Study of Michel parameters in τ decays at Belle

Denis Epifanov on behalf of Belle collaboration

Novosibirsk State University, Budker Institute of Nuclear Physics, Novosibirsk Russia

1. Introduction: Michel parameters

In the SM charged weak interaction is described by the exchange of $W^\pm$ with a pure vector coupling to only left-handed fermions ("V-A" Lorentz structure). Deviations from "V-A" indicate New Physics. $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau$ ($\ell = e, \mu$) decays provide clean laboratory to probe electroweak couplings.
The most general, Lorentz invariant four-lepton interaction matrix element:

$$\mathcal{M} = \frac{4G}{\sqrt{2}}\sum_{\substack{N=S,V,T\\ i,j=L,R}} g^N_{ij}\Big[\bar{u}_i(l^-)\Gamma^N v_n(\bar{\nu}_l)\Big]\Big[\bar{u}_m(\nu_\tau)\Gamma_N u_j(\tau^-)\Big],\ \Gamma^S = 1,\ \Gamma^V = \gamma^\mu,\ \Gamma^T = \frac{i}{2\sqrt{2}}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

Ten couplings g^N_{ij}, in the SM the only non-zero constant is $g^V_{LL} = 1$

Four bilinear combinations of g^N_{ij}, which are called as Michel parameters (MP): ρ, η, ξ and δ appear in the energy spectrum of the outgoing lepton:

$$\frac{d\Gamma(\tau^\mp)}{d\Omega dx} = \frac{4G_F^2M_\tau E^4_{\max}}{(2\pi)^4}\sqrt{x^2 - x_0^2}\Big(x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)$$
$$\mp\frac{1}{3}P_\tau cos\theta_\ell\xi\sqrt{x^2 - x_0^2}\Big[1 - x + \frac{2}{3}\delta\big(4x - 4 + \sqrt{1 - x_0^2}\big)\Big]\Big),\ x = \frac{E_\ell}{E_{\max}},\ x_0 = \frac{m_\ell}{E_{\max}},\ E_{\max} = \frac{M_\tau}{2}\Big(1 + \frac{m_\ell^2}{M_\tau^2}\Big)$$

In the SM: $\rho = \frac{3}{4}$, $\eta = 0$, $\xi = 1$, $\delta = \frac{3}{4}$

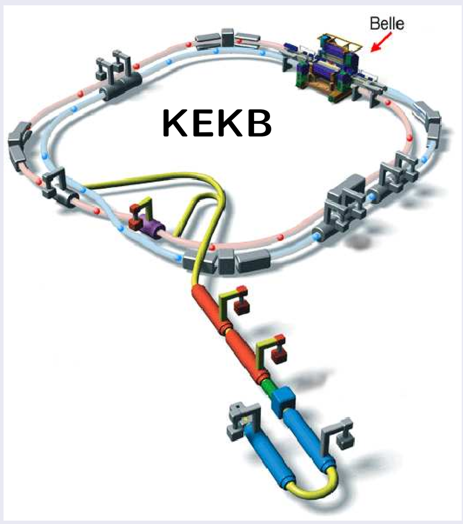
Michel par.	Measured value	Experiment	SM value
ρ (e or μ)	$0.747 \pm 0.010 \pm 0.006$ 1.2%	CLEO-97	0.75
η (e or μ)	$0.012 \pm 0.026 \pm 0.004$ 2.6%	ALEPH-01	0
ξ (e or μ)	$1.007 \pm 0.040 \pm 0.015$ 4.3%	CLEO-97	1
$\xi\delta$ (e or μ)	$0.745 \pm 0.026 \pm 0.009$ 2.8%	CLEO-97	0.75
$\xi_{\rm h}$ (all hadr.)	$0.992 \pm 0.007 \pm 0.008$ 1.1%	ALEPH-01	1

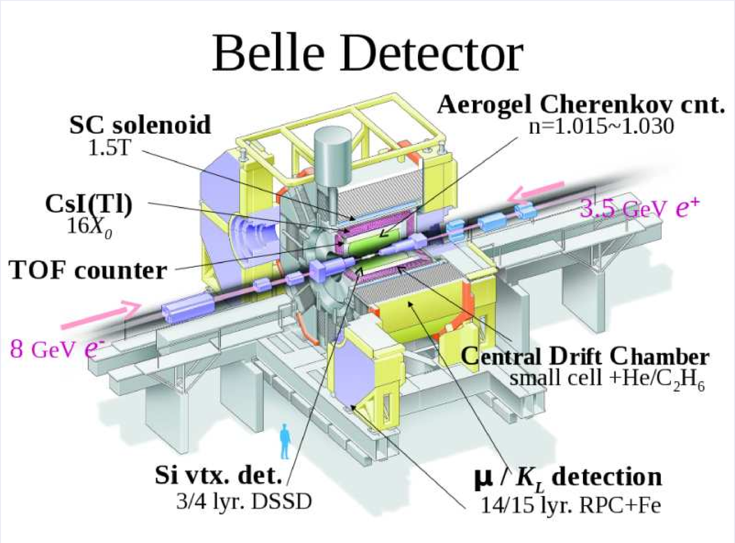
Search for New Physics in leptonic τ decays

- **Type II 2HDM:** $\eta_\mu(\tau) = \frac{m_\mu M_\tau}{2}\Big(\frac{tan^2\beta}{M^2_{H^\pm}}\Big)^2$; $\frac{\eta_\mu(\tau)}{\eta_e(\mu)} = \frac{M_\tau}{m_e} \approx 3500$
- **Tensor interaction:** $\mathcal{L} = \frac{g}{2\sqrt{2}}W^\mu\Big\{\bar{\nu}\gamma_\mu(1-\gamma^5)\tau + \frac{\kappa^W_\tau}{2m_\tau}\partial^\nu\Big(\bar{\nu}\sigma_{\mu\ nu}(1-\gamma^5)\tau\Big)\Big\}$, $-0.096 < \kappa^W_\tau < 0.037$: DELPHI Abreu EPJ C16 (2000) 229.
- **Unparticles:** Moyotl PRD 84 (2011) 073010, Choudhury PLB 658 (2008) 148.
- **Lorentz and CPTV:** Hollenberg PLB 701 (2011) 89
- **Heavy Majorana neutrino:** M. Doi et al., Prog. Theor. Phys. 118 (2007) 1069.

With $\times 300$ Belle statistics we can improve MP uncertainties by one order of magnitude

2. Belle Experiment



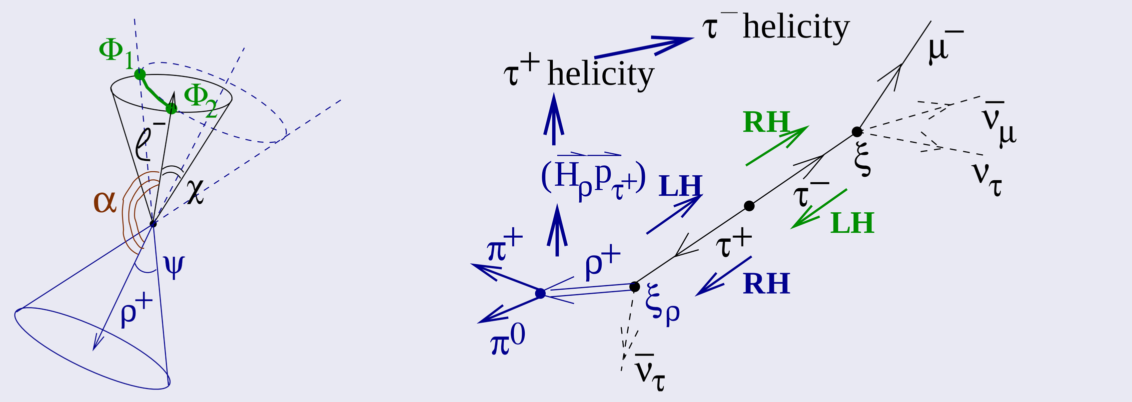


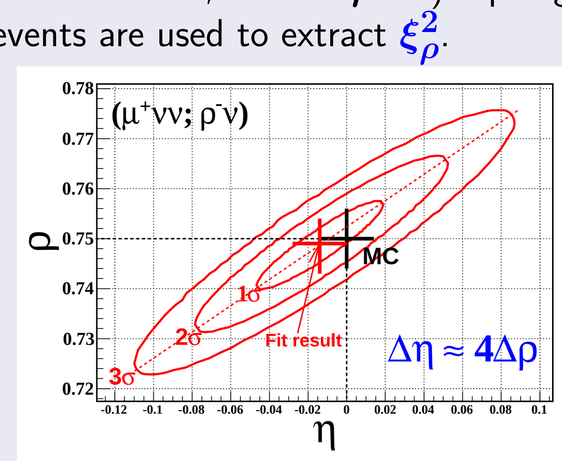
Process	σ, nb
$e^+e^- \to e^+e^-(\gamma)$ $15^o \le \theta \le 165^o$	123.5
$e^+e^- \to \mu^+\mu^-(\gamma)$	1.005
$e^+e^- \to q\bar{q}$ ($q = u, d, s, c$)	3.39
$e^+e^- \to b\bar{b}$	1.05
$e^+e^- \to e^+e^-f\bar{f}$ ($f = u, d, s, c, e, \mu, \tau$)	72.6
$e^+e^- \to \tau^+\tau^-(\gamma)$	0.919

- $E_{e^-} = 8$ GeV, $E_{e^+} = 3.5$ GeV
- Peak luminosity: $L = 2.11 \times 10^{34}$ cm^{-2}s^{-1}
- Integrated luminosity: $\int Ldt \simeq 1$ ab^{-1}, $N_{\tau\tau} \simeq 10^9$
- B-factory is also τ-factory

3. Method, study of $(\ell\nu\nu;\ \rho\nu)$ and $(\rho\nu;\ \rho\nu)$ events

Effect of τ spin-spin correlation is used to measure ξ and δ MP. Events of $(\tau^\mp \to \ell^\mp\nu\nu;\ \tau^\pm \to \rho^\pm\nu)$ topology are used to measure: ρ, η, $\xi_\rho\xi$ and $\xi_\rho\xi\delta$, while $(\tau^\mp \to \rho^\mp\nu;\ \tau^\pm \to \rho^\pm\nu)$ events are used to extract ξ^2_ρ.





$$\frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dE^*_\ell d\Omega^*_\ell d\Omega^*_\rho dm^2_{\pi\pi}d\tilde{\Omega}_\pi d\Omega_\tau} = A_0 + \rho A_1 + \eta A_2 + \xi_\rho\xi A_3 + \xi_\rho\xi\delta A_4 = \sum_{i=0}^4 A_i\Theta_i,\ \mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm^2_{\pi\pi}d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2}\frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dE^*_\ell d\Omega^*_\ell d\Omega^*_\rho dm^2_{\pi\pi}d\tilde{\Omega}_\pi d\Omega_\tau}\left|\frac{\partial(E^*_\ell,\Omega^*_\ell,\Omega^*_\rho,\Omega_\tau)}{\partial(p_\ell,\Omega_\ell,p_\rho,\Omega_\rho,\Phi_\tau)}\right|d\Phi_\tau$$

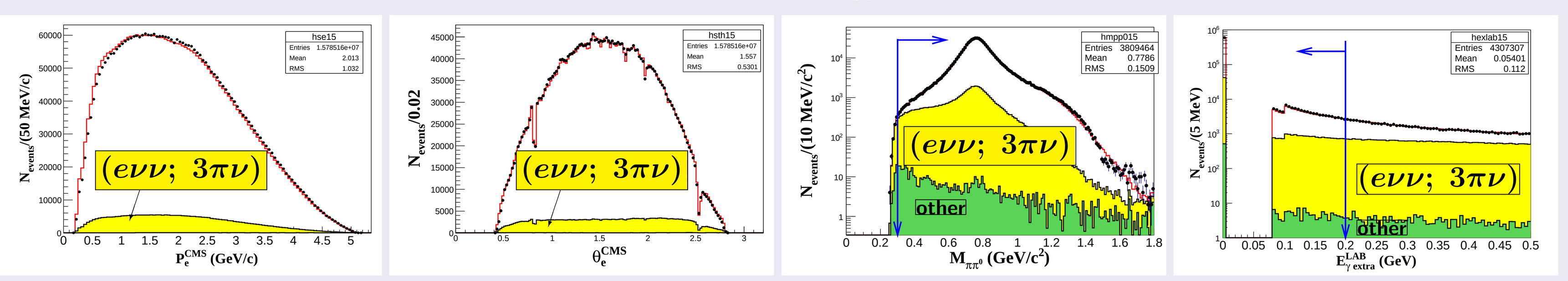
$$L = \prod_{k=1}^N\mathcal{P}^{(k)},\ \mathcal{P}^{(k)} = \mathcal{F}(\vec{z}^{(k)})/\mathcal{N}(\vec{\Theta}),\ \mathcal{F}(\vec{z}) = \sum_{i=0}^4\mathcal{F}_i(\vec{z})\Theta_i,\ \mathcal{N}(\vec{\Theta}) = \int\mathcal{F}(\vec{z})d\vec{z} = \sum_{i=0}^4\mathcal{N}_i\Theta_i,\ \mathcal{N}_i = \int\mathcal{F}_i(\vec{z})d\vec{z},\ \vec{\Theta} = (1, \rho, \eta, \xi_\rho\xi_\ell, \xi_\rho\xi_\ell\delta_\ell)$$

MP are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu;\ \rho\nu)$ events in the 9D phase space
$\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m^2_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$

4. Selection criteria

- After the standard preselections we take events with two oppositely charged tracks, one of them is identified as lepton ($eID, \mu ID > 0.9$) and the other one as pion ($PID(\pi/K) > 0.4$).
- π^0 candidate is reconstructed from the pair of gammas ($E^{\rm LAB}_\gamma > 80$ MeV) satisfying 115 MeV/$c^2 < M_{\gamma\gamma} <$ 150 MeV/c^2, $P^{\rm CMS}_{\pi^0} > 0.3$ GeV/c.
- $\cos(\vec{P}_{\rm lep}, \vec{P}_\pi) < 0$, $\cos(\vec{P}_{\rm lep}, \vec{P}_{\pi^0}) < 0$, 0.3 GeV/$c^2 < M_{\pi\pi^0} <$ 1.8 GeV/c^2.
- $E^{\rm LAB}_{\rm rest\gamma} < 0.2$ GeV

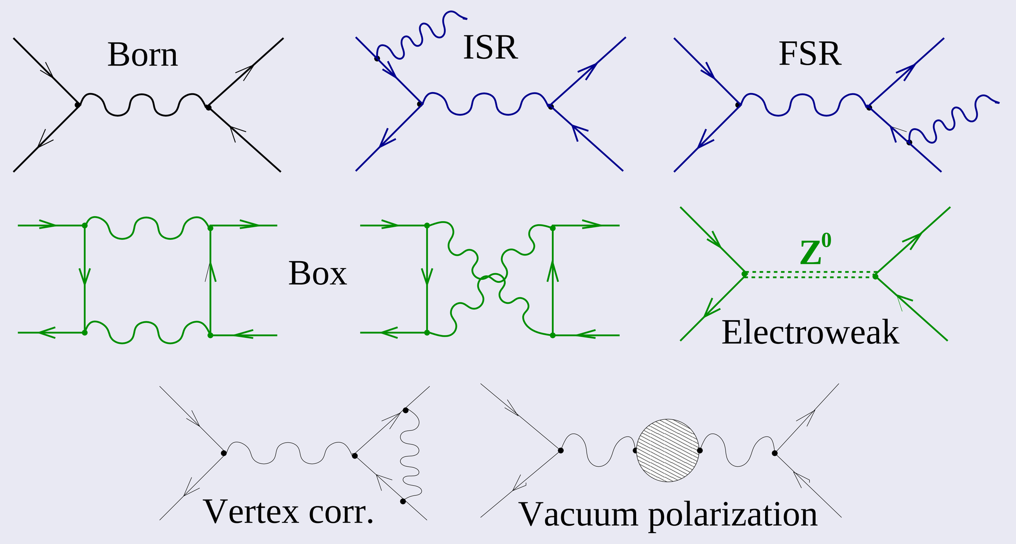
Detection efficiency $\varepsilon_{\rm det} \simeq 12\%$



5. Physical corrections, detector effects

Physical corrections:
- All $\mathcal{O}(\alpha^3)$ QED and electroweak higher order corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$ are included
- Radiative leptonic decays $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau\gamma$
- Radiative decay $\tau^- \to \pi^-\pi^0\nu_\tau\gamma$

Detector effects:
- Track momentum resolution
- γ energy and angular resolution
- Effect of external bremsstrahlung for $e - \rho$ events
- Beam energy spread
- Data/MC efficiency corrections (trigger, track rec., π^0 rec., lepton ID, πID)



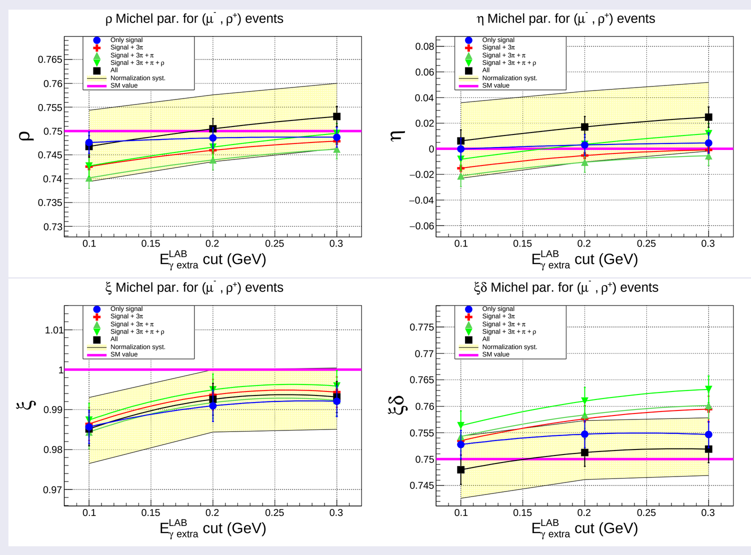
Background:
The main background comes from $(\ell\nu\nu;\ \pi2\pi^0\nu)(\sim 10\%)$, $(\pi\nu;\ \pi\pi^0\nu)(\sim 1.5\%)$ and $(\rho^+\nu;\ \rho^-\nu)(\sim 0.5\%)$ events, it is included in PDF analytically. The remaining background($\sim 2.0\%$) is taken into account using MC-based approach.
Background from the non-$\tau\tau$ events is $\lesssim 0.1\%$.

6. Description of background

$$\mathcal{P}(x) = \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\Big((1 - \sum_i\lambda_i)\frac{S(x)}{\int\frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}S(x)dx} + \lambda_{3\pi}\frac{\tilde{B}_{3\pi}(x)}{\int\frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\tilde{B}_{3\pi}(x)dx} + \lambda_\pi\frac{\tilde{B}_\pi(x)}{\int\frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\tilde{B}_\pi(x)dx} + \lambda_\rho\frac{\tilde{B}_\rho(x)}{\int\frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\tilde{B}_\rho(x)dx} + (1 - \sum_i\lambda_i)\frac{N^{\rm sel}_{\rm rest}(x)}{N^{\rm sel}_{\rm sig}(x)}S_{\rm SM}(x)\Big)$$

$$\tilde{B}_{3\pi}(x) = \int 2(1 - \varepsilon_{\pi^0}(y))\varepsilon_{\rm add}(y)B_{3\pi}(x,y)dy,\ \tilde{B}_\pi(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)}B_\pi(x),\ \tilde{B}_\rho(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)}\int(1 - \varepsilon_{\pi^0}(y))\varepsilon_{\rm add}(y)B_\rho(x,y)dy$$

- $x = (p_\ell,\ \Omega_\ell,\ p_\rho,\ \Omega_\rho,\ m^2_{\pi\pi},\ \tilde{\Omega}_\pi)$; $y = (p_{\pi^0},\ \Omega_{\pi^0})$;
- $S(x)$ - theoretical density of signal $(\ell^\mp\nu\nu,\ \rho^\pm\nu)$ events;
- $B_{3\pi}(x,y)$ - theoretical density of background $(\ell^\mp\nu\nu,\ \pi^\pm2\pi^0\nu)$ events;
- $B_\pi(x)$ - theoretical density of background $(\pi^\mp\nu,\ \rho^\pm\nu)$ events;
- $B_\rho(x)$ - theoretical density of background $(\rho^\mp\nu,\ \rho^\pm\nu)$ events;
- $\varepsilon(x)$ - detection efficiency for signal events (common multiplier);
- $\overline{\varepsilon(x)} = \epsilon_{\rm corr}(x)\varepsilon(x)$ - corrected detection efficiency;
- $\epsilon_{\rm corr}(x)$ - Data/MC efficiency corrections;
- $N^{\rm sel}_{\rm rest}(x)/N^{\rm sel}_{\rm sig}(x)$ - number of the selected (remaining/signal) MC events in the multidimensional cell around "x";
- λ_i - i-th background fraction (from MC)
- $\varepsilon_{\pi^0}(y)$ - π^0 detection efficiency (tabulated from MC);
- $\varepsilon_{\rm add}(y) = \varepsilon^{3\pi}_{\rm add}(y)/\varepsilon^{\rm sig}_{\rm add}$ - ratio of the $E^{\rm LAB}_{\gamma\rm rest}$ cut efficiencies (tabulated from MC);
- $\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)/\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)$ is tabulated from MC.



7. Validation of the fitter

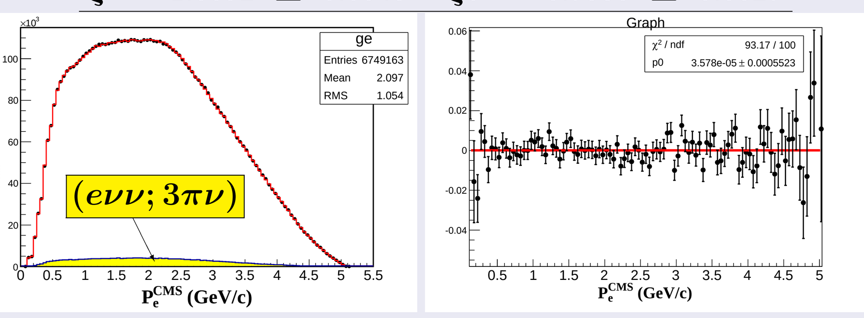
For each configuration 5M MC sample is fitted. The other statistically independent 5M MC sample was used to calculate normalization.

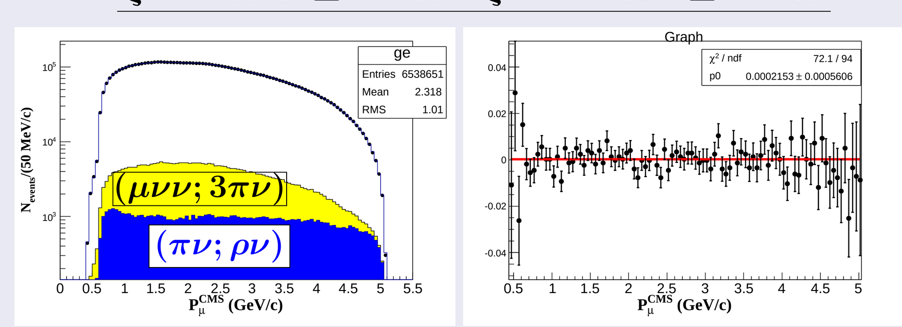
$(e^+\nu\nu;\ \rho^-\nu)$

ρ	$= 0.7517 \pm 0.0010$	η	$=$	$0\ -$	fixed
ξ	$= 1.0092 \pm 0.0043$	$\xi\delta$	$=$	0.7538 ± 0.0027	

$(\mu^+\nu\nu;\ \rho^-\nu)$

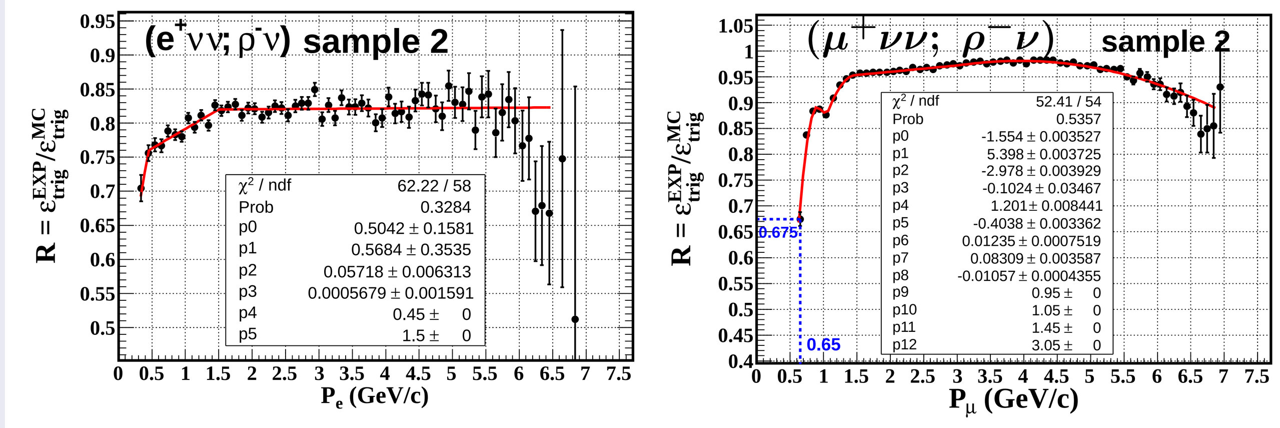
ρ	$= 0.7494 \pm 0.0027$	η	$= 0.0052 \pm 0.0101$
ξ	$= 0.9995 \pm 0.0050$	$\xi\delta$	$= 0.7519 \pm 0.0033$



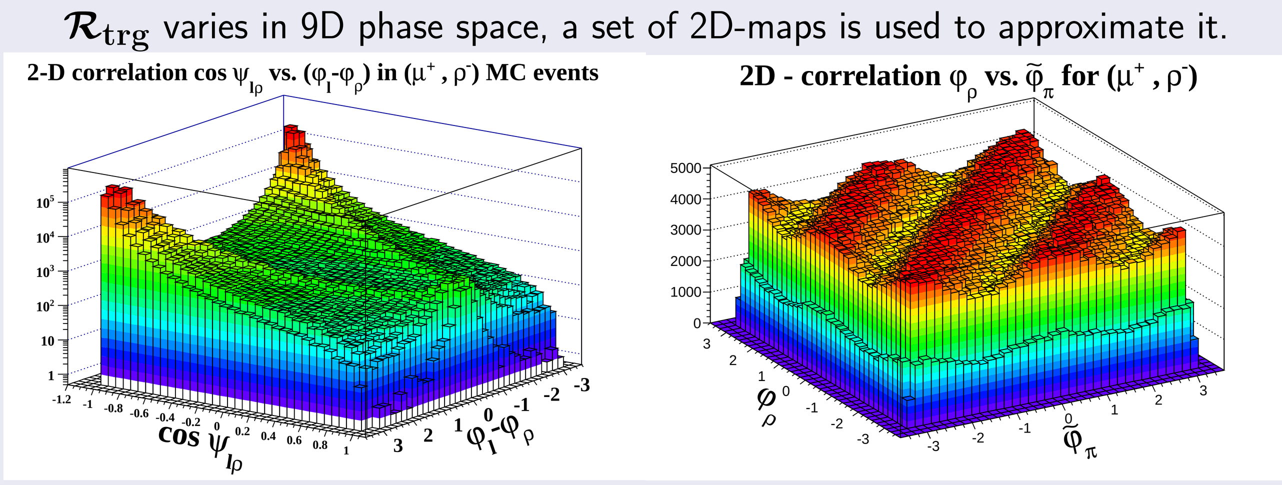


8. Data/MC efficiency corrections

We found that the **Data/MC trigger efficiency correction, $\mathcal{R}_{\rm trg}$**, is the dominant one.
Two independent subtriggers (energy trigger and track trigger) are used to evaluate it.



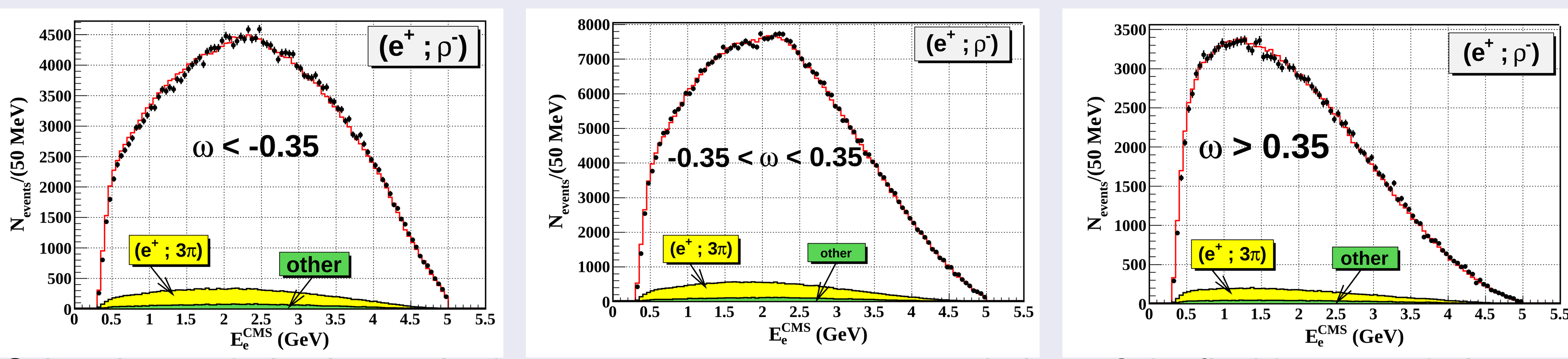
$\mathcal{R}_{\rm trg}$ varies in 9D phase space, a set of 2D-maps is used to approximate it.



The track reconstruction efficiencies are different for the energy and track triggers, the combined procedure is under development.

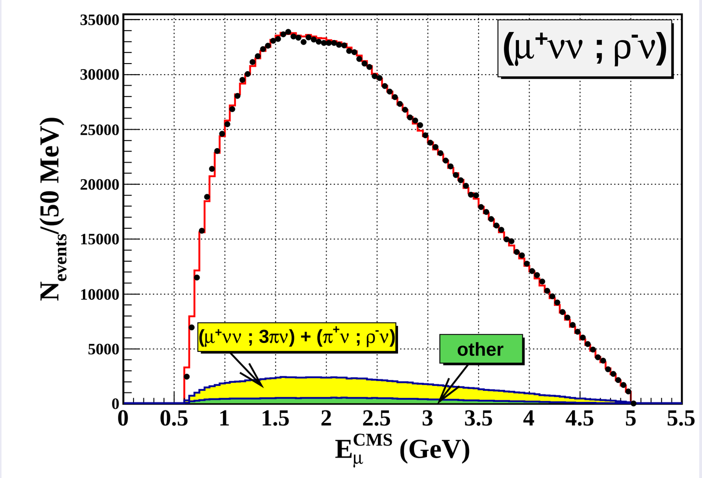
9. Fit of the data, systematic uncertainties

Helicity sensitive variable $\omega = \frac{1}{\Phi_2 - \Phi_1}\int_{\Phi_1}^{\Phi_2}(\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm})d\Phi =< (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) >_{\Phi_\tau}$



Spin-spin correlation is seen in the momentum-momentum correlations of the final lepton and pions

Source	$\Delta(\rho)$, %	$\Delta(\eta)$, %	$\Delta(\xi_\rho\xi)$, %	$\Delta(\xi_\rho\xi\delta)$, %
Physical corrections				
ISR+$\mathcal{O}(\alpha^3)$	0.10	0.30	0.20	0.15
$\tau \to \ell\nu\nu\gamma$	0.03	0.10	0.09	0.08
$\tau \to \rho\nu\gamma$	0.06	0.16	0.11	0.02
Background	0.20	0.60	0.20	0.20
Apparatus corrections				
Resolution $\oplus$ brems.	0.10	0.33	0.11	0.19
$\sigma(E_{\rm beam})$	0.07	0.25	0.03	0.15
Normalization				
$\Delta\mathcal{N}$	0.11	0.50	0.17	0.13
without Data/MC corr.	**0.29**	**0.95**	**0.38**	**0.38**
trigger eff. corr.	~ 1	~ 2	~ 3	~ 3



We are working on the Data/MC efficiency corrections (trigger, lepton ID, track rec., π^0 rec.).

10. Summary

- The procedure to measure 4 Michel parameters (MP) $(\rho,\ \eta,\ \xi,\ \xi\delta)$ in leptonic τ decays at B factory has been developed and tested. It is based on the analysis of the $(\ell^\mp\nu\nu;\ \rho^\pm\nu)$, $\ell = e,\ \mu$ events and utilizes spin-spin correlation of tau leptons.
- We confirmed that with the whole Belle data sample the statistical accuracy of MP is by one order of magnitude better than in the previous best measurements (CLEO, ALEPH).
- The main background components ($(\ell\nu\nu;\ \pi2\pi^0\nu)$, $(\pi\nu;\ \rho\nu)$, $(\rho\nu;\ \rho\nu)$) are described analytically in the fitter, the remaining background (with the fraction of about 2.0%) is described with help of the MC-based method. We reached acceptable description of the backgrounds in the PDF.
- Various Data/MC efficiency corrections provide the dominant contribution to the systemtic uncertainties of MP. **The largest contribution comes from the trigger efficiency correction (1–3)%.** We are working to improve this uncertainty.

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