Study of Michel parameters in τ decays at Belle

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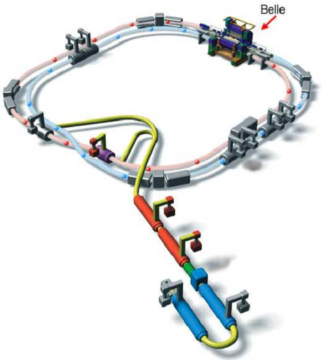
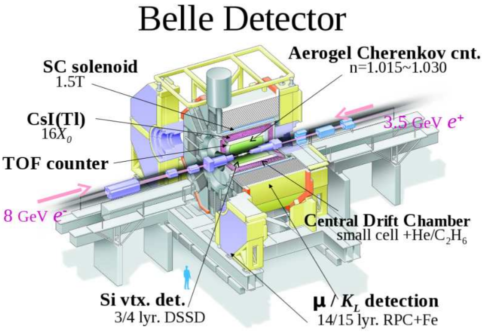




Process	σ, nb
$e^+e^- \to e^+e^-(\gamma)$ $15^o \leq \theta \leq 165^o$	123.5
$e^+e^- \to \mu^+\mu^-(\gamma)$	1.005
$e^+e^- \to q\overline{q}$ $(q = u, d, s, c)$	3.39
$e^+e^- \to b\overline{b}$	1.05
$e^+e^- \to e^+e^- f\overline{f}$ $(f = u, d, s, c, e, \mu, \tau)$	72.6
$e^+e^- \to \tau^+\tau^-(\gamma)$	0.919

- $E_{e^-} = 8$ **GeV,** $E_{e^+} = 3.5$ **GeV**
- **Peak luminosity:** $L = 2.11 \times 10^{34}$ **cm**$^{-2}$**s**$^{-1}$
- **Integrated luminosity:** $\int Ldt \simeq 1$ **ab**$^{-1}$, $N_{\tau\tau} \simeq 10^9$
- B-factory is also τ-factory

Introduction: Michel parameters

In the SM charged weak interaction is described by the exchange of $W^{\pm}$ with a pure vector coupling to only left-handed fermions ("V-A" Lorentz structure). Deviations from "V-A" indicate New Physics. $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ ($\ell = e, \mu$) decays provide clean laboratory to probe electroweak couplings.

The most general, Lorentz invariant four-lepton interaction matrix element:

$$\mathcal{M} = \frac{4G}{\sqrt{2}} \sum_{\substack{N=S,V,T \\ i,j=L,R}} g_{ij}^N \left[\bar{u}_i(l^-)\Gamma^N v_n(\bar{\nu}_l)\right] \left[\bar{u}_m(\nu_\tau)\Gamma_N u_j(\tau^-)\right],$$

$$\Gamma^S = 1,\ \Gamma^V = \gamma^\mu,\ \Gamma^T = \frac{i}{2\sqrt{2}}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

Ten couplings g_{ij}^N, in the SM the only non-zero constant is $g_{LL}^V = 1$

Four bilinear combinations of g_{ij}^N, which are called as Michel parameters (MP): ρ, η, ξ and δ appear in the energy spectrum of the outgoing lepton:

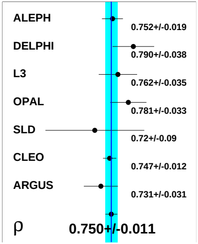
$$\frac{d\Gamma(\tau^\mp)}{d\Omega dx} = \frac{4G_F^2 M_\tau E_{\max}^4}{(2\pi)^4}\sqrt{x^2 - x_0^2}\left(x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)\right.$$

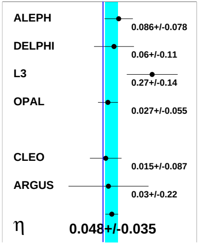
$$\left.\mp\frac{1}{3}P_\tau cos\theta_\ell \xi\sqrt{x^2 - x_0^2}\left[1 - x + \frac{2}{3}\delta\left(4x - 4 + \sqrt{1 - x_0^2}\right)\right]\right),\ x = \frac{E_\ell}{E_{\max}},\ x_0 = \frac{m_\ell}{E_{\max}}$$

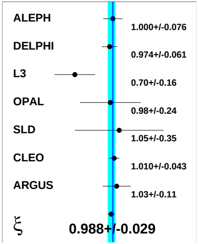
In the SM: $\rho = \frac{3}{4}$, $\eta = 0$, $\xi = 1$, $\delta = \frac{3}{4}$

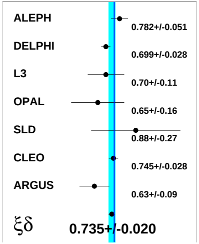
Status of Michel parameters in τ decays

Michel par.	Measured value	Experiment	SM value
ρ (e or μ)	$0.747 \pm 0.010 \pm 0.006$ 1.2%	CLEO-97	3/4
η (e or μ)	$0.012 \pm 0.026 \pm 0.004$ 2.6%	ALEPH-01	0
ξ (e or μ)	$1.007 \pm 0.040 \pm 0.015$ 4.3%	CLEO-97	1
$\xi\delta$ (e or μ)	$0.745 \pm 0.026 \pm 0.009$ 2.8%	CLEO-97	3/4
ξ_h (all hadr.)	$0.992 \pm 0.007 \pm 0.008$ 1.1%	ALEPH-01	1









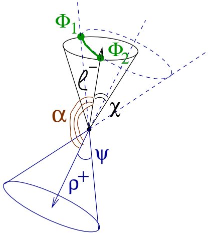
Status of Michel parameters in τ decays

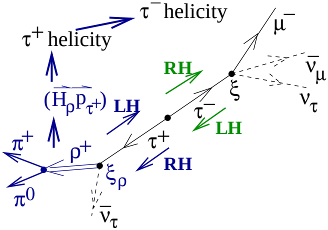
With Belle statistics, which is about 300 times larger than the previous experimental $\tau\tau$ data samples, we can improve MP uncertainties by one order of magnitude.
In BSM models the couplings to τ are expected to be larger than those to μ. Contribution from New Physics in τ decays can be enhanced by a factor of $(\frac{m_\tau}{m_\mu})^2$.

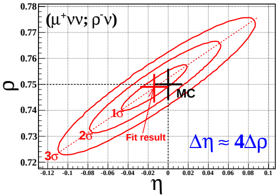
- **Type II 2HDM:** $\eta_\mu(\tau) = \frac{m_\mu M_\tau}{2}\left(\frac{\tan^2\beta}{M^2_{H\pm}}\right)^2$; $\frac{\eta_\mu(\tau)}{\eta_e(\mu)} = \frac{M_\tau}{m_e} \approx 3500$
- **Tensor interaction:**
  $\mathcal{L} = \frac{g}{2\sqrt{2}} W^\mu \left\{ \bar{\nu}\gamma_\mu(1-\gamma^5)\tau + \frac{\kappa^W_\tau}{2m_\tau}\partial^\nu \left(\bar{\nu}\sigma_{\mu\ nu}(1-\gamma^5)\tau \right) \right\}$,
  $-0.096 < \kappa^W_\tau < 0.037$: DELPHI Abreu EPJ C16 (2000) 229.
- **Unparticles:** Moyotl PRD 84 (2011) 073010, Choudhury PLB 658 (2008) 148.
- **Lorentz and CPTV:** Hollenberg PLB **701** (2011) 89
- **Heavy Majorana neutrino:** M. Doi *et al.*, Prog. Theor. Phys. **118** (2007) 1069.

Method, study of $(\ell\nu\nu;\ \rho\nu)$ and $(\rho\nu;\ \rho\nu)$ events

Effect of τ spin-spin correlation is used to measure ξ and δ MP.
Events of $(\tau^{\mp} \to \ell^{\mp}\nu\nu;\ \tau^{\pm} \to \rho^{\pm}\nu)$ topology are used to measure: ρ, η, $\xi_\rho\xi$ and $\xi_\rho\xi\delta$, while $(\tau^{\mp} \to \rho^{\mp}\nu;\ \tau^{\pm} \to \rho^{\pm}\nu)$ events are used to extract ξ_ρ^2.







$$\frac{d\sigma(\ell^{\mp}\nu\nu,\rho^{\pm}\nu)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = A_0 + \rho A_1 + \eta A_2 + \xi_\rho\xi A_3 + \xi_\rho\xi\delta A_4 = \sum_{i=0}^{4} A_i \Theta_i$$

$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^{\mp}\nu\nu,\rho^{\pm}\nu)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^{\mp}\nu\nu,\rho^{\pm}\nu)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} \left| \frac{\partial(E_\ell^*, \Omega_\ell^*, \Omega_\rho^*, \Omega_\tau)}{\partial(p_\ell, \Omega_\ell, p_\rho, \Omega_\rho, \Phi_\tau)} \right| d\Phi_\tau$$

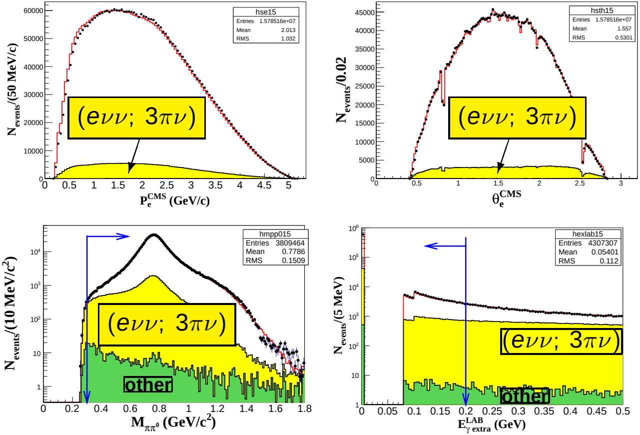
$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)},\ \mathcal{P}^{(k)} = \mathcal{F}(\vec{z}^{(k)})/\mathcal{N}(\vec{\Theta}),\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z},\ \vec{\Theta} = (1, \rho, \eta, \xi_\rho\xi_\ell, \xi_\rho\xi_\ell\delta_\ell)$$

MP are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu;\ \rho\nu)$ events in the 9D phase space $\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m_{\pi\pi}^2,\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$ in CMS.

Selection criteria

- After the standard preselections we take events with two oppositely charged tracks, one of them is identified as lepton (eID, $\mu ID > 0.9$) and the other one as pion ($PID(\pi/K) > 0.4$).
- π^0 candidate is reconstructed from the pair of gammas ($E_\gamma^{\rm LAB} > 80$ MeV) satisfying 115 MeV/$c^2 < M_{\gamma\gamma} <$ 150 MeV/c^2, $P_{\pi^0}^{\rm CMS} > 0.3$ GeV/c.
- $\cos(\vec{P}_{\rm lep}, \vec{P}_\pi) < 0$, $\cos(\vec{P}_{\rm lep}, \vec{P}_{\pi^0}) < 0$, 0.3 GeV/$c^2 < M_{\pi\pi^0} <$ 1.8 GeV/c^2.
- $E_{{\rm rest}\,\gamma}^{\rm LAB} < 0.2$ GeV

Detection efficiency $\varepsilon_{\rm det} \simeq 12\%$



Corrections, detector effects, background

Physical corrections:

- All $\mathcal{O}(\alpha^3)$ QED and electroweak higher order corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$ are included
- Radiative leptonic decays $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau\gamma$
- Radiative decay $\tau^- \to \pi^-\pi^0\nu_\tau\gamma$

Detector effects:

- Track momentum resolution
- γ energy and angular resolution
- Effect of external bremsstrahlung for $e-\rho$ events
- Beam energy spread
- EXP/MC efficiency corrections (trigger, track rec., π^0 rec., ℓID, πID)

Background:

The main background comes from $(\ell\nu\nu;\ \pi 2\pi^0\nu)$(∼10%), $(\pi\nu;\ \pi\pi^0\nu)$(∼1.5%) and $(\rho^+\nu;\ \rho^-\nu)$(∼0.5%) events, it is included in PDF analytically. The remaining background(∼2.0%) is taken into account using MC-based approach.

Background from the non-$\tau\tau$ events is $\lesssim 0.1\%$.

Description of background

Total PDF

$$\mathcal{P}(x) = \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\Bigg((1-\sum_i \lambda_i)\frac{S(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} S(x)dx} + \lambda_{3\pi}\frac{\tilde{B}_{3\pi}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\tilde{B}_{3\pi}(x)dx} + \lambda_{\pi}\frac{\tilde{B}_{\pi}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\tilde{B}_{\pi}(x)dx} + \lambda_{\rho}\frac{\tilde{B}_{\rho}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\tilde{B}_{\rho}(x)dx} +$$

$$+(1-\sum_i \lambda_i)\frac{N^{\rm sel}_{\rm rest}(x)}{N^{\rm sel}_{\rm sig}(x)} S_{\rm SM}(x)\Bigg)$$

$$\tilde{B}_{3\pi}(x) = \int 2(1-\varepsilon_{\pi^0}(y))\varepsilon_{\rm add}(y)B_{3\pi}(x,y)dy,\ \tilde{B}_{\pi}(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)}B_\pi(x)$$

$$\tilde{B}_{\rho}(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)}\int (1-\varepsilon_{\pi^0}(y))\varepsilon_{\rm add}(y)B_\rho(x,y)dy,\ \overline{\varepsilon(x)} = \epsilon^{\rm trg}_{\rm corr}(x)\epsilon^{\ell{\rm ID}}_{\rm corr}(x)\varepsilon(x)$$

- $x = (p_\ell,\ \Omega_\ell,\ p_\rho,\ \Omega_\rho,\ m^2_{\pi\pi},\ \tilde{\Omega}_\pi)$; $y = (p_{\pi^0},\ \Omega_{\pi^0})$;
- $S(x)$ - theoretical density of signal ($\ell^\mp\nu\nu,\ \rho^\pm\nu$) events;
- $B_{3\pi}(x,y)$ - theoretical density of background ($\ell^\mp\nu\nu,\ \pi^\pm 2\pi^0\nu$) events;
- $B_\pi(x)$ - theoretical density of background ($\pi^\mp\nu,\ \rho^\pm\nu$) events;
- $B_\rho(x)$ - theoretical density of background ($\rho^\mp\nu,\ \rho^\pm\nu$) events;
- $\varepsilon(x)$ - detection efficiency for signal events (**common multiplier**);
- $N^{\rm sel}_{\rm rest}(x)/N^{\rm sel}_{\rm sig}(x)$ - number of the selected (remaining/signal) MC events in the multidimensional cell around "x". Admixture of the remaining background is $(1\div 2)\%$.
- λ_i - i-th background fraction (from MC)
- $\varepsilon_{\pi^0}(y)$ - π^0 detection efficiency (tabulated from MC);
- $\varepsilon_{\rm add}(y) = \varepsilon^{3\pi}_{\rm add}(y)/\varepsilon^{\rm sig}_{\rm add}$ - ratio of the $E^{\rm LAB}_{\gamma\rm rest}$ cut efficiencies (tabulated from MC);
- $\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)/\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)$ is tabulated from MC;
- $\epsilon^{\rm trg}_{\rm corr}(x),\ \epsilon^{\ell{\rm ID}}_{\rm corr}(x)$ - EXP/MC efficiency corrections.

MC fit (with only dominant background)

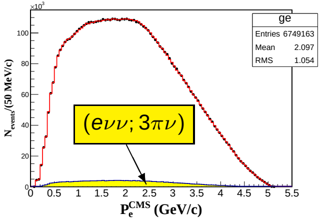
For each configuration 5M MC sample is fitted. The other, statistically independent, 5M MC sample was used to calculate normalization.

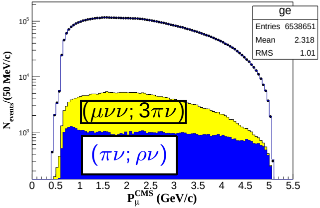
$(e^+\nu\nu;\rho^-\nu)$

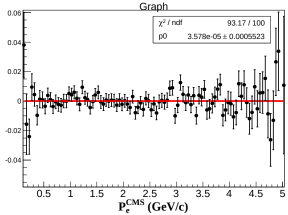
ρ	=	0.7517	$\pm$	0.0010
η	=	0	–	fixed
ξ	=	1.0092	$\pm$	0.0043
$\xi\delta$	=	0.7538	$\pm$	0.0027

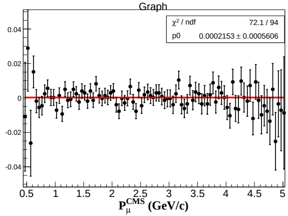
$(\mu^+\nu\nu;\rho^-\nu)$

ρ	=	0.7494	$\pm$	0.0027
η	=	0.0052	$\pm$	0.0101
ξ	=	0.9995	$\pm$	0.0050
$\xi\delta$	=	0.7519	$\pm$	0.0033



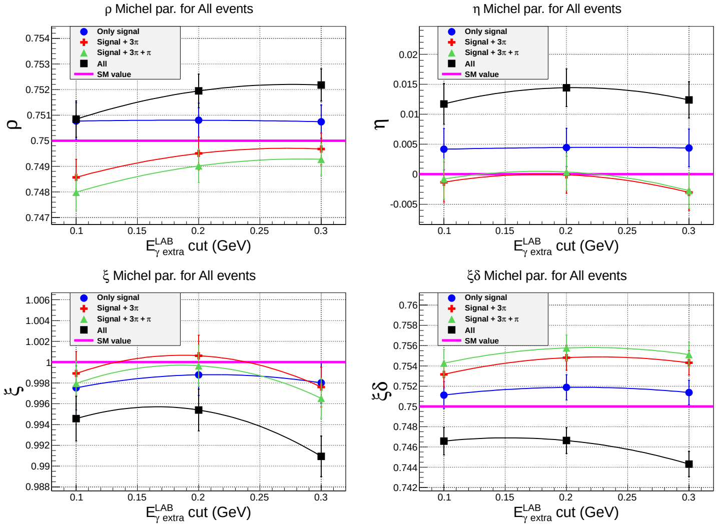






Full MC fit

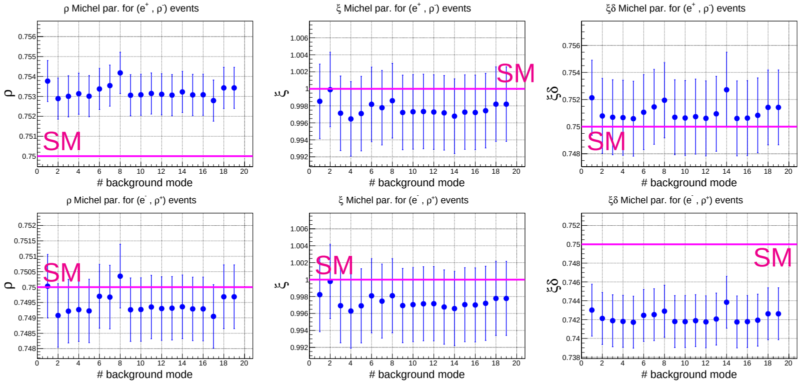
Simultaneous fit of MC $(e^{\pm}\nu\nu; \rho^{\mp}\nu)$, $(\mu^{\pm}\nu\nu; \rho^{\mp}\nu)$ events



All parameters agree with their SM expectations within systematic uncertainties (ρ(0.3%), η(0.8%), ξ(0.5%), $\xi\delta$(0.4%))

Remaining background in $(e\nu\nu;\ \rho\nu)$

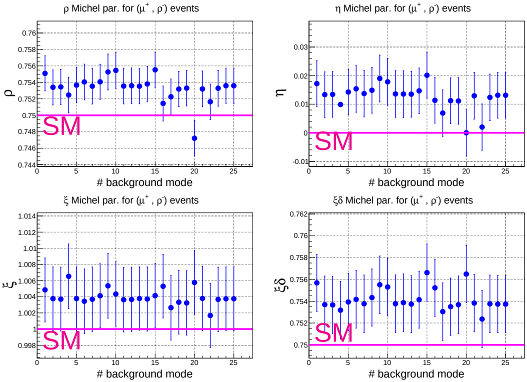
The remaining background in the $(e\nu\nu;\ \rho\nu)$ events is populated mainly by 18 different τ decay modes (see backup slides)



Switching off each mode, one by one, in the fitted MC sample we (conservatively) tested the impact of the MC description of all modes on Michel parameters ($\lesssim 0.2\%$).

Remaining background in $(\mu\nu\nu;\ \rho\nu)$

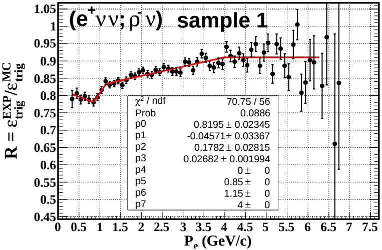
The remaining background in the $(\mu\nu\nu;\ \rho\nu)$ events is populated mainly by 24 different τ decay modes (see backup slides)

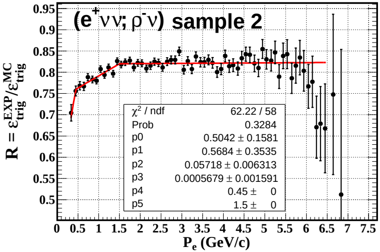


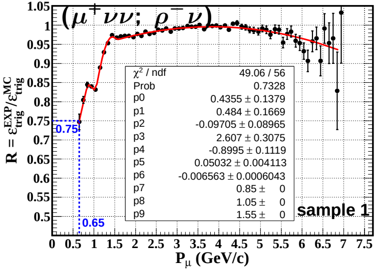
Switching off each mode, one by one, in the fitted MC sample we (conservatively) tested the impact of the MC description of all modes on Michel parameters ($\lesssim$ 0.5%).

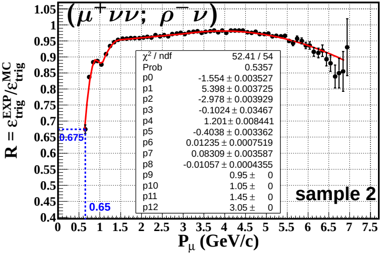
Trigger EXP/MC efficiency corrections (I)

We found that the EXP/MC trigger efficiency correction, $\mathcal{R}_{\rm trg}$, is the dominant one. Two independent subtriggers (energy trigger and track trigger) are used to evaluate it.





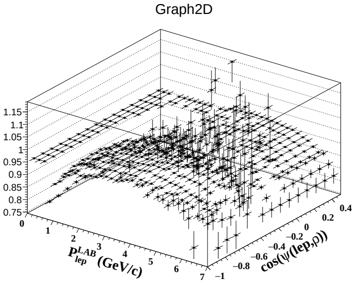


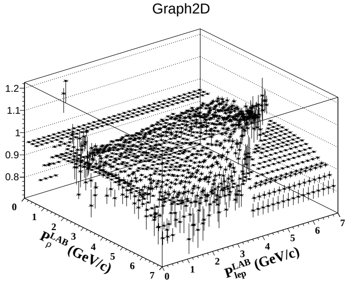


Trigger EXP/MC efficiency corrections (II)

$\mathcal{R}_{\rm trg}$ varies in 9D phase space, appropriate parametrization is needed. One of many parametrizations, which we tested is:

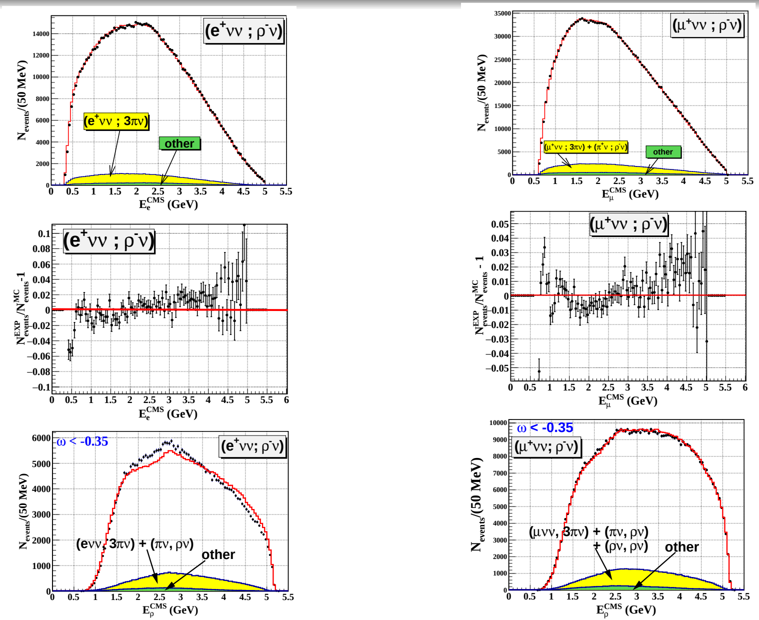
$$\mathcal{R}_{\rm trg} = \frac{\mathcal{R}_{\rm trg}(P_\ell, \cos\psi_{\ell\rho})\mathcal{R}_{\rm trg}(P_\ell, P_\rho)}{\mathcal{R}_{\rm trg}(P_\ell)}$$





Still, we have notable $\mathcal{R}_{\rm trg}$-related systematic uncertainty: ξ, $\xi\delta$ ($\sim$ 3%), η ($\sim$ 2%), ρ ($\sim$ 1%).

Fit of the experimental data, $(\ell^+\nu\nu;\rho^-\nu)$



Experimental data sample of $485\ \mathrm{fb}^{-1}$ (446 $\times 10^6\ \tau^+\tau^-$) was analyzed

Systematic uncertainties

Source	$\Delta(\rho)$, %	$\Delta(\eta)$, %	$\Delta(\xi_\rho\xi)$, %	$\Delta(\xi_\rho\xi\delta)$, %
Physical corrections				
ISR+$\mathcal{O}(\alpha^3)$	0.10	0.30	0.20	0.15
$\tau \to \ell\nu\nu\gamma$	0.03	0.10	0.09	0.08
$\tau \to \rho\nu\gamma$	0.06	0.16	0.11	0.02
Background	0.20	0.60	0.20	0.20
Apparatus corrections				
Resolution $\oplus$ brems.	0.10	0.33	0.11	0.19
$\sigma(E_{\rm beam})$	0.07	0.25	0.03	0.15
Normalization				
$\Delta\mathcal{N}$	0.21	0.60	0.38	0.26
without EXP/MC corr.	0.33	1.0	0.51	0.45
$\mathcal{R}_{\rm trg}$	~ 1	~ 2	~ 3	~ 3

We are working on the EXP/MC trigger efficiency correction, $\mathcal{R}_{\rm trg}$.

Summary

- The procedure to measure 4 Michel parameters (MP) (ρ, η, ξ, $\xi\delta$) in leptonic τ decays at B factory has been developed and tested. It is based on the analysis of the ($\ell^{\mp}\nu\nu;\ \rho^{\pm}\nu$), $\ell = e,\ \mu$ events and utilizes spin-spin correlation of tau leptons.
- We confirmed that with the whole Belle data sample the statistical accuracy of MP is by one order of magnitude better than in the previous best measurements (CLEO, ALEPH).
- The main background components (($\ell\nu\nu;\ \pi 2\pi^0\nu$), ($\pi\nu;\ \rho\nu$), ($\rho\nu;\ \rho\nu$)) are described analytically in the fitter, the remaining background (with the fraction of about 2.0%) is described with help of the MC-based method. We reached acceptable description of the backgrounds in the PDF.
- Various EXP/MC efficiency corrections provide the dominant contribution to the systemtic uncertainties of MP. **The largest contribution comes from the trigger efficiency correction (1–3)%**. We are working to improve this uncertainty.
- The analysis is going on.

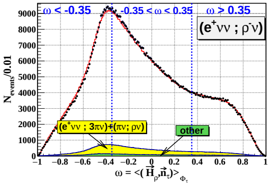
Backup slides

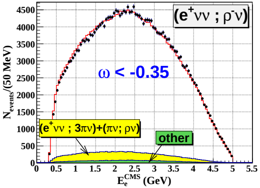
Helicity sensitive variable ω

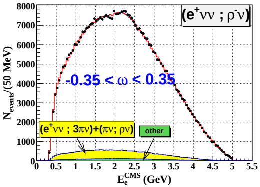
M. Davier *et. al* Phys. Lett. B **306** (1993) 411.

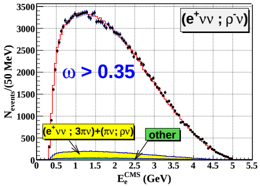
Helicity sensitive variable ω is introduced as:

$$\omega = \frac{1}{\Phi_2 - \Phi_1}\int_{\Phi_1}^{\Phi_2} (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) d\Phi = < (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) >_{\Phi_\tau}$$









Spin-spin correlation manifests itself through momentum-momentum correlations of final lepton and pions.

Multidimensional unbinned maximum likelihood fit

4 Michel parameters ($\vec{\Theta} = (1,\ \rho,\ \eta,\ \xi_\rho\xi_\ell,\ \xi_\rho\xi_\ell\delta_\ell)$) are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu;\ \rho\nu)$ events in the 9D phase space in CMS,
$\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$. The PDF for individual k-th event is written in the form:

$$\mathcal{P}^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})},\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z}$$

Likelihood function for N events:

$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)},\ \mathcal{L} = -\ln L = N\ln\mathcal{N}(\vec{\Theta}) - \sum_{k=1}^{N} \ln\mathcal{F}^{(k)},\ \mathcal{F}^{(k)} = \mathcal{F}(\vec{z}^{(k)})$$

$$\mathcal{F}^{(k)} = A_0^{(k)}\Theta_0 + A_1^{(k)}\Theta_1 + A_2^{(k)}\Theta_2 + A_3^{(k)}\Theta_3 + A_4^{(k)}\Theta_4 = \sum_{i=0}^{4} A_i^{(k)}\Theta_i$$

$$\mathcal{N} = C_0\Theta_0 + C_1\Theta_1 + C_2\Theta_2 + C_3\Theta_3 + C_4\Theta_4,\ C_j = \frac{1}{N}\sum_{k=1}^{N} C_j^{(k)},\ C_j^{(k)} = \frac{A_j^{(k)}}{\sum_{i=0}^{4} A_i^{(k)}\Theta_i^{MC}}$$

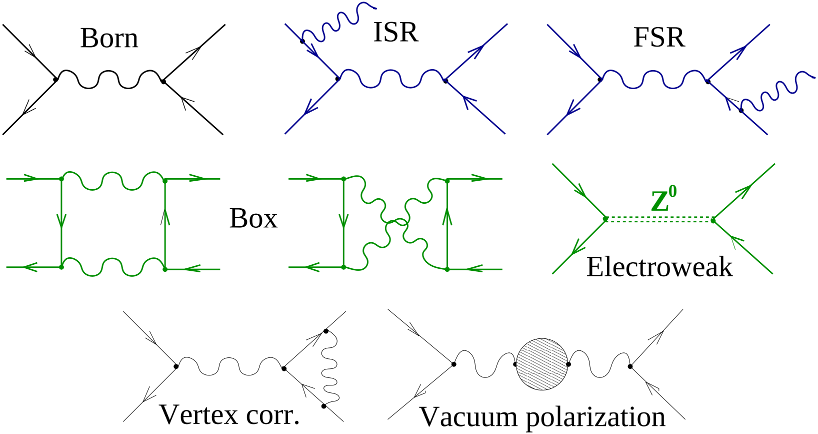
$$\vec{\Theta}^{MC} = (1,\ 0.75,\ 0,\ 1,\ 0.75),\ \mathcal{L} = N\ln\left(\sum_{j=0}^{4} C_j\Theta_j\right) - \sum_{k=1}^{N}\ln\left(\sum_{i=0}^{4} A_i^{(k)}\Theta_i\right)$$

As a result fitted statistics is represented by a set of $5\times N$ values of $A_i^{(k)}$ $(k = 1 \div N,\ i = 0 \div 4)$, which is calculated only once.

C_i $(i = 0 \div 4)$ are calculated using MC simulation.

In ideal case (no rad. corr., $\varepsilon = 100\%$): $C_0 = 1$, $C_2 = 4m_\ell/m_\tau$, $C_{1,3,4} = 0$

$\mathcal{O}(\alpha^3)$ corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$



S. Jadach and Z. Was, Acta Phys. Polon. B **15** (1984) 1151 [Erratum-ibid. B **16** (1985) 483].
A. B. Arbuzov *et al* JHEP **9710** (1997) 001.

Charge-odd part of the cross section comes from the interference of the ISR and FSR diagrams as well as box and Born diagrams, and Z^0-exchange and Born diagrams.

Remaining background for the $(e^{\pm}\nu\nu,\ \rho^{\mp}\nu)$ events

#	$(e/h_1\nu(\nu);\ h_2\nu)$ mode	$E_{\gamma\ rest}^{LAB} < 0.1$ GeV	$E_{\gamma\ rest}^{LAB} < 0.3$ GeV
1	$(e\nu\nu;\ \text{other})$	11.5%	13.2%
2	$(e\nu\nu;\ \pi\nu)$	8.3%	7.6%
3	$(e\nu\nu;\ 3\pi\nu)$	2.4%	2.1%
4	$(e\nu\nu;\ \pi K_S\nu)$	4.9%	7.0%
5	$(e\nu\nu;\ \pi K_L\nu)$	1.5%	1.3%
6	$(e\nu\nu;\ K\pi^0\nu)$	11.6%	8.5%
7	$(e\nu\nu;\ 3\pi\pi^0\nu)$	9.9%	8.5%
8	$(e\nu\nu;\ \pi 3\pi^0\nu)$	8.8%	15.9%
9	$(e\nu\nu;\ \pi K_S K_L\nu)$	0.7%	1.0%
10	$(e\nu\nu;\ K\pi^0 K_S\nu)$	0.3%	0.3%
11	$(e\nu\nu;\ K\pi^0 K_L\nu)$	1.4%	1.2%
12	$(e\nu\nu;\ K 2\pi^0\nu)$	0.4%	0.5%
13	$(e\nu\nu;\ \pi\pi^0 K_S\nu)$	3.5%	3.2%
14	$(e\nu\nu;\ \pi\pi^0 K_L\nu)$	22.1%	17.1%
15	$(e\nu\nu;\ \pi 4\pi^0\nu)$	0.2%	0.4%
16	$(e\nu\nu;\ \pi\omega\pi^0\nu)$	0.2%	0.3%
17	$(\pi\nu;\ \pi 2\pi^0\nu)$	0.5%	0.7%
18	$(\rho\nu;\ \rho\nu)$	2.0%	2.8%
	sum	97.0%	96.7%
	rest	3.0%	3.3%

Remaining background for the $(\mu^{\pm};\ \rho^{\mp})$ events

#	$(\ell/h_1\nu(\nu);\ \ell/h_2\nu(\nu))$ mode	$E_{\gamma\ rest}^{LAB} < 0.1$ GeV	$E_{\gamma\ rest}^{LAB} < 0.3$ GeV
1	$(\mu\nu\nu;\ \text{other})$	9.1%	9.3%
2	$(\mu\nu\nu;\ e\nu\nu)$	0.8%	0.6%
3	$(\mu\nu\nu;\ \mu\nu\nu)$	0.6%	0.4%
4	$(\mu\nu\nu;\ \pi\nu)$	7.6%	6.3%
5	$(\mu\nu\nu;\ 3\pi\nu)$	1.7%	1.5%
6	$(\mu\nu\nu;\ \pi K_S\nu)$	3.9%	4.9%
7	$(\mu\nu\nu;\ \pi K_L\nu)$	1.4%	1.1%
8	$(\mu\nu\nu;\ K\pi^0\nu)$	8.9%	6.2%
9	$(\mu\nu\nu;\ 3\pi\pi^0\nu)$	8.8%	7.0%
10	$(\mu\nu\nu;\ \pi 3\pi^0\nu)$	6.6%	10.6%
11	$(\mu\nu\nu;\ \pi K_S K_L\nu)$	0.5%	0.7%
12	$(\mu\nu\nu;\ KK_L\pi^0\nu)$	1.0%	0.7%
13	$(\mu\nu\nu;\ K2\pi^0\nu)$	0.3%	0.3%
14	$(\mu\nu\nu;\ \pi K_S\pi^0\nu)$	2.2%	2.0%
15	$(\mu\nu\nu;\ \pi K_L\pi^0\nu)$	15.5%	11.5%
16	$(\pi\nu;\ \pi 2\pi^0\nu)$	4.4%	5.5%
17	$(\rho\nu;\ \pi 2\pi^0\nu)$	0.9%	1.8%
18	$(3\pi\nu;\ \rho\nu)$	0.5%	0.4%
19	$(\pi 2\pi^0\nu;\ \rho\nu)$	0.6%	1.1%
20	$(K\nu;\ \rho\nu)$	7.2%	4.9%
21	$(K\nu;\ \pi 2\pi^0\nu)$	0.5%	0.6%
22	$(\pi K_L\nu;\ \rho\nu)$	1.4%	1.1%
23	$(K\pi^0\nu;\ \rho\nu)$	0.4%	0.5%
24	$(KK_L\nu;\ \rho\nu)$	0.4%	0.3%
	sum	95.8%	95.3%
	rest	4.2%	4.7%