int_{\Phi_1}^{\Phi_2} (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) d\Phi = < (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) >_{\Phi_\tau}$$







Spin-spin correlation manifests itself through momentum-momentum correlations of final lepton and pions.

Multidimensional unbinned maximum likelihood fit

4 Michel parameters ($\vec{\Theta} = (1,\ \rho,\ \eta,\ \xi_\rho\xi_\ell,\ \xi_\rho\xi_\ell\delta_\ell)$) are extracted in the unbinned maximum likelihood fit of $\ell - \rho$ events in the 9D phase space in CMS,
$\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$. The PDF for individual k-th event is written in the form:

$$\mathcal{P}^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})},\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z}$$

Likelihood function for N events:

$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)},\ \mathcal{L} = -\ln L = N\ln\mathcal{N}(\vec{\Theta}) - \sum_{k=1}^{N} \ln\mathcal{F}^{(k)},\ \mathcal{F}^{(k)} = \mathcal{F}(\vec{z}^{(k)})$$

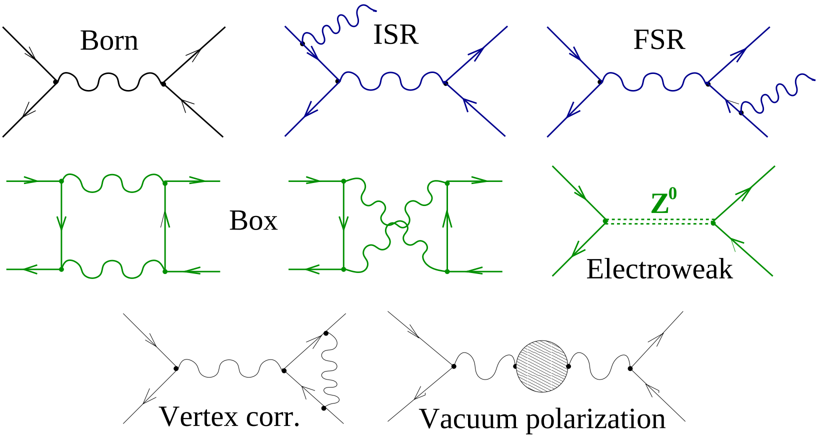
$$\mathcal{F}^{(k)} = A_0^{(k)}\Theta_0 + A_1^{(k)}\Theta_1 + A_2^{(k)}\Theta_2 + A_3^{(k)}\Theta_3 + A_4^{(k)}\Theta_4 = \sum_{i=0}^{4} A_i^{(k)}\Theta_i$$

$$\mathcal{N} = C_0\Theta_0 + C_1\Theta_1 + C_2\Theta_2 + C_3\Theta_3 + C_4\Theta_4,\ C_j = \frac{1}{N}\sum_{k=1}^{N} C_j^{(k)},\ C_j^{(k)} = \frac{A_j^{(k)}}{\sum_{i=0}^{4} A_i^{(k)}\Theta_i^{MC}}$$

$$\vec{\Theta}^{MC} = (1,\ 0.75,\ 0,\ 1,\ 0.75),\ \mathcal{L} = N\ln\left(\sum_{j=0}^{4} C_j\Theta_j\right) - \sum_{k=1}^{N}\ln\left(\sum_{i=0}^{4} A_i^{(k)}\Theta_i\right)$$

As a result fitted statistics is represented by a set of $5 \times N$ values of $A_i^{(k)}$ ($k = 1 \div N,\ i = 0 \div 4$), which is calculated only once.
C_i ($i = 0 \div 4$) are calculated using MC simulation.
In ideal case (no rad. corr., $\varepsilon = 100\%$): $C_0 = 1$, $C_2 = 4m_\ell/m_\tau$, $C_{1,3,4} = 0$

$\mathcal{O}(\alpha^3)$ corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$



S. Jadach and Z. Was, Acta Phys. Polon. B **15** (1984) 1151 [Erratum-ibid. B **16** (1985) 483].
A. B. Arbuzov *et al* JHEP **9710** (1997) 001.

Charge-odd part of the cross section comes from the interference of the ISR and FSR diagrams as well as box and Born diagrams, and Z^0-exchange and Born diagrams.

Contents of the remaining background

$(e,\ h)$ mode	$E_{\gamma\ rest}^{LAB} < 0.1$ GeV	$E_{\gamma\ rest}^{LAB} < 0.3$ GeV
$(e,\ \pi\pi^0 K_L)$	22.1%	17.1%
$(e,\ \pi 3\pi^0)$	8.8%	15.9%
$(e,\ 3\pi\pi^0)$	9.8%	8.5%
$(e,\ K\pi^0)$	11.6%	8.5%
$(e,\ \pi)$	8.3%	7.6%
$(e,\ \pi K_S(\to \pi^0\pi^0))$	4.9%	7.1%
$(e,\ \pi\pi^0 K_S(\to \pi^0\pi^0))$	3.5%	3.2%
$(\pi,\ \pi\pi^0)$	6.9%	4.9%
$(\pi\pi^0,\ \pi\pi^0)$	2.0%	2.8%
sum	77.9%	75.6%
rest	22.1%	24.4%

$(\mu,\ h)$ mode	$E_{\gamma\ rest}^{LAB} < 0.1$ GeV	$E_{\gamma\ rest}^{LAB} < 0.3$ GeV
$(\mu,\ \pi\pi^0 K_L)$	15.5%	11.5%
$(\mu,\ \pi 3\pi^0)$	6.5%	10.6%
$(\mu,\ 3\pi\pi^0)$	8.8%	7.0%
$(\mu,\ K\pi^0)$	8.9%	6.2%
$(\mu,\ \pi)$	7.6%	6.3%
$(\mu,\ \pi K_S(\to \pi^0\pi^0))$	3.9%	4.9%
$(\pi\pi^0,\ \pi\pi^0)$	10.8%	15.9%
$(\pi,\ \pi 2\pi^0)$	4.4%	5.5%
$(K,\ \pi\pi^0)$	7.2%	4.9%
sum	73.6%	72.8%
rest	26.4%	27.2%