Super Charm-Tau factory in Russia

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4. Future studies of hadronic τ decays
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Introduction: τ physics

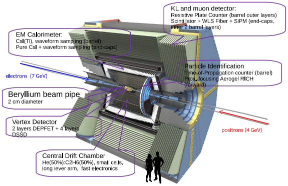
- In the SM τ decays due to the charged weak interaction described by the exchange of $W^{\pm}$ with a pure vector coupling to only left-handed fermions. There are two main classes of tau decays:
  - Decays with leptons, like: $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$, $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma$, $\tau^- \to \ell^- \ell'^+ \ell'^- \bar{\nu}_\ell \nu_\tau$; $\ell, \ell' = e, \mu$. They provide very clean laboratory to probe electroweak couplings, which is complementary/competitive to precision studies with muon (in experiments with muon beam). Plenty of New Physics models can be tested/constrained in the precision studies of the dynamics of decays with leptons.
  - Hadronic decays of τ offer unique tools for the precision study of low energy QCD.
- The world largest statistics of τ leptons collected by e^+e^- B factories (Belle and *BABAR*) opens new era in the precision tests of the Standard Model (SM).
  Still, many interesting and important studies with τ lepton will be done using Belle/*BABAR* statistics.
- **Belle II and, possibly, Super Charm-Tau Factory (SCTF) are the next players in this area.**

Belle II with unpolarized beams

Planned integrated luminosity is 50 ab^{-1}

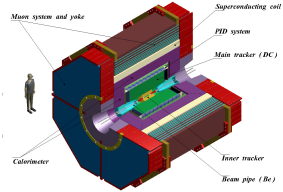
$$\sigma(b\bar{b}) = 1.05\ \text{nb} \quad N_{b\bar{b}} = 53 \times 10^9$$
$$\sigma(c\bar{c}) = 1.30\ \text{nb} \quad N_{c\bar{c}} = 65 \times 10^9$$
$$\boldsymbol{\sigma(\tau\tau) = 0.92\ \text{nb} \quad N_{\tau\tau} = 46 \times 10^9}$$

Belle II detector



SCTF with polarized e^- beam

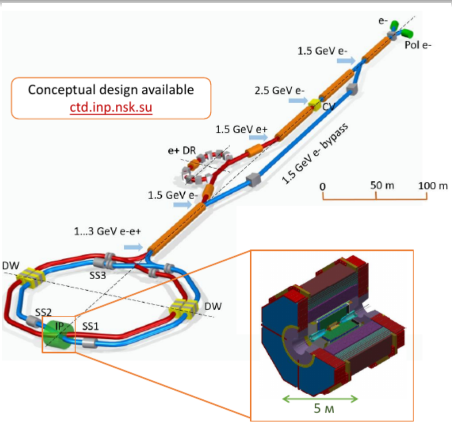
In five c.m.s. energy points
($2E = 3.554,\ 3.686,\ 3.770,\ 4.170,\ 4.650$ GeV)
it is planned to accumulate 7 ab^{-1}, which
corresponds to $\boldsymbol{N_{\tau\tau} = 21 \times 10^9}$

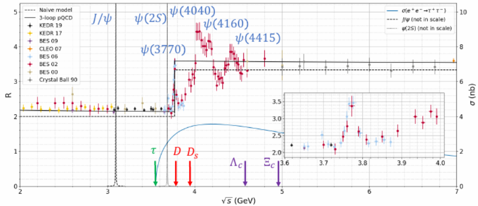


The polarized e^- beam results in the nonzero average polarization of single τ, which provide advantages in some studies with τ lepton, where the τ spin-dependent effects dominate

The SCTF in Russia

- **Electron-positron collider**
  - Center-of-mass energy range $\sqrt{s} = 2E_{\rm beam} = (3 - 7)$ GeV
  - Luminosity $L = 10^{35}\ 1/\mathrm{cm}^2/\mathrm{s}$ at $2E = 4$ GeV
  - Longitudinal e^- beam polarization ($\mathcal{P}_e > 0.5$)
- **Universal particle detector**
  - Advanced inner and main tracker
  - Electromagnetic calorimeter
  - PID system
  - Magnet and Muon system
  - **DAQ with the trigger rate up to 300 kHz at the J/Ψ peak**





$2E$, GeV	N/year
$3.55 \div 4.3$	$2 \cdot 10^9\ \tau^+\tau^-$
3.1	$10^{12}\ J/\Psi$
3.69	$10^{11}\ \psi(2S)$
3.77	$10^9\ D\bar{D}$
4.17	$10^8\ D_s\bar{D}_s$
4.65	$10^8\ \Lambda_c^+\Lambda_c^-$

SCTF physics program

- J/Ψ **and** $\psi(2S)$ **factory**
  - Big samples of light charmonia (η_c, h_c, χ_{cJ}), precise measurement of their parameters, study of (radiative) transitions between them.
  - J/Ψ hadronic decays, observation of weak decays ($J/\Psi \to \phi\phi$)
  - Search for LFV J/Ψ decays, c-quark EDM ($J/\Psi \to \phi\phi\gamma$)
- **Study of exotic charmonia**
- **Clean** $D\bar{D}$ **production**, lower multiplicity and background, precise measurement of the absolute branching fractions.
- **Coherent production of** $D^0\bar{D}^0$
  - mixing of D mesons
  - measurement of various strong phases (needed also by Belle II)
  - CPV in D decays
- **Study of charmed baryons**
- **Study of** $e^+e^- \to \mathrm{hadrons}$
- **Two-photon physics**
- τ **factory**
  - Near threshold kinematics ($\tau^- \to \pi^-/K^-\nu_\tau(\gamma)$), suppression of ISR background (essential in the searches for LFV $\tau \to \mu\gamma$).
  - e^- beam polarization $\to$ polarized single $\tau \to$ tau spin-dependent effects: (Michel par., EDM, CPV in hadronic decays, etc)

Michel parameters in τ decays

In the SM, charged weak interaction is described by the exchange of $W^{\pm}$ with a pure vector coupling to only left-handed fermions ("V-A" Lorentz structure). Deviations from "V-A" indicate New Physics. $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ ($\ell = e, \mu$) decays provide clean laboratory to probe electroweak couplings.

The most general, Lorentz invariant four-lepton interaction matrix element:

$$\mathcal{M} = \frac{4G}{\sqrt{2}} \sum_{\substack{N=S,V,T \\ i,j=L,R}} g_{ij}^N \left[\bar{u}_i(l^-)\Gamma^N v_n(\bar{\nu}_l)\right] \left[\bar{u}_m(\nu_\tau)\Gamma_N u_j(\tau^-)\right],$$

$$\Gamma^S = 1,\ \Gamma^V = \gamma^\mu,\ \Gamma^T = \frac{i}{2\sqrt{2}}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

Ten couplings g_{ij}^N, in the SM the only non-zero constant is $g_{LL}^V = 1$

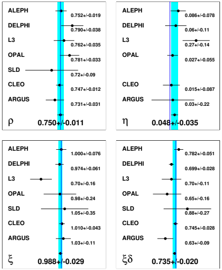
Four bilinear combinations of g_{ij}^N, which are called as Michel parameters (MP): $\boldsymbol{\rho}$, $\boldsymbol{\eta}$, $\boldsymbol{\xi}$ and $\boldsymbol{\delta}$ appear in the energy spectrum of the outgoing lepton:

$$\frac{d\Gamma(\tau^\mp)}{d\Omega dx} = \frac{4G_F^2 M_\tau E_{\rm max}^4}{(2\pi)^4}\sqrt{x^2 - x_0^2}\Bigg(x(1-x) + \frac{2}{9}\boldsymbol{\rho}(4x^2 - 3x - x_0^2) + \boldsymbol{\eta} x_0(1-x)$$

$$\mp \frac{1}{3} P_\tau cos\theta_\ell \boldsymbol{\xi} \sqrt{x^2 - x_0^2}\left[1 - x + \frac{2}{3}\boldsymbol{\delta}\left(4x - 4 + \sqrt{1 - x_0^2}\right)\right]\Bigg),\ x = \frac{E_\ell}{E_{\rm max}},\ x_0 = \frac{m_\ell}{E_{\rm max}}$$

In the SM: $\boldsymbol{\rho} = \frac{3}{4}$, $\boldsymbol{\eta} = 0$, $\boldsymbol{\xi} = 1$, $\boldsymbol{\delta} = \frac{3}{4}$

Michel par.	Measured value	Experiment	SM value
ρ (e or μ)	$0.747 \pm 0.010 \pm 0.006$ 1.2%	CLEO-97	3/4
η (e or μ)	$0.012 \pm 0.026 \pm 0.004$ 2.6%	ALEPH-01	0
ξ (e or μ)	$1.007 \pm 0.040 \pm 0.015$ 4.3%	CLEO-97	1
$\xi\delta$ (e or μ)	$0.745 \pm 0.026 \pm 0.009$ 2.8%	CLEO-97	3/4
$\xi_{\rm h}$ (all hadr.)	$0.992 \pm 0.007 \pm 0.008$ 1.1%	ALEPH-01	1



In BSM models the couplings to τ are expected to be enhanced in comparison with μ.

- **Type II 2HDM:** $\eta_\mu(\tau) = \frac{m_\mu M_\tau}{2}\left(\frac{\tan^2\beta}{M^2_{H\pm}}\right)^2$; $\frac{\eta_\mu(\tau)}{\eta_e(\mu)} = \frac{M_\tau}{m_e} \approx 3500$
- **Tensor interaction:** $\mathcal{L} = \frac{g}{2\sqrt{2}} W^\mu \left\{ \bar\nu\gamma_\mu(1-\gamma^5)\tau + \frac{\kappa^W_\tau}{2m_\tau}\partial^\nu\left(\bar\nu\sigma_{\mu\ nu}(1-\gamma^5)\tau\right)\right\}$,
  $-0.096 < \kappa^W_\tau < 0.037$: DELPHI Abreu EPJ C16 (2000) 229.
- **Unparticles:** Moyotl PRD 84 (2011) 073010, Choudhury PLB 658 (2008) 148.
- **Lorentz and CPTV:** Hollenberg PLB **701** (2011) 89
- **Heavy Majorana neutrino:** M. Doi *et al.*, Prog. Theor. Phys. **118** (2007) 1069.

Effect of the e^- beam polarization at the SCTF

At the SCTF with polarized electron beam the average polarization of single τ is nonzero, hence the differential decay probability will contain both, τ spin-dependent and spin-independent parts.

$$\frac{d\sigma(\vec{\zeta}^-,\vec{\zeta}^+)}{d\Omega_\tau} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i^-\zeta_j^+ + \mathcal{P}_e(F_i^-\zeta_i^- + F_j^+\zeta_j^+))$$

$$D_0 = 1+\cos^2\theta + \frac{1}{\gamma_\tau^2}\sin^2\theta,\ \mathcal{P}_e = \frac{N_e(+)-N_e(-)}{N_e(+)+N_e(-)}$$

$$D_{ij} = \begin{pmatrix} (1+\frac{1}{\gamma_\tau^2})\sin^2\theta & 0 & \frac{1}{\gamma_\tau}\sin 2\theta \\ 0 & -\beta_\tau^2\sin^2\theta & 0 \\ \frac{1}{\gamma_\tau}\sin 2\theta & 0 & 1+\cos^2\theta - \frac{1}{\gamma_\tau^2}\sin^2\theta \end{pmatrix}$$

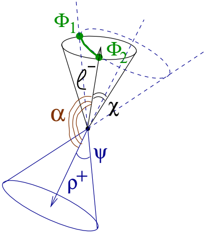
Single τ studies at the SCTF:

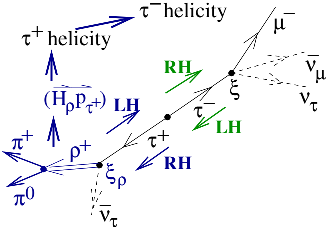
$$\frac{d\sigma(\vec{\zeta}^-)}{d\Omega_\tau} = \frac{\alpha^2}{32E_\tau^2}\beta_\tau(D_0 + \mathcal{P}_e F_i^-\zeta_i^-)$$

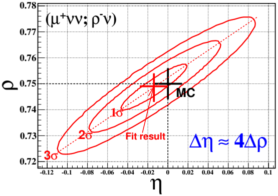
As a result, there are two methods to measure MP:

- **(I) Unbinned fit of the $(\ell,\ \rho)$ events in 9D phase space (spin-spin correlations + polarized e^- beam)**
- **(II) Unbinned fit of the $(\ell,\ \mathrm{all})$ events in 3D lepton phase space (only polarized e^- beam)**

Effect of τ spin-spin correlation is used to measure ξ and δ MP.
Events of the $(\tau^\mp \to \ell^\mp \nu\nu;\ \tau^\pm \to \rho^\pm \nu)$ topology are used to measure: $\rho,\ \eta,\ \xi_\rho \xi$ and $\xi_\rho \xi \delta$, while $(\tau^\mp \to \rho^\mp \nu;\ \tau^\pm \to \rho^\pm \nu)$ events are used to extract ξ_ρ^2.







$$\frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dE^*_\ell\, d\Omega^*_\ell\, d\Omega^*_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi\, d\Omega_\tau} = A_0 + \rho A_1 + \eta A_2 + \xi_\rho \xi A_3 + \xi_\rho \xi \delta A_4 = \sum_{i=0}^{4} A_i \Theta_i$$

$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dp_\ell\, d\Omega_\ell\, dp_\rho\, d\Omega_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dE^*_\ell\, d\Omega^*_\ell\, d\Omega^*_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi\, d\Omega_\tau} \left| \frac{\partial(E^*_\ell, \Omega^*_\ell, \Omega^*_\rho, \Omega_\tau)}{\partial(p_\ell, \Omega_\ell, p_\rho, \Omega_\rho, \Phi_\tau)} \right| d\Phi_\tau$$

$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)},\ \mathcal{P}^{(k)} = \mathcal{F}(\vec{z}^{(k)})/\mathcal{N}(\vec{\Theta}),\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z},\ \vec{\Theta} = (1, \rho, \eta, \xi_\rho\xi_\ell, \xi_\rho\xi_\ell\delta_\ell)$$

$$\boldsymbol{\mathcal{P}}_{\mathbf{total}} = (1 - \sum_{i=1}^{4}\lambda_i)\boldsymbol{\mathcal{P}}^{\ell-\rho}_{signal} + \lambda_1 \mathcal{P}^{\ell-3\pi}_{bg} + \lambda_2 \mathcal{P}^{\pi-\rho}_{bg} + \lambda_3 \mathcal{P}^{\rho-\rho}_{bg} + \lambda_4 \boldsymbol{\mathcal{P}}^{other}_{bg}\ \mathbf{(MC)}$$

MP are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu;\ \rho\nu)$ events in the 9D phase space $\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m^2_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$ in CMS.

Analysis of $(\ell,\ \text{all})$ events in 3D

$$\frac{d\sigma(\vec{\zeta})}{d\Omega_\tau} = \frac{\alpha^2}{32E_\tau^2}\beta_\tau(D_0 + \mathcal{P}_e F_i\zeta_i)$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*) \to \ell^\mp\nu\nu)}{dx^* d\Omega_\ell^*} = \kappa_\ell(A(x^*) \mp \xi_\ell \vec{n}_\ell^* \vec{\zeta}^* B(x^*)),\ x^* = E_\ell^*/E_{\ell max}^*$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*),\ B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\sigma(\ell^\mp)}{dE_\ell^* d\Omega_\ell^* d\Omega_\tau} = \kappa_\ell \frac{\alpha^2\beta_\tau}{32E_\tau^2}(D_0 A(E_\ell^*) \mp \mathcal{P}_e\xi_\ell F_i n_{\ell i}^* B(E_\ell^*))$$

$$\frac{d\sigma(\ell^\mp)}{dp_\ell d\Omega_\ell} = \int\limits_{\Omega_\tau-\text{sector}} \frac{d\sigma(\ell^\mp)}{dE_\ell^* d\Omega_\ell^* d\Omega_\tau}\left|\frac{\partial(E_\ell^*, \Omega_\ell^*)}{\partial(p_\ell, \Omega_\ell)}\right| d\Omega_\tau$$

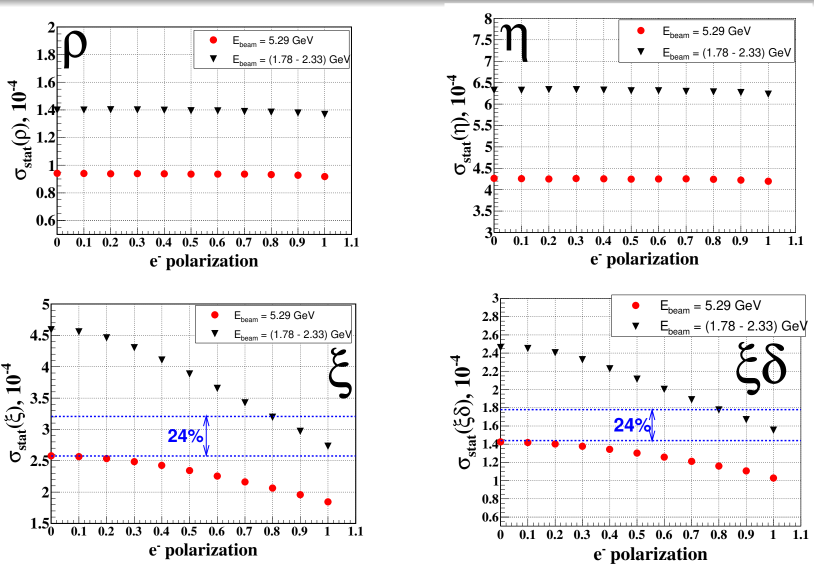
Ω_τ-sector is determined by the kinematical constraint $m_{\nu\nu} > 0$

- All Michel parameters (ρ, η, $\mathcal{P}_e\xi$, $\mathcal{P}_e\xi\delta$) are measured in the unbinned maximum likelihood fit of ($\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau;\ \tau^+ \to \text{all}$) events in the **3D** phase space.
- The reduced 3D phase space allows one to tabulate various EXP/MC corrections to the detection efficiency more precisely.
- **The crucial point in this method is to have high-efficiency 1-track trigger.**

Toy MC studies of the effect of polarized e^- beam

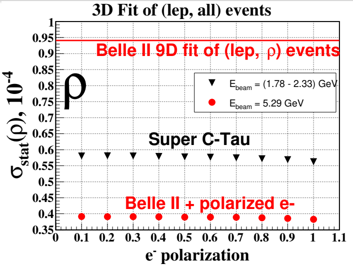
- 66 10M $(\mu,\ \rho)$ samples, at 6 center-of-mass (c.m.s.) energies (according to Table 1.1 in SCTF CDR part I) : $2E = 3.554$ GeV ($\tau^+\tau^-$ production threshold), $2E = 3.686$ GeV ($\psi(2S)$), $2E = 3.770$ GeV ($\psi(3770)$), $2E = 4.170$ GeV ($\psi(4160)$), $2E = 4.650$ GeV (maximum of the $\sigma(e^+e^- \to \Lambda_c^+\Lambda_c^-)$), $2E = 10.58$ GeV (Belle II), for 11 values of e^- beam polarization: 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, were generated for the calculation of the normalizations. 66 statistically independent 1M samples at the same energies and polarizations were generated for the fit.
- To evaluate MP sensitivities (rescaling the sensitivities obtained in the fits of 1M samples) we took the detection efficiency of $(\mu,\ \rho)$ events to be 20% (to be compared with 12% efficiency obtained at Belle, where the π^0 rec. efficiency is only 40%). The detection efficiency of (μ, all) events was taken to be 30%.
- To measure ρ, ξ and $\xi\delta$ MP, samples with $\ell = e,\ \mu$ were taken into account, while η MP is measured in samples with $\ell = \mu$ only.

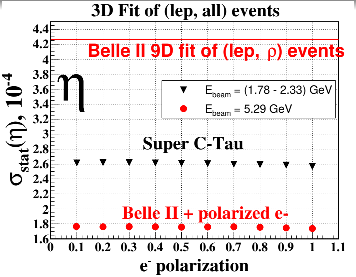
Fit of (ℓ, ρ) in 9D at Belle II and SCTF

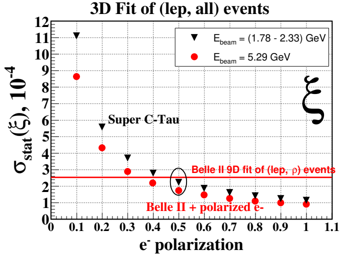


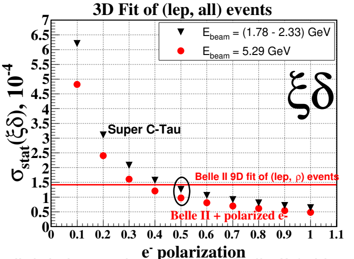
Sensitivities to Michel par. at Belle II (with unpolarized e^- beam) are slightly better (by a factor of 1.2–1.5) than those at the SCTF.

Fit of $(\ell,\ \mathrm{all})$ in 3D at Belle II and SCTF









The sensitivities to all Michel par. at the SCTF become slightly better than those at Belle II (with unpolarized e^- beam) for $\mathcal{P}_e > 0.5$.

Expected MP stat. uncertainties are $\sim 10^{-4}$, to reach the same level systematic uncertainty, the NNLO corrections ($\mathcal{O}(\alpha^4)$) to the differential $e^+e^- \to \tau^+\tau^-$ cross section are mandatory.

Tau decays with leptons at Belle II and SCTF

- Precise study of the radiative leptonic decays $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma$.
  Measured branching ratios (*BABAR*) agree with the LO predictions however the LO+NLO theoretical prediction for the $\tau \to e\nu\nu\gamma$ differs from the experimental result by **3.5σ**:
  J. P. Lees *et al.*, Phys. Rev. D **91** (2015) 051103.
  M. Fael, L. Mercolli and M. Passera, JHEP **1507** (2015) 153.
  It is important to embed NLO corrections to TAUOLA to calculate the detection efficiency correctly.
- Precise study of the doubly radiative decay $\tau^- \to \ell^- \nu\nu\gamma\gamma$.
- Measurement of Michel parameters in the radiative leptonic τ decays:
  A. B. Arbuzov and T. V. Kopylova, JHEP **1609** (2016) 109. ($m_\ell \neq 0$)
  First experimental result from Belle:
  N. Shimizu et al., PTEP **2018** (2018) no.2, 023C01.
  Statistical improvement at Belle II and SCTF. At the SCTF the polarized τ allows one further to inrease sensitivity to $\xi\kappa$ parameter.
- Decay-in-flight muon polarization measurement ($\tau \to \mu\nu\nu$, $\mu \to e\nu\nu$) to measure ξ' Michel parameter:
  D. Bodrov, Physics of Atomic Nuclei **84**(2) (2021) 212.
  This analysis is now ongoing at Belle, it allows one to measure $\xi\kappa$ parameter with much better sensitivity than in the radiative leptonic decay. At Belle II and SCTF with the improved track rec. algorithm in the drift chamber of the detector and higher statistics it will be possible to improve the accuracy.
- Precise study of the five-body leptonic τ decays $\tau \to \ell\ell'^+\ell'^-\nu\nu$:
  A. Flores-Tlalpa, G. Lopez Castro and P. Roig, JHEP **1604** (2016) 185.
  While $\tau^- \to e^-e^+e^-2\nu$ and $\tau^- \to \mu^-e^+e^-2\nu$ modes can be studies already at B factories, the $\tau^- \to e^-\mu^+\mu^-2\nu$ and $\tau^- \to \mu^-\mu^+\mu^-2\nu$ modes can be discovered at Belle II and SCTF.
  Possibility to search for T-odd correlations of the form $\vec{s}_\tau(\vec{p}_i \times \vec{p}_j)$.

Test of LFU at Belle II and SCTF

B. Aubert et al., Phys. Rev. Lett. **105** (2010) 051602.

BABAR collaboration performed the test of lepton flavor universality (LFU) in leptonic and simple hadronic τ decays

$\tau \to P\nu$ and $P \to \mu\nu$, $(P = \pi,\ K)$

$$(g_\tau/g_\mu)_P^2 = \frac{\mathcal{B}(\tau \to P\nu_\tau)}{\mathcal{B}(P \to \mu\nu_\mu)} \frac{2m_P m_\mu^2 \tau_P}{(1+\delta_P)m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_P^2}{1 - m_P^2/m_\tau^2}\right)^2,$$

$$\delta_\pi = (0.16 \pm 0.14)\%, \quad \delta_K = (0.90 \pm 0.22)\%$$

$$\boldsymbol{(g_\tau/g_\mu)_\pi = 0.9856 \pm 0.0057,\ (g_\tau/g_\mu)_K = 0.9827 \pm 0.0086}$$

$$\boldsymbol{(g_\tau/g_\mu)_{\pi\&K} = 0.9850 \pm 0.0054} \text{ (}2.8\sigma\text{ away from SM)}$$

$$\boldsymbol{(g_\tau/g_\mu)_{\tau+\pi+K} = 0.9999 \pm 0.0014} \text{ (HFLAV2018)}$$

- The correct determination of δ_P is not enough. The proper description of $\tau \to P\nu\gamma$ and virt. corrections to $\tau \to P\nu$ are important to embed in TAUOLA for the reliable estimation of the detection efficiency. As a result, to improve LFU test at Belle II and SCTF, a high statistics study of $\tau \to P\nu\gamma$ is needed:
  Z. H. Guo and P. Roig, Phys. Rev. D **82** (2010) 113016.
  The structure of $W - P - \gamma$ vertex can be studied also in the $\tau^- \to P^-\nu_\tau\ell^+\ell^-$ decays:
  A. Guevara, G. Lopez Castro and P. Roig, Phys. Rev. D **88** (2013) no.3, 033007.
  The first experimental study of $\tau^- \to \pi^-\nu_\tau\ell^+\ell^-$ at Belle:
  Y. Jin et al., Phys. Rev. D **100** (2019) no.7, 071101.
  At Belle II and SCTF the $\tau \to P\nu\gamma$ and $\tau^- \to P^-\nu_\tau\ell^+\ell^-$ decays will be studied with better accuracy, the decay mechanism will be established and LFU test can be improved.
- At the SCTF at the $\tau^+\tau^-$ production threshold (τ is at rest) the pion/kaon from $\tau \to \pi/K\nu$ can be easily separated via their momentum difference (of about 63 MeV). The radiative decay $\tau \to P\nu\gamma$ is easier to study near the $\tau^+\tau^-$ production threshold due to the lack of ISR background.

CPV in hadronic τ decays at B factories

- CPV has not been observed in lepton decays
- It is strongly suppressed in the SM ($A_{\rm SM}^{\rm CP} \lesssim 10^{-12}$) and observation of large CPV in lepton sector would be clean sign of New Physics
- τ lepton provides unique possibility to search for CPV effects, as it is the only lepton decaying to hadrons, so that the associated strong phases allows us to visualize CPV in hadronic τ decays.

I. CPV in $\tau^- \to \pi^- K_S(\geq 0\pi^0)\nu_\tau$ at BaBar (Phys. Rev. D 85, 031102 (2012))

Data sample of $\int Ldt = 476$ fb^{-1} was analyzed

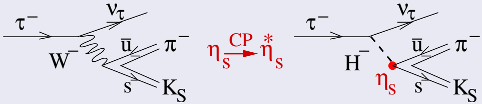
$$A_{\rm CP} = \frac{\Gamma(\tau^+ \to \pi^+ K_S^0(\geq 0\pi^0)\bar{\nu}_\tau) - \Gamma(\tau^- \to \pi^- K_S^0(\geq 0\pi^0)\nu_\tau)}{\Gamma(\tau^+ \to \pi^+ K_S^0(\geq 0\pi^0)\bar{\nu}_\tau) + \Gamma(\tau^- \to \pi^- K_S^0(\geq 0\pi^0)\nu_\tau)} = (-0.36 \pm 0.23 \pm 0.11)\%$$

2.8σ deviation from the SM expectation: $A_{\rm CP}^{K^0} = (+0.36 \pm 0.01)\%$

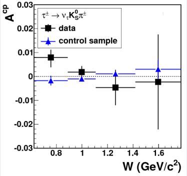
II. CPV in $\tau^- \to K_S^0\pi^-\nu_\tau$ at Belle (Phys. Rev. Lett. 107, 131801 (2011)) $\int$Ldt=699 fb^{-1}

Angular distributions were analyzed, $A_{\rm CP}(W = M_{K_S\pi})$ was measured ($d\omega = d\cos\beta d\cos\theta$):

$$A_{\rm CP}(W) = \frac{\int \cos\beta\cos\psi\left(\frac{d\Gamma_{\tau^-}}{d\omega} - \frac{d\Gamma_{\tau^+}}{d\omega}\right)d\omega}{\frac{1}{2}\int\left(\frac{d\Gamma_{\tau^-}}{d\omega} + \frac{d\Gamma_{\tau^+}}{d\omega}\right)d\omega} \simeq \langle\cos\beta\cos\psi\rangle_{\tau^-} - \langle\cos\beta\cos\psi\rangle_{\tau^+}$$



$|Im(\eta_S)| < 0.026$



CPV in $\tau^\pm \to K_S \pi^\pm \nu_\tau$ with polarized τ lepton

At the center-of-mass energies close to the $\tau^+\tau^-$ production threshold the τ lepton is produced with the polarization

$|\vec{P}_\tau| = P_e \frac{2E_{\rm beam}\sqrt{p^2_{\rm beam}\cos^2\theta + M^2_\tau}}{E^2_{\rm beam} + M^2_\tau + p^2_{\rm beam}\cos^2\theta} \approx P_e$ along electron beam polarization

$((P_\tau)_Z = P_e \frac{E_{\rm beam}\cos^2\theta + M_\tau \sin^2\theta}{\sqrt{p^2_{\rm beam}\cos^2\theta + M^2_\tau}} \approx P_e)$.

In case of New Physics contribution, the amplitudes for the decays $\tau^- \to (K\pi)^-\nu_\tau$ and $\tau^+ \to (K\pi)^+\bar{\nu}_\tau$ are:

$$\mathcal{A} = A_1 + A_2 e^{i\phi} e^{i\delta}, \ \ \overline{\mathcal{A}} = A_1 + A_2 e^{-i\phi} e^{i\delta}$$

where ϕ and δ are relative weak (CP-odd) and strong (CP-even) phases. CPV is studied comparing $|\mathcal{A}|^2$ and $|\overline{\mathcal{A}}|^2$, there are three possibilities to construct CPV asymmetry:

- decay rate asymmetry $\sim \sin\delta \sin\phi$
- weighted rate asymmetry $\sim \sin\delta \sin\phi$
- asymmetry based on $\vec{P}_\tau(\vec{p}_K \times \vec{p}_\pi)$ triple product $\sim \cos\delta \sin\phi$

At the Super Charm-Tau factory, with nonzero single τ polarization, nonzero strong-phase difference, δ, is not needed to measure CPV.

Future precise studies of hadronic τ decays

Analysis of the $(\tau^\mp \to (K\pi)^\mp\nu\,;\ \tau^\pm \to \rho^\pm\nu)$ events, search for CPV in $\tau^- \to (K\pi)^-\nu_\tau$.
CPV parameter $\eta_{\rm CP}$ is extracted in the simultaneous unbinned maximum likelihood fit of the $((K\pi)^-, \rho^+)$ and $((K\pi)^+, \rho^-)$ events in the 12D phase space.

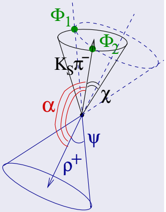
$$\frac{d\sigma(\vec{\zeta}^*, \vec{\zeta'}^*)}{d\Omega_\tau} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i^*\zeta_j'^*),\quad \frac{d\Gamma(\tau^\pm(\vec{\zeta'}^*) \to \rho^\pm\nu)}{dm^2_{\pi\pi}\,d\Omega^*_\rho\,d\tilde{\Omega}_\pi} = A' \mp \vec{B'}\vec{\zeta'}^*$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*) \to (K\pi)^\mp\nu)}{dm^2_{K\pi}\,d\Omega^*_{K\pi}\,d\tilde{\Omega}_\pi} = \begin{matrix}(A_0 + \eta_{CP}A_1) + (\vec{B}_0 + \eta_{CP}\vec{B}_1)\vec{\zeta}^* \\ (A_0 + \eta^*_{CP}A_1) - (\vec{B}_0 + \eta^*_{CP}\vec{B}_1)\vec{\zeta}^*\end{matrix}$$

$$\frac{d\sigma((K\pi)^\mp, \rho^\pm)}{dm^2_{K\pi}\,d\Omega^*_{K\pi}\,d\tilde{\Omega}_\pi\,dm^2_{\pi\pi}\,d\Omega^*_\rho\,d\tilde{\Omega}_\pi\,d\Omega_\tau} = \frac{\alpha^2\beta_\tau}{64E_\tau^2}\begin{pmatrix}\mathcal{F} + \eta_{CP}\mathcal{G} \\ \mathcal{F} + \eta^*_{CP}\mathcal{G}\end{pmatrix}$$

$$\mathcal{F} = D_0A_0A' - D_{ij}B_{0i}B'_j,\ \mathcal{G} = D_0A_1A' - D_{ij}B_{1i}B'_j$$

$$\frac{d\sigma((K\pi)^\mp, \rho^\pm)}{dp_{K\pi}\,d\Omega_{K\pi}\,dm^2_{K\pi}\,d\tilde{\Omega}_\pi\,dp_\rho\,d\Omega_\rho\,dm^2_{\pi\pi}\,d\tilde{\Omega}_\pi} = \sum_{\Phi_1,\Phi_2}\frac{d\sigma((K\pi)^\mp, \rho^\pm)}{dm^2_{K\pi}\,d\Omega^*_{K\pi}\,d\tilde{\Omega}_\pi\,dm^2_{\pi\pi}\,d\Omega^*_\rho\,d\tilde{\Omega}_\pi\,d\Omega_\tau}\left|\frac{\partial(\Omega^*_{K\pi}, \Omega^*_\rho, \Omega_\tau)}{\partial(p_{K\pi}, \Omega_{K\pi}, p_\rho, \Omega_\rho)}\right|$$



Such kind of analysis was done only once by CLEO for the $(\tau \to \ell\nu\nu;\ \tau \to \pi\pi^0\pi^0\nu)$ events to study the dynamics of $\tau \to \pi\pi^0\pi^0\nu$ decay.

If we pretend on the $\lesssim 1\%$ level in the studies of the dynamics of hadronic τ decays, the research program for any hadronic τ decay should be:

- Measure hadronic structure functions on the signal side following Kuhn & Co. approach.
- Perform the unbinned fit of the full event configuration $(\tau \to \text{signal};\ \tau \to \text{tag})$, where the dynamics of the $\tau \to \text{tag}$ is well known (for example, leptonic tag). Identify the structure of the "remnant" from the spin-spin correlation term and correct the hadronic structure functions, measured on the first step.
- Extract CPV parameter in the simulataneous approximation of the $(\tau^- \to \text{signal}^-;\ \tau^+ \to \text{tag}^+)$ and $(\tau^+ \to \text{signal}^+;\ \tau^- \to \text{tag}^-)$ events.

Summary, physics

- The world largest statistics of τ leptons collected by Belle and *BABAR* opens new era in the precision tests of the Standard Model, search for the effects of New Physics and precision studies of low energy QCD.
  Belle II is the main player in τ studies in the nearest future.
- Nonzero average polarization of single τ at the SCTF make it to be competitive to Belle II player in τ lepton studies regardless smaller expected statistics (by a factor of 2.2).
- Vast research program of the proper precise study of hadronic τ decays taking into account spin-spin correlation term requires great effort and competition between at least two e^+e^- experiments (Belle II and SCTF).
- The physics program of the SCTF is further developed to unveil rich potential of the e^- beam polarization option.

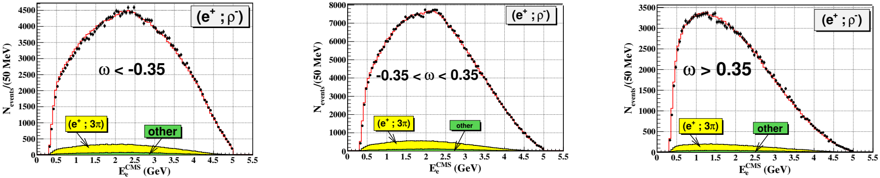
Summary, status of the SCTF project

- In 2017 included in the plan for the implementation of the first phase of the Russian Strategy for Science and Technology Development.
- In 2019 updated conceptual design report was submitted to Russian government.
- Submitted proposal to the Update of European Strategy for Particle Physics.
- Working groups on the detector subsystems were established, the work is going on.
- In 2020 the EU project CREMLINplus was launched, provides funds for European groups working on R&D for SCTF detector.
- The IAC of the SCTF project was formed, four International Workshops on the SCTF project were held, the nearest one will be held in BINP in November 15-17, 2021:
  https://indico.inp.nsk.su/event/62/
- From 2020 the SCTF is discussed in the context of the ”Large Sarov” project (under patronage of ROSATOM Corp.):
  http://www.vniief.ru/en/presscenter/Sarovcity/
- In 2021-2022 we are waiting for the decision on SCTF implementation in Sarov, Russia.
- **Collaboration of the SCTF project is growing, we invite interested physicists.**

Backup slides

Michel par. at Belle, systematic uncertainties

Helicity sensitive variable $\omega = \frac{1}{\Phi_2 - \Phi_1} \int_{\Phi_1}^{\Phi_2} (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) d\Phi = < (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) >_{\Phi_\tau}$



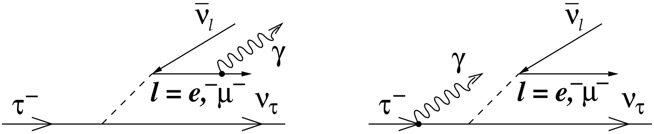
Spin-spin correlation manifests itself through momentum-momentum correlations of final lepton and pions.

Source	$\Delta(\rho)$, %	$\Delta(\eta)$, %	$\Delta(\xi_\rho\xi)$, %	$\Delta(\xi_\rho\xi\delta)$, %
Physical corrections				
ISR+$\mathcal{O}(\alpha^3)$	0.10	0.30	0.20	0.15
$\tau \to \ell\nu\nu\gamma$	0.03	0.10	0.09	0.08
$\tau \to \rho\nu\gamma$	0.06	0.16	0.11	0.02
Background	0.20	0.60	0.20	0.20
Apparatus corrections				
Resolution $\oplus$ brems.	0.10	0.33	0.11	0.19
$\sigma(E_{\rm beam})$	0.07	0.25	0.03	0.15
Normalization				
$\Delta\mathcal{N}$	0.11	0.50	0.17	0.13
without Data/MC corr.	**0.29**	**0.95**	**0.38**	**0.38**
trigger eff. corr.	**$\sim$ 1**	**$\sim$ 2**	**$\sim$ 3**	**$\sim$ 3**

At Belle, various EXP/MC (L1, L3/L4 software trigger, track and π^0 rec., πID and ℓID) efficiency corrections produce the systematic uncertainties in MP of about few percent.

Michel parameters in $\tau \to \ell\nu\nu\gamma$ at Belle (I)

A. B. Arbuzov and T. V. Kopylova, JHEP **1609** (2016) 109. ($m_\ell \neq 0$)



Photon carries information about spin state of outgoing lepton, as a result two additional parameters, $\bar{\eta}$ and $\xi\kappa$, can be extracted.

These parameters were measured in τ decays at Belle for the first time.

$$\frac{d\Gamma(\tau^\mp \to \ell^\mp \nu_\ell \nu_\tau \gamma)}{dx\, dy\, d\Omega_\ell\, d\Omega_\gamma} = \Gamma_0 \frac{\alpha}{64\pi^3} \frac{\beta_\ell}{y} \Big[F(x,y,d) \pm P_\tau \left(\beta_\ell \cos\theta_\ell G(x,y,d) + \cos\theta_\gamma H(x,y,d) \right) \Big],$$

$$\Gamma_0 = G_F^2 m_\tau^5/192\pi^3,\ \beta_\ell = \sqrt{1 - m_\ell^2/E_\ell^2},\ x = 2E_\ell/m_\tau,\ y = 2E_\gamma/m_\tau,\ d = 1 - \beta_\ell \cos\theta_{\ell\gamma}$$

$$F = F_0 + \bar{\eta} F_1,\ G = G_0 + \xi\kappa G_1,\ H = H_0 + \xi\kappa H_1,\ \frac{d\sigma(\ell^\mp \nu\nu\gamma,\, \rho^\pm \nu)}{dE_\ell^*\, d\Omega_\ell^*\, dE_\gamma^*\, d\Omega_\gamma^*\, d\Omega_\rho^*\, dm_{\pi\pi}^2\, d\tilde{\Omega}_\pi\, d\Omega_\tau} = A_0 + \bar{\eta} A_1 + \xi\kappa A_2$$

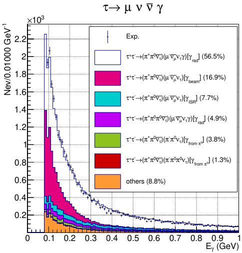
$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp \nu\nu\gamma,\, \rho^\pm \nu)}{dp_\ell\, d\Omega_\ell\, dp_\gamma\, d\Omega_\gamma\, dp_\rho\, d\Omega_\rho\, dm_{\pi\pi}^2\, d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp \nu\nu\gamma,\, \rho^\pm \nu)}{dE_\ell^*\, d\Omega_\ell^*\, dE_\gamma^*\, d\Omega_\gamma^*\, d\Omega_\rho^*\, dm_{\pi\pi}^2\, d\tilde{\Omega}_\pi\, d\Omega_\tau} |\text{JACOBIAN}|\, d\Phi_\tau$$

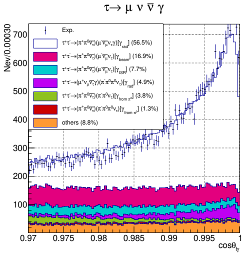
$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)},\ \mathcal{P}^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})} = \frac{\mathcal{F}_0 + \mathcal{F}_1 \bar{\eta} + \mathcal{F}_2 \xi\delta}{\mathcal{N}_0 + \mathcal{N}_1 \bar{\eta} + \mathcal{N}_2 \xi\delta},\ \mathcal{N}_k = \int \mathcal{F}_k(\vec{z}) d\vec{z},\ (k = 0, 1, 2)$$

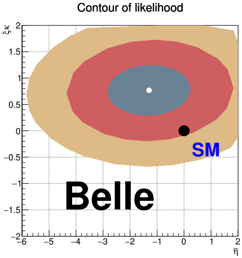
$\bar{\eta}$ and $\xi\delta$ are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu\gamma;\ \rho\nu)$ events in the 12D phase space in CMS.

Michel parameters in $\tau \to \ell\nu\nu\gamma$ at Belle (II)

$N_{\tau\tau} = 646 \times 10^6$, **selected: 71171** $(\mu\nu\nu\gamma;\ \rho\nu)$ **and 776834** $(e\nu\nu\gamma;\ \rho\nu)$ **events**







Source	$\sigma^{e}_{\bar\eta}$	$\sigma^{e}_{\xi\kappa}$	$\sigma^{\mu}_{\bar\eta}$	$\sigma^{\mu}_{\xi\kappa}$
Normalization	4.3	0.94	0.15	0.04
Background PDF	2.5	0.24	0.67	0.22
Branching ratios	3.8	0.05	0.25	0.01
Cluster merge in ECL	2.2	0.46	0.02	0.06
Detector resolution	0.74	0.20	0.22	0.02
Data/MC eff. corr.	1.9	0.14	0.04	0.04
Total	7.0	1.1	0.76	0.24

Belle result

$\bar\eta = \mathbf{-1.3 \pm 1.5 \pm 0.8}$

$\xi\kappa = \mathbf{0.5 \pm 0.4 \pm 0.2}$

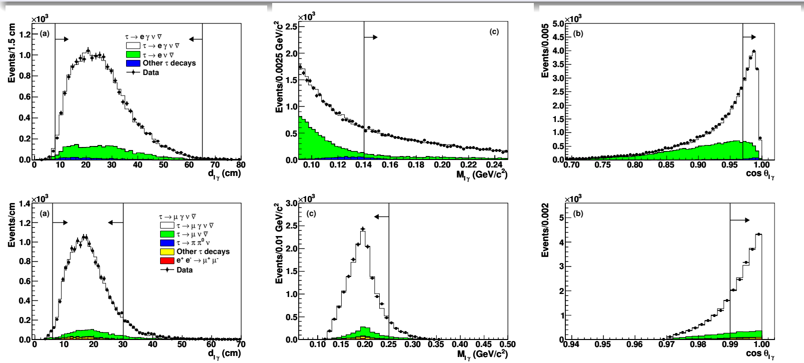
N. Shimizu *et al.* [Belle Collab.], PTEP **2018** (2018) no.2, 023C01.

Measurement of $\mathcal{B}(\tau \to \ell\nu\nu\gamma)$ at *BABAR* (I)

$$\int Ldt = 431\ \mathrm{fb}^{-1}$$

Selections:

- 2-track events with zero net charge and 1 photon with $E_\gamma > 50$ MeV;
- 0.9<thrust<0.995, signal hemisphere: $\ell + \gamma$, tag hemisphere: track+neutrals;
- reject $\ell^\mp - \ell^\pm$ events, $E_{\rm tot} < 9$ GeV, distance between track and photon clusters $d_{\ell\gamma} < 100$ cm.



$e\nu\nu\gamma$	$0.22 \le E_\gamma \le 2.0$ GeV, $M_{e\gamma} \ge 0.14$ GeV/c^2, $\cos\theta_{e\gamma} \ge 0.97$, $8 \le d_{e\gamma} \le 65$ cm
$\mu\nu\nu\gamma$	$0.10 \le E_\gamma \le 2.5$ GeV, $M_{\mu\gamma} \le 0.25$ GeV/c^2, $\cos\theta_{\mu\gamma} \ge 0.99$, $6 \le d_{\mu\gamma} \le 30$ cm

$$N_{\rm sel}(\mu\nu\nu\gamma) = 15688 \pm 125 \quad N_{\rm sel}(e\nu\nu\gamma) = 18149 \pm 135$$

Measurement of $\mathcal{B}(\tau \to \ell\nu\nu\gamma)$ at *BABAR* (II)

$$\mathcal{B} = \frac{N_{\rm sel}(1 - f_{\rm bg})}{2\sigma_{\tau\tau}\mathcal{L}\varepsilon}$$

	$\mu\nu\nu\gamma$	$e\nu\nu\gamma$
ε (%)	0.480 ± 0.010	0.105 ± 0.003
$f_{\rm bg}$	0.102 ± 0.002	0.156 ± 0.003

	$\tau \to \mu\nu\nu\gamma$	$\tau \to e\nu\nu\gamma$
Photon efficiency	1.8	1.8
Particle identification	1.5	1.5
Background evaluation	0.9	0.7
BF	0.7	0.7
Luminosity and cross section	0.6	0.6
MC statistics	0.5	0.6
Selection criteria	0.5	0.5
Trigger selection	0.5	0.6
Track reconstruction	0.3	0.3
Total	2.8	2.8

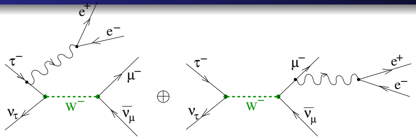
$$\mathcal{B}(\tau \to \mu\nu\nu\gamma)[E^*_\gamma > 10\,\mathrm{MeV}] = (3.69 \pm 0.03 \pm 0.10) \times 10^{-3}$$

$$\mathcal{B}(\tau \to e\nu\nu\gamma)[E^*_\gamma > 10\,\mathrm{MeV}] = (1.847 \pm 0.015 \pm 0.052) \times 10^{-2}$$

Measured branching ratios agree with the LO predictions ($\mathcal{B}(\mu\nu\nu\gamma) = 3.663 \times 10^{-3}$, $\mathcal{B}(e\nu\nu\gamma) = 1.834 \times 10^{-2}$), however the LO+NLO prediction for the $\tau \to e\nu\nu\gamma$ ($\mathcal{B}(e\nu\nu\gamma) = 1.645 \times 10^{-2}$) differs from the experimental result by **3.5σ**.
It is important to embed NLO corrections to the MC generator (TAUOLA) of the radiative leptonic decay. Also background from the doubly-radiative leptonic decays should be properly studied and subtracted.

M. Fael, L. Mercolli and M. Passera, JHEP 1507 (2015) 153.

Tau decays into 5 leptons



D. A. Dicus and R. Vega, Phys. Lett. B **338** (1994) 341.

M. S. Alam *et al.* [CLEO Collaboration], Phys. Rev. Lett. **76** (1996) 2637.

A. Flores-Tlalpa, G. Lopez Castro and P. Roig, JHEP 1604 (2016) 185.

Mode	$\mathcal{B}_{\rm theory}$	$\mathcal{B}_{\rm CLEO}$
$e^\mp e^+ e^- 2\nu$	$(4.21 \pm 0.01) \times 10^{-5}$	$(2.7^{+1.6}_{-1.2}) \times 10^{-5}$
$\mu^\mp e^+ e^- 2\nu$	$(1.984 \pm 0.004) \times 10^{-5}$	$< 3.2 \times 10^{-5}$(90% CL)
$e^\mp \mu^+ \mu^- 2\nu$	$(1.247 \pm 0.001) \times 10^{-7}$	
$\mu^\mp \mu^+ \mu^- 2\nu$	$(1.183 \pm 0.001) \times 10^{-7}$	

A. Kersch, N. Kraus and R. Engfer [SINDRUM], Nucl. Phys. A **485** (1988) 606.

$$\frac{d\Gamma(\tau)}{d\mathcal{PS}} = Q_{LL} d_1 + Q_{LR} d_2 + Q_{RL} d_3 + Q_{RR} d_4 + B_{RL} d_5 + B_{LR} d_6$$

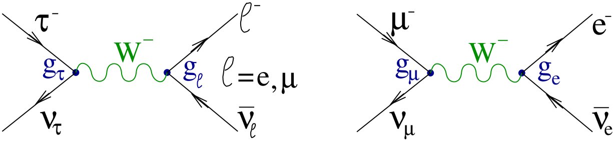
Up to now Q_{LL}, Q_{LR}, Q_{RL}, Q_{RR}, B_{RL}, B_{LR} were measured only in muon decays ($\mu^- \to e^- e^- e^+ \nu_\mu \bar{\nu}_e$) with the accuracy of about $10 \div 20\%$.

Michel parameters can be measured in two ways: in the study of the dynamics and from the measurement of the branching fraction:

$$\mathcal{B}_{\rm exp}/\mathcal{B}_{\rm SM} = Q_{LL} + \alpha_{LR} Q_{LR} + \alpha_{RL} Q_{RL} + \alpha_{RR} Q_{RR} + \beta_{RL} B_{RL} + \beta_{LR} B_{LR}$$

Lepton universality in the SM

$g_e = g_\mu = g_\tau$



$$\Gamma(L^- \to \ell^- \bar{\nu}_\ell \nu_L(\gamma)) = \frac{\mathcal{B}(L^- \to \ell^- \bar{\nu}_\ell \nu_L(\gamma))}{\tau_L} = \frac{g_L^2 g_\ell^2}{32 M_W^4} \frac{m_L^5}{192\pi^3} F_{\rm corr}(m_L, m_\ell)$$

$$F_{\rm corr}(m_L, m_\ell) = f(x)\left(1 + \frac{3}{5}\frac{m_L^2}{M_W^2}\right)\left(1 + \frac{\alpha(m_L)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right)$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x, \; x = m_\ell / m_L$$

$$\mathcal{B}(\mu^- \to e^- \bar{\nu}_e \nu_\mu(\gamma)) = 1$$

$$\frac{g_\tau}{g_e} = \sqrt{\mathcal{B}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau(\gamma)) \frac{\tau_\mu}{\tau_\tau} \frac{m_\mu^5}{m_\tau^5} \frac{F_{\rm corr}(m_\mu, m_e)}{F_{\rm corr}(m_\tau, m_\mu)}}, \quad \frac{\mathbf{g}_\tau}{\mathbf{g_e}} = \mathbf{1.0029 \pm 0.0015} \text{ (HFAG2017)}$$

$$\frac{g_\tau}{g_\mu} = \sqrt{\mathcal{B}(\tau^- \to e^- \bar{\nu}_e \nu_\tau(\gamma)) \frac{\tau_\mu}{\tau_\tau} \frac{m_\mu^5}{m_\tau^5} \frac{F_{\rm corr}(m_\mu, m_e)}{F_{\rm corr}(m_\tau, m_e)}}, \quad \frac{\mathbf{g}_\tau}{\mathbf{g}_\mu} = \mathbf{1.0010 \pm 0.0015} \text{ (HFAG2017)}$$

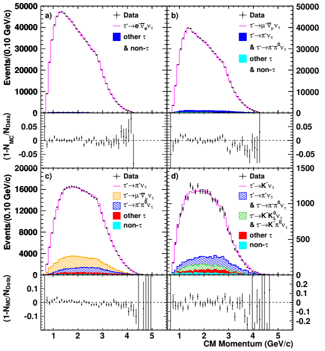
$$\frac{g_\mu}{g_e} = \sqrt{\frac{\mathcal{B}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau(\gamma))}{\mathcal{B}(\tau^- \to e^- \bar{\nu}_e \nu_\tau(\gamma))} \frac{F_{\rm corr}(m_\tau, m_e)}{F_{\rm corr}(m_\tau, m_\mu)}}, \quad \frac{\mathbf{g}_\mu}{\mathbf{g_e}} = \mathbf{1.0019 \pm 0.0014} \text{ (HFAG2017)}$$

Test of lepton universality at *BABAR*

$\int Ldt = 467\ \mathrm{fb}^{-1}$

Selections:

- 4-track events with zero net charge;
- $0.1\sqrt{s} < E_{\rm miss}^{\rm CMS} < 0.7\sqrt{s}$, $|\cos(\theta_{\rm miss}^{\rm CMS})| < 0.7$
- thrust> 0.9, signal hemisphere: $\ell/h(\ell = e, \mu;\ h = \pi, K)$, tag hemisphere: $\tau \to \pi\pi\pi\nu$;
- signal hemisphere: $E_{\rm extra\gamma}^{LAB} < \{1.0,\ 0.5,\ 0.2,\ 0.2\}$ GeV for $\{e,\ \mu,\ \pi,\ K\}$, respectively



	μ	π	K
N$^{\rm D}$	731102	369091	25123
Purity	97.3%	78.7%	76.6%
Total Efficiency	0.485%	0.324%	0.330%
Particle ID Efficiency	74.5%	74.6%	84.6%
Systematic uncertainties:			
Particle ID	0.32	0.51	0.94
Detector response	0.08	0.64	0.54
Backgrounds	0.08	0.44	0.85
Trigger	0.10	0.10	0.10
$\pi^-\pi^-\pi^+$ modelling	0.01	0.07	0.27
Radiation	0.04	0.10	0.04
$\mathcal{B}(\tau^- \to \pi^-\pi^-\pi^+\nu_\tau)$	0.05	0.15	0.40
$\mathcal{L}\sigma_{\tau\tau}$	0.02	0.39	0.20
Total [%]	**0.36**	**1.0**	**1.5**

$\boldsymbol{\tau \to e\nu\nu}$: $N_{\rm sel} = 884426$, $\varepsilon = (0.589 \pm 0.010)\%$, purity is $(99.69 \pm 0.06)\%$

Test of lepton universality

$$R_\mu = \frac{\mathcal{B}(\tau \to \mu\nu\nu)}{\mathcal{B}(\tau \to e\nu\nu)} = 0.9796 \pm 0.0016 \pm 0.0036$$

$$R_\pi = \frac{\mathcal{B}(\tau \to \pi\nu)}{\mathcal{B}(\tau \to e\nu\nu)} = 0.5945 \pm 0.0014 \pm 0.0061$$

$$R_K = \frac{\mathcal{B}(\tau \to K\nu)}{\mathcal{B}(\tau \to e\nu\nu)} = 0.03882 \pm 0.00032 \pm 0.00057$$

$$\left(g_\mu/g_e\right)_\tau = \sqrt{R_\mu \frac{F_{\rm corr}(m_\tau, m_e)}{F_{\rm corr}(m_\tau, m_\mu)}} = 1.0036 \pm 0.0020$$

$$(g_\tau/g_\mu)^2_h = \frac{\mathcal{B}(\tau \to h\nu_\tau)}{\mathcal{B}(h \to \mu\nu_\mu)} \frac{2m_h m_\mu^2 \tau_h}{(1+\delta_h) m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2}\right)^2$$

$$\left(g_\tau/g_\mu\right)_\pi = 0.9856 \pm 0.0057, \ \left(g_\tau/g_\mu\right)_K = 0.9827 \pm 0.0086$$

$$\left(g_\tau/g_\mu\right)_h = 0.9850 \pm 0.0054 \ (2.8\sigma \text{ away from SM})$$

$$\left(g_\tau/g_\mu\right)_{\tau+\pi+K} = 1.0000 \pm 0.0014 \ (\text{HFAG2017})$$

At the Super Charm-Tau factory at the $\tau^+\tau^-$ production threshold (τ is at rest) the pion from $\tau \to \pi\nu$ and kaon from $\tau \to K\nu$ can be easily separated via their momentum difference (of about 63 MeV). The clean sample of $\tau \to K\nu$ is also used to measure precisely $f_K V_{\rm us}$.

Hadronic τ decays

Cabibbo-allowed decays ($\mathcal{B} \sim \cos^2\theta_c$)

$\mathcal{B}(S=0) = (61.85 \pm 0.11)\%$ (PDG)

Cabibbo-suppressed decays ($\mathcal{B} \sim \sin^2\theta_c$)

$\mathcal{B}(S=-1) = (2.88 \pm 0.05)\%$ (PDG)

$$iM_{\rm fi}\left\{\begin{array}{c} S=0 \\ S=-1 \end{array}\right\} = \frac{G_F}{\sqrt{2}}\bar{u}_{\nu_\tau}\gamma^\mu(1-\gamma^5)u_\tau \cdot \left\{\begin{array}{c} \cos\theta_c \cdot \langle \mathrm{hadrons}(q^\mu)|\hat{J}_\mu^{S=0}(q^2)|0\rangle \\ \sin\theta_c \cdot \langle \mathrm{hadrons}(q^\mu)|\hat{J}_\mu^{S=-1}(q^2)|0\rangle \end{array}\right\},\ q^2 \le M_\tau^2$$

The main tasks

- Measurement of branching fractions with highest possible accuracy
- Measurement of low-energy hadronic spectral functions
  - Determination of the decay mechanism (what are intermediate mesons and their contributions)
  - Precise measurement of masses and widths of the intermediate mesons
- Search for CP violation
- Comparison with hadronic formfactors from e^+e^- experiments to check CVC theorem
- Measurement of $\Gamma_{\rm inclusive}(S=0)$ to determine α_s
- Measurement of $\Gamma_{\rm inclusive}(S=-1)$ to determine s-quark mass and $V_{\rm us}$:

$$|V_{\rm us}| = \sqrt{\frac{R_{\rm strange}}{\frac{R_{\rm non-strange}}{|V_{\rm ud}|^2} - \delta R_{\rm theory}}}$$

- $R_{\rm strange} = \mathcal{B}_{\rm strange}/\mathcal{B}_e$
- $R_{\rm non-strange} = \mathcal{B}_{\rm non-strange}/\mathcal{B}_e$
- $\delta R_{\rm theory}$ - SU(3)-breaking contribution

CPV in $\tau^\pm \to K_S\pi^\pm\nu_\tau$ with polarized τ lepton (I)

S. Y. CHOI *et al.*, PLB **437**, 191 (1998).

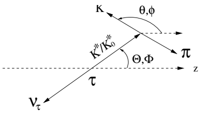
$$M_\sigma = \frac{G_F}{\sqrt{2}}\sin\theta_c\Big[(1+\chi)\bar{u}_\nu(k,-)\gamma^\mu(1-\gamma^5)u(p,\sigma)J_\mu + \eta\bar{u}_\nu(k,-)(1+\gamma^5)u(p,\sigma)J_S\Big]$$

$$J_\mu = \langle(K\pi)^-|\bar{s}\gamma_\mu u|0\rangle = F_V(q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)(q_1-q_2)^\nu + F_S(q^2)q^\mu$$

$$J_S = \langle(K\pi)^-|\bar{s}u|0\rangle = \frac{q^2}{m_s-m_u}F_S(q^2)$$

- $\sigma = \pm1$ – helicity of τ^-;
- χ,η parametrize BSM contribution, $\xi = \frac{m^2_{K_0^*}}{(m_s-m_u)m_\tau}\left(\frac{\eta}{1+\chi}\right)$
- τ^-: $M_\pm(\chi,\xi)$, τ^+: $\overline{M}_\pm(\chi^*,\xi^*)$

If χ and η are real: $M_\pm(\Theta;q^2;\theta,\phi) = \mp\overline{M}_\mp(\Theta;q^2;\theta,-\phi)$
τ is polarized in the $(\theta_p,\ \phi_p)$ direction:
$|\theta_p,\ \phi_p\rangle = \cos\frac{\theta_p}{2}|+\rangle + \sin\frac{\theta_p}{2}|-\rangle$
$\langle\Theta,\ \Phi|\theta_p,\ \phi_p\rangle = \cos\frac{\theta_p}{2}M_+ + \sin\frac{\theta_p}{2}M_-$



$$d\Gamma = \frac{1}{2}d(\Gamma_{++}+\Gamma_{--}) + P_\tau\left(\frac{1}{2}\cos_p d(\Gamma_{++}-\Gamma_{--}) + \sin_p\cos(\phi_p-\Phi)d\mathrm{Re}\Gamma_{+-} - \sin_p\sin(\phi_p-\Phi)d\mathrm{Im}\Gamma_{+-}\right)$$

$$d\Gamma_{\sigma\sigma'} = \frac{1}{(2\pi)^3}\frac{1}{32m_\tau}\left(1-\frac{q^2}{m_\tau^2}\right)M_\sigma M^*_{\sigma'}P_K d\Phi_3 d\Phi,\quad d\Phi_3 = d\sqrt{q^2}d\cos d\phi d\cos$$

CPV in $\tau^\pm \to K_S \pi^\pm \nu_\tau$ with polarized τ lepton (II)

After integration on Φ (and $P_\tau = 1$):

$$\frac{d\Gamma_1}{d\Phi_3} = \frac{d(\Gamma_{++} + \Gamma_{--})}{d\Phi_3},\ \frac{d\Gamma_2}{d\Phi_3} = \frac{d(\Gamma_{++} - \Gamma_{--})}{d\Phi_3},\ \frac{d\Gamma_3}{d\Phi_3} = 2\mathrm{Re}\left(\frac{d\Gamma_{+-}}{d\Phi_3}\right),\ \frac{d\Gamma_4}{d\Phi_3} = 2\mathrm{Im}\left(\frac{d\Gamma_{+-}}{d\Phi_3}\right)$$

$$\frac{d\Gamma_i}{d\Phi_3} = \frac{1}{2}(\Sigma_i + \Delta_i),\quad \Sigma_i/\Delta_i - \mathrm{CP\ even/odd\ part},\quad i = 1 \div 4$$

$$\Sigma_1 = \frac{d(\Gamma_1 + \bar{\Gamma}_1)}{d\Phi_3},\ \Sigma_2 = \frac{d(\Gamma_2 - \bar{\Gamma}_2)}{d\Phi_3},\ \Sigma_3 = \frac{d(\Gamma_3 - \bar{\Gamma}_3)}{d\Phi_3},\ \Sigma_4 = \frac{d(\Gamma_4 + \bar{\Gamma}_4)}{d\Phi_3},$$

$$\Delta_1 = \frac{d(\Gamma_1 - \bar{\Gamma}_1)}{d\Phi_3},\ \Delta_2 = \frac{d(\Gamma_2 + \bar{\Gamma}_2)}{d\Phi_3},\ \Delta_3 = \frac{d(\Gamma_3 + \bar{\Gamma}_3)}{d\Phi_3},\ \Delta_4 = \frac{d(\Gamma_4 - \bar{\Gamma}_4)}{d\Phi_3}$$

CP even: $\Sigma_1 \gg \Sigma_2, \Sigma_3, \Sigma_4$,
CP odd: Δ_1 – P_τ-independent part, $\Delta_{2,3,4}$ – P_τ-dependent part.
Four optimal variables to search for CPV are: $w_i^{\mathrm{opt}} = \Delta_i/\Sigma_1$.

P_τ-independent $\boldsymbol{w_1^{\mathrm{opt}}}$ was used at Belle, while 3 P_τ-dependent $\boldsymbol{w_{2\div 4}^{\mathrm{opt}}}$ can be additionally measured at the Super Charm-Tau factory:

$$\boldsymbol{w_1^{\mathrm{opt}} = A_1(q^2;\Theta,\theta,\phi)\mathrm{Im}(\xi)\mathrm{Im}(F_V F_S^*)},$$

$$\boldsymbol{w_{2\div 4}^{\mathrm{opt}} = A_{2\div 4}(q^2;\Theta,\theta,\phi)\mathrm{Im}(\xi)\mathrm{Im}(F_V F_S^*) + B_{2\div 4}(q^2;\Theta,\theta,\phi)\ \mathrm{Im}(\xi)\mathrm{Re}(F_V F_S^*)}$$

At the Super Charm-Tau factory CPV search doesn't depend on $F_V F_S^*$ phase.