Tau-lepton analysis dissection

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Outline:

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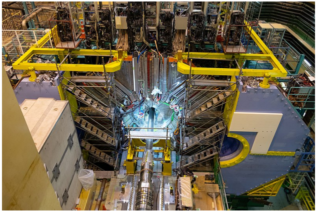
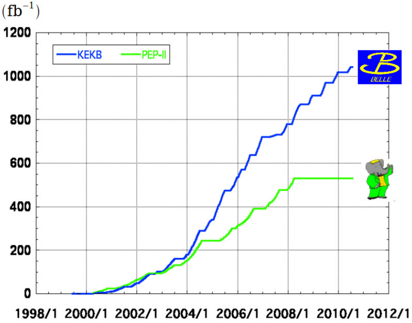
3. **CPV in $\tau^- \to (K\pi)^-\nu_\tau$**
4. **Further studies at Belle II**
5. **Summary**



Introduction: τ physics

- In the SM τ decays due to the charged weak interaction described by the exchange of $W^{\pm}$ with a pure vector coupling to only left-handed fermions. There are two main classes of tau decays:
  - Decays with leptons, like: $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$, $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma$, $\tau^- \to \ell^- \ell'^+ \ell'^- \bar{\nu}_\ell \nu_\tau$; $\ell, \ell' = e, \mu$. They provide very clean laboratory to probe electroweak couplings, which is complementary/competitive to precision studies with muon (in experiments with muon beam). Plenty of New Physics models can be tested/constrained in the precision studies of the dynamics of decays with leptons.
  - Hadronic decays of τ offer unique tools for the precision study of low energy QCD.
- The world largest statistics of τ leptons collected by e^+e^- B factories (Belle and *BABAR*) opens new era in the precision tests of the Standard Model (SM).
  Still, many interesting and important studies with τ lepton will be done using Belle/*BABAR* statistics.
- **Belle II is the new active and very promising player in this area.**

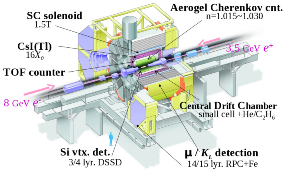
Integrated luminosity of B factories



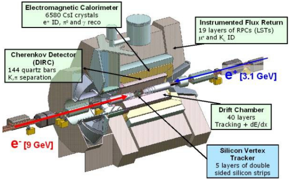
> 1 ab^{-1}
On resonance:
$\Upsilon(5S)$: 121 fb^{-1}
$\Upsilon(4S)$: 711 fb^{-1}
$\Upsilon(3S)$: 3 fb^{-1}
$\Upsilon(2S)$: 25 fb^{-1}
$\Upsilon(1S)$: 6 fb^{-1}
Off reson./scan:
$\sim$ 100 fb^{-1}

$\sim$ 550 fb^{-1}
On resonance:
$\Upsilon(4S)$: 433 fb^{-1}
$\Upsilon(3S)$: 30 fb^{-1}
$\Upsilon(2S)$: 14 fb^{-1}
Off resonance:
$\sim$ 54 fb^{-1}

Belle Detector



BABAR detector



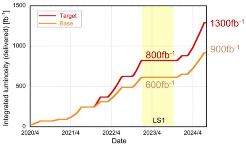
$$\sigma(b\bar{b}) = 1.05 \text{ nb} \quad N_{b\bar{b}} = 1.2 \times 10^9$$
$$\sigma(c\bar{c}) = 1.30 \text{ nb} \quad N_{c\bar{c}} = 2.0 \times 10^9$$
$$\sigma(\tau\tau) = 0.92 \text{ nb} \quad N_{\tau\tau} = 1.4 \times 10^9$$

B-factories are also charm- and τ-factories !

Unique experiment at the HEP intensity frontier

Projections of Integrated Luminosity Delivered by SuperKEKB to Belle II

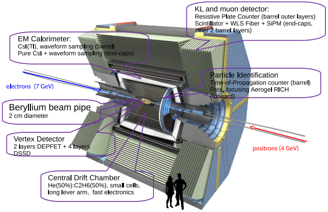
- Target scenario: extrapolation from early 2021 run including expected improvements
- Base scenario: conservative extrapolation of SuperKEKB parameters from early 2021 run



Long Shutdown 1 (LS1) is currently scheduled to start January 2023

If SuperKEKB performance indicates that insufficient integrated luminosity will be collected before LS1 or COVID-19 travel restrictions persist, the option exists to postpone the start of LS1 to July 2023

Belle II detector

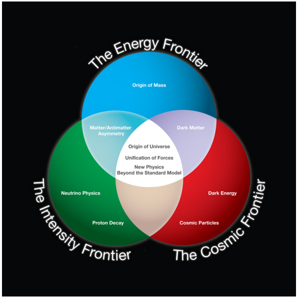


Planned integrated luminosity is 50 ab^{-1}

$\sigma(b\bar{b}) = 1.05$ nb $\quad N_{b\bar{b}} = 53 \times 10^9$

$\sigma(c\bar{c}) = 1.30$ nb $\quad N_{c\bar{c}} = 65 \times 10^9$

$\boldsymbol{\sigma(\tau\tau) = 0.92}$ **nb** $\quad \boldsymbol{N_{\tau\tau} = 46 \times 10^9}$



Introduction: τ properties at B factories

- **Tau mass:**
  BES3: $m_\tau = (1776.91 \pm 0.12(\mathrm{stat}) \pm {}^{0.10}_{0.13}(\mathrm{syst}))$ MeV/c^2; PRD 90, 012001 (2014)
  KEDR: $m_\tau = (1776.81 \pm {}^{0.25}_{0.23}(\mathrm{stat}) \pm 0.15(\mathrm{syst}))$ MeV/c^2; JETPL 85, 347 (2007)
  Belle: $m_\tau = (1776.61 \pm 0.13(\mathrm{stat}) \pm 0.35(\mathrm{syst}))$ MeV/c^2; PRL 99, 011801 (2007)
  BABAR: $m_\tau = (1776.68 \pm 0.12(\mathrm{stat}) \pm 0.41(\mathrm{syst}))$ MeV/c^2; PRD 80, 092005 (2009)
- **Tau lifetime:**
  Belle: $\tau_\tau = (290.17 \pm 0.53(\mathrm{stat}) \pm 0.33(\mathrm{syst}))$ fs; PRL 112, 031801 (2014)
  BABAR(prelim.): $\tau_\tau = (289.40 \pm 0.91(\mathrm{stat}) \pm 0.90(\mathrm{syst}))$ fs; Nucl. Phys. B 144, 105 (2005)
- **Michel parameters in $\tau \to \ell\nu\nu$ (ρ, η, ξ, δ):**
  Belle: Systematic uncertainties are about $(1 \div 3)$%; arXiv:1409.4969
- **Study of the radiative leptonic decays $\tau \to \ell\nu\nu\gamma$:**
  BABAR: Measurement of $\mathcal{B}(\tau \to \ell\nu\nu\gamma)$; PRD 91, 051103(R) (2015)
  Belle: $\bar{\eta} = -1.3 \pm 1.5 \pm 0.8$, $\xi\kappa = 0.5 \pm 0.4 \pm 0.2$; arXiv:1709.08833
- **Study of the 5-lepton decays $\tau \to \ell\ell'^+\ell'^-\nu\nu$:**
  CLEO: $\mathcal{B}(\tau \to ee e\nu\nu) = (2.8 \pm 1.5) \times 10^{-5}$,
  $\mathcal{B}(\tau \to \mu ee\nu\nu) < 3.6 \times 10^{-5} (CL = 90\%)$; PRL 76, 2637 (1996)
  Belle: statistical uncertainties are about $(3 \div 5)$%; J. Phys. Conf. Ser. **912** (2017) no.1, 012002.
- **Lepton universality with $\tau \to \ell\nu\nu$ and $\tau \to h\nu$ (h=π,K):**
  BABAR : $\left(\frac{g_\mu}{g_e}\right)_\tau = 1.0036 \pm 0.0020$, $\left(\frac{g_\tau}{g_\mu}\right)_\mathrm{h} = 0.9850 \pm 0.0054$; PRL 105, 051602 (2010)
- **Tau electric dipole moment (EDM):**
  Belle: $\mathrm{Re}(d_\tau) = (-0.62 \pm 0.63) \times 10^{-17}$ e·cm, $\mathrm{Im}(d_\tau) = (-0.40 \pm 0.32) \times 10^{-17}$ e·cm; submitted to JHEP in 2021 ($\int Ldt = 833$ fb^{-1})
- **Hadronic contribution to a_μ ($\tau^- \to \pi^-\pi^0\nu_\tau$):**
  Belle: $a_\mu^{\pi\pi} = (523.5 \pm 1.1(\mathrm{stat}) \pm 3.7(\mathrm{syst})) \times 10^{-10}$; PRD 78, 072006 (2008)

Introduction: hadronic τ decays

Cabibbo-allowed decays ($\mathcal{B} \sim \cos^2\theta_c$)

$\mathcal{B}(S=0) = (61.85 \pm 0.11)\%$ (PDG)

Cabibbo-suppressed decays ($\mathcal{B} \sim \sin^2\theta_c$)

$\mathcal{B}(S=-1) = (2.88 \pm 0.05)\%$ (PDG)

$$iM_{\rm fi}\left\{\begin{array}{c} S=0 \\ S=-1 \end{array}\right\} = \frac{G_F}{\sqrt{2}}\bar{u}_{\nu_\tau}\gamma^\mu(1-\gamma^5)u_\tau \cdot \left\{\begin{array}{c} \cos\theta_c \cdot \langle \mathrm{hadrons}(q^\mu)|\hat{J}_\mu^{S=0}(q^2)|0\rangle \\ \sin\theta_c \cdot \langle \mathrm{hadrons}(q^\mu)|\hat{J}_\mu^{S=-1}(q^2)|0\rangle \end{array}\right\},\ q^2 \le M_\tau^2$$

The main tasks

- Measurement of branching fractions with highest possible accuracy
- Measurement of low-energy hadronic spectral functions
  - Determination of the decay mechanism (what are intermediate mesons and their contributions)
  - Precise measurement of masses and widths of the intermediate mesons
- Search for CP violation (CPV)
- Comparison with hadronic formfactors from e^+e^- experiments to check CVC theorem
- Measurement of $\Gamma_{\rm inclusive}(S=0)$ to determine α_s
- Measurement of $\Gamma_{\rm inclusive}(S=-1)$ to determine s-quark mass and V_{us}:

$$|V_{us}| = \sqrt{\frac{R_{\rm strange}}{\frac{R_{\rm non-strange}}{|V_{ud}|^2} - \delta R_{\rm theory}}}$$

- $R_{\rm strange} = \mathcal{B}_{\rm strange}/\mathcal{B}_e$
- $R_{\rm non-strange} = \mathcal{B}_{\rm non-strange}/\mathcal{B}_e$
- $\delta R_{\rm theory}$ - SU(3)-breaking contribution

Study of the $\tau^- \to K_S^0\pi^-\nu_\tau,\ K^-\pi^0\nu_\tau$ decays

- **Measurement of $\mathcal{B}(\tau \to K_S^0\pi\nu_\tau)$ and $\mathcal{B}(\tau \to K^-\pi^0\nu_\tau)$:** $\tau \to K\pi\nu_\tau$ has the largest $\mathcal{B}$ among decays with one kaon, so, it provides the dominant contribution to the s-quark mass sensitive total strange hadronic spectral function.
- **Study of the $K\pi$ dynamics (mass spectrum):**
  M. Finkemeier, E. Mirkes, Z. Phys. C **72**, 619 (1996).
  The hadronic current in the case of two pseudoscalar hadrons with $q^\mu_{1,2}$:

  $$J^\mu = F_V(q^2)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)(q_1 - q_2)_\nu + F_S(q^2)q^\mu,\ q^\mu = q_1^\mu + q_2^\mu$$

  - F_V: $K^*(892)^\pm$, $K^*(1410)^\pm$, $K^*(1680)^\pm$;
  - F_S: $K^*(800)^\pm(\kappa)$, $K^*(1430)^\pm$;
  - Precision measurement of $M(K^*(892)^\pm)$ and $\Gamma(K^*(892)^\pm)$.
- **CPV in $\tau \to K_S^0\pi\nu_\tau,\ K^-\pi^0\nu_\tau$**
  - in the mode with the K_S^0 there is additional CPV asymmetry related to the known CPV in the system of neutral kaons
  - J.Kuhn, E.Mirkes,Phys. Lett. **B398**, 407 (1997).
  - Y. Grossman and Y. Nir, JHEP **1204**, 002 (2012).
  - J. P. Lees *et al.* [BABAR], Phys. Rev. D **85**, 031102 (2012).
  - M. Bischofberger *et al.* [Belle], Phys. Rev. Lett. **107**, 131801 (2011).

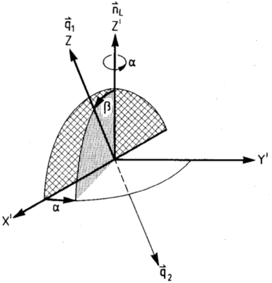
$\tau \to K(q_1)\pi(q_2)\nu_\tau$ hadronic spectral functions (I)

$$d\Gamma = \frac{G_F^2}{256\pi^3 m_\tau}\sin^2\theta_c \{L_{\mu\nu}H^{\mu\nu}\}\left(1-\frac{q^2}{m_\tau^2}\right)|\vec{q}_1|\frac{dq^2}{\sqrt{q^2}}\frac{d\alpha}{2\pi}\frac{d\cos\beta}{2}\frac{d\cos\theta}{2}$$

$$L_{\mu\nu}H^{\mu\nu} = 2m_\tau^2\left(1-\frac{q^2}{m_\tau^2}\right)(\overline{L}_B W_B + \overline{L}_{SA}W_{SA} + \overline{L}_{SF}W_{SF}),\ q = q_1 + q_2,$$

$$W_B = 4|\vec{q}_1|^2|F_V|^2,\ W_{SA} = q^2|F_S|^2,\ W_{SF} = 4\sqrt{q^2}|\vec{q}_1|\mathrm{Re}[F_V F_S^*]$$

$$\overline{L}_B = \frac{1}{3}\left(2+\frac{m_\tau^2}{q^2}\right)-\frac{1}{6}\left(1-\frac{m_\tau^2}{q^2}\right)(3\cos^2\psi-1)(3\cos^2\beta-1),\ \overline{L}_{SA} = \frac{m_\tau^2}{q^2},\ \overline{L}_{SF} = -\frac{m_\tau^2}{q^2}\cos\psi\cos\beta$$



$$\cos\beta = -\vec{n}_q\cdot\frac{\vec{q}_1}{|\vec{q}_1|}$$

$$\cos\theta = \frac{(2\frac{E_{K\pi}}{E_\tau}-1-\frac{q^2}{m_\tau^2})}{(1-\frac{q^2}{m_\tau^2})\sqrt{1-m_\tau^2/E_\tau^2}}$$

$$\cos\psi = \frac{\frac{E_{K\pi}}{E_\tau}(m_\tau^2+q^2)-2q^2}{(m_\tau^2-q^2)\sqrt{(E_{K\pi}^2-q^2)/E_\tau^2}}$$

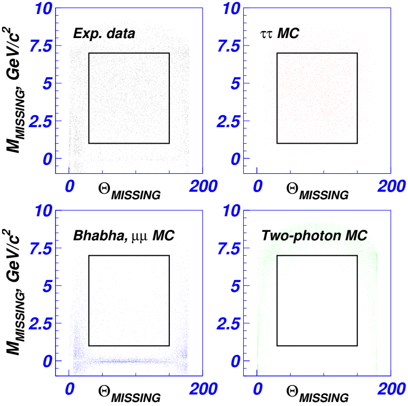
$\tau \to K\pi\nu_\tau$ hadronic spectral functions (II)

- β - angle between $\vec{q}_1$ (kaon) and direction to CMS frame in the $K\pi$ rest frame
- ψ - angle between $\vec{p}_\tau$ and direction to CMS frame in the $K\pi$ rest frame
- θ - angle between $\vec{p}_\tau$ in CMS and momentum of $K\pi$ in τ rest frame (correlated with ψ)

The form factors (or hadronic spectral functions) can be extracted by averaging particular functions of β, ψ and θ angles in bins of q^2. In this case the detection efficiency dependence on α, $\cos\beta$ and $\cos\theta$ should be taken into account.

Selection of $\tau^+\tau^-$ events at *B* factories

- General preselection of low-multiplicity events
- Selection on the 2D plot $\theta^{\rm CMS}_{\rm missing}$ vs. $M_{\rm missing}$
- Tag one τ by 1-prong or leptonic mode, and reconstruct the decay products (except neutrino(s)) of the signal tau. At *B* factories the decay products of the oppositely charged taus almost don't overlap (they are located in the opposite hemispheres).



Background from $B\bar{B}$, $q\bar{q}$ ($q = u, d, s, c$), two-photon, Bhabha, $\mu\mu(\gamma)$ is

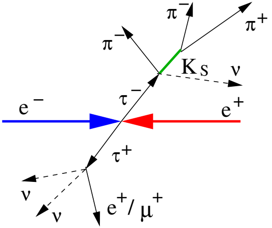
Measurement of $\mathcal{B}(\tau^- \to K^0_S \pi^- \nu_\tau)$

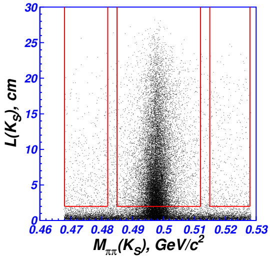
D. EPIFANOV *et al.* [BELLE], PHYS. LETT. B **654**, 65 (2007).

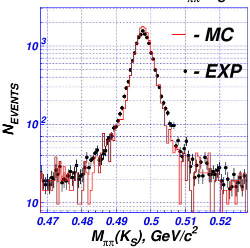
Statistics: $\int Ldt = 351$ fb^{-1}, $N_{\tau\tau} = 323 \times 10^6$

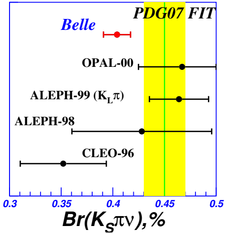
53110 signal events with efficiency $\varepsilon_{\rm det} \simeq 6\%$.

Two-lepton ($\tau \to e\nu\nu, \tau \to \mu\nu\nu$) events are used for normalization.

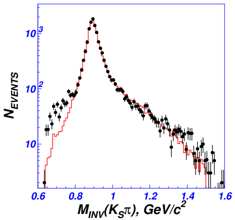








Fit with K*(892) only



Mode	Contents, %
$K_S\pi\nu$	79
$K_S\pi K_L\nu$	9
$K_S\pi\pi^0\nu$	4
$K_S K\nu$	2
$3\pi\nu$	5
non-$\tau\tau$	1

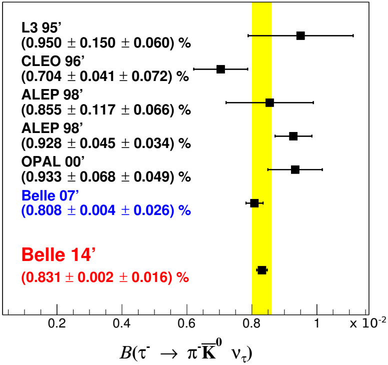
$$\mathcal{B}(\tau^- \to K_S\pi^-\nu_\tau) = (0.404 \pm 0.002(\text{stat.}) \pm 0.013(\text{syst.}))\%$$

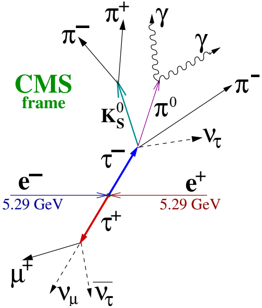
Study of $\tau^- \to K_S^0 X^- \nu_\tau$ decays at Belle

S. Ryu *et al.* [Belle], Phys. Rev. D **89**, 072009 (2014)

Data sample of $\int Ldt = 669$ fb^{-1} with $N_{\tau\tau} = 616 \times 10^6$ was used to study inclusive decay $\tau^- \to K_S^0 X^- \nu_\tau$ as well as 6 exclusive modes:

$$\pi^- K_S^0 \nu_\tau \qquad K^- K_S^0 \nu_\tau \qquad \pi^- K_S^0 K_S^0 \nu_\tau$$
$$\pi^- K_S^0 \pi^0 \nu_\tau \qquad K^- K_S^0 \pi^0 \nu_\tau \qquad \pi^- K_S^0 K_S^0 \pi^0 \nu_\tau$$

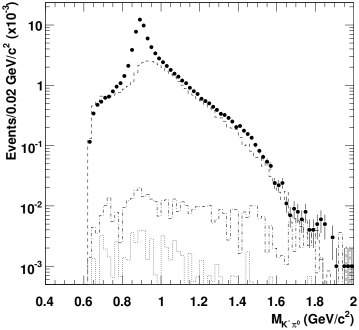




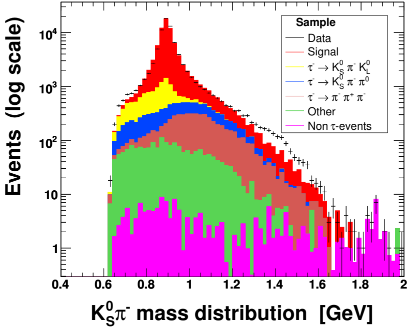
$$\mathcal{B}(\tau^- \to K_S^0 \pi^- \nu_\tau) = (4.16 \pm 0.01 \pm 0.08) \times 10^{-3}$$
$$\mathcal{B}(\tau^- \to K_S^0 X^- \nu_\tau) = (9.14 \pm 0.01 \pm 0.22) \times 10^{-3}$$

B. Aubert *et al.* [BaBar], Phys. Rev. D **76**, 051104 (2007).
B. Aubert *et al.* [BaBar], Nucl. Phys. Proc. Suppl. **189**, 193 (2009).

$\tau^- \to K^-\pi^0\nu_\tau$



$\tau^- \to K_S^0\pi^-\nu_\tau$



$$\mathcal{B}(\tau^- \to K^-\pi^0\nu_\tau) = (0.416 \pm 0.003(\text{stat.}) \pm 0.018(\text{syst.}))\%$$
$$\mathcal{B}(\tau^- \to K_S^0\pi^-\nu_\tau) = (0.420 \pm 0.002(\text{stat.}) \pm 0.012(\text{syst.}))\%\ (\text{preliminary})$$

Study of the $K_S^0\pi$ mass spectrum at Belle (I)

$$\frac{d\Gamma}{d\sqrt{s}} \sim \frac{1}{s}\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+2\frac{s}{M_\tau^2}\right)P\left\{P^2|F_V|^2+\frac{3(M_K^2-M_\pi^2)^2}{4s(1+2\frac{s}{M_\tau^2})}|F_S|^2\right\}$$

$$s=q^2=M^2_{\rm inv}(K_S^0\pi)$$

$$\mathrm{F_V}=\frac{\mathrm{BW_{K^*(892)}}+\mathrm{a(K^*(1410))}\cdot\mathrm{BW_{K^*(1410)}}+\mathrm{a(K^*(1680))}\cdot\mathrm{BW_{K^*(1680)}}}{1+\mathrm{a(K^*(1410))}+\mathrm{a(K^*(1680))}}$$

$$\mathrm{F_S}=\mathrm{a(K_0^*(800))}\cdot\mathrm{BW_{K_0^*(800)}}+\mathrm{a(K_0^*(1430))}\cdot\mathrm{BW_{K_0^*(1430)}}$$

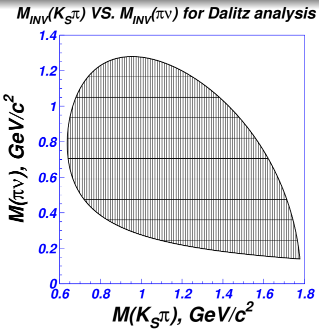
$$\mathrm{BW_X}=\frac{\mathrm{M_X^2}}{\mathrm{M_X^2-s-i\sqrt{s}\Gamma_X(s)}}$$

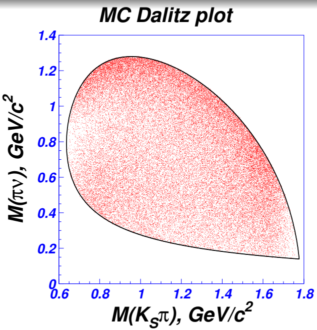
$$\Gamma_X(s)=\Gamma_X\frac{\mathrm{M_X^2}}{\mathrm{s}}\left(\frac{\mathrm{P(s)}}{\mathrm{P(M_X^2)}}\right)^{2\ell+1}\cdot\mathrm{F_R^{\ell 2}}$$

$$\mathrm{P(s)}=\frac{\sqrt{\mathrm{(s-(M_K+M_\pi)^2)(s-(M_K-M_\pi)^2)}}}{2\sqrt{\mathrm{s}}}$$

Spin ℓ	Blatt-Weisskopf factor $\mathrm{F_R^\ell}$
0	1
1	$\sqrt{\frac{1+R^2P^2(M_X^2)}{1+R^2P^2(s)}}$
2	$\sqrt{\frac{9+3R^2P^2(M_X^2)+R^4P^4(M_X^2)}{9+3R^2P^2(s)+R^4P^4(s)}}$

Study of the $K_S^0\pi$ mass spectrum at Belle (II)





To take into account the detector apparatus function we introduce 100×1000 efficiency matrix:

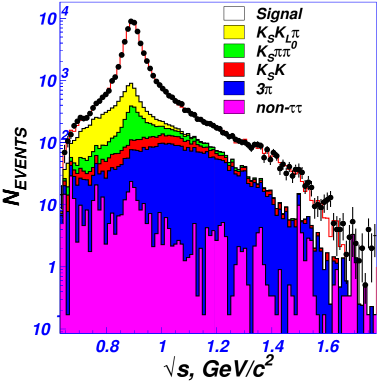
$$\varepsilon_{ij}^{\mathrm{MC}} = \frac{N_i^{\mathrm{MC}}(\mathrm{sel})}{N_j^{\mathrm{MC}}(\mathrm{gen})},\ i = 1 \div 100,\ j = 1 \div 1000$$

$$\chi^2 = \sum_{\mathrm{bins}} \frac{(N_i^{\mathrm{EXP}} - \varepsilon_{ij}^{\mathrm{MC}} N_j^{\mathrm{THEORY}})^2}{N_i^{\mathrm{EXP}} + \sigma_{\varepsilon N}^2}$$

$$N_j^{\mathrm{THEORY}} = \int_j \frac{d\Gamma}{dm_{12}dm_{23}} dm_{12} dm_{23},\ m_{12} = M(K_S^0\pi) = \sqrt{s},\ m_{23} = M(\pi\nu)$$

$K_0^*(800) + K^*(892) + K^*(1410)$

The $K^*(892)$ alone is not sufficient to describe the $K_S^0\pi$ spectrum



$M_{K^*(892)} = 895.47 \pm 0.20\ \mathrm{MeV/c^2}$

$\Gamma_{K^*(892)} = 46.19 \pm 0.57\ \mathrm{MeV}$

$|\mathrm{a(K^*(1410))}| = (75 \pm 6) \times 10^{-3}$

$\arg(\mathrm{a(K^*(1410))}) = 1.44 \pm 0.15$

$|\mathrm{a(K_0^*(800))}| = 1.57 \pm 0.23$

$\chi^2/\mathrm{Ndf} = 90.2/84,\ P(\chi^2) = 30\%$

We take $K_0^*(800)$ parameters:
$M_{K_0^*(800)} = (878 \pm 23 \pm 60)\ \mathrm{MeV/c^2},\ \Gamma_{K_0^*(800)} = (499 \pm 52 \pm 71)\ \mathrm{MeV/c^2}$ from:
M. ABLIKIM *et al.*, [BES COLLABORATION], PHYS. LETT. B **633**, 681 (2006).
There is large systematic uncertainty in the near $K_S^0\pi$ production threshold part of the spectrum due to the large background from the $\tau^- \to K_S^0\pi^- K_L^0\nu_\tau$ decay, whose dynamics is not precisely known.

$K_0^*(800) + K^*(892) + K_0^*(1430)$

	solution 1	solution 2
$M_{K^*(892)}$, MeV/c^2	895.42 ± 0.19	895.50 ± 0.22
$\Gamma_{K^*(892)}$, MeV	46.14 ± 0.55	46.20 ± 0.69
$\lvert a(K_0^*(1430))\rvert$	0.954 ± 0.081	1.92 ± 0.20
$arg(a(K_0^*(1430)))$	0.62 ± 0.34	4.03 ± 0.09
$a(K_0^*(800))$	1.27 ± 0.22	2.28 ± 0.47
χ^2/ndf	$86.5/84$	$95.1/84$
$P(\chi^2), \%$	41	19
$\mathcal{B}(K_0^*(1430) \to K_S\pi)$	$1/3$	$1/3$
$\mathcal{B}(\tau \to K_0^*(1430)\nu_\tau)$	$(13 \pm^{3}_{2}) \times 10^{-5}$	$(54 \pm^{18}_{9}) \times 10^{-5}$

M. Z. Yang, "Testing the structure of the scalar meson $K_0^*(1430)$ in $\tau \to K_0^*(1430)\nu_\tau$ decay", Mod. Phys. Lett. A **21**, 1625 (2006) [arXiv:hep-ph/0509102]:

$$\mathcal{B}(\tau \to K_0^*(1430)\nu_\tau) = (7.9 \pm 3.1) \times 10^{-5}$$

From the $M_{\rm inv}(K_S^0\pi)$ fit only it is not possible to extract precisely the $K_0^*(1430)$ component due to the multiple solutions for the $K_0^*(800)$ and $K_0^*(1430)$ amplitudes in the scalar form factor F_S.

Multiple solutions (two Breit-Wigner amplitides) (I)

$$|A|^2(s|a_1,a_2,\varphi) = \left| a_1 \frac{m_1^2}{s-m_1^2+im_1\Gamma_1} + a_2 e^{i\varphi} \frac{m_2^2}{s-m_2^2+im_2\Gamma_2} \right|^2, \ s=m^2.$$

In the case of constant widths for each set of parameters (a_1,a_2,φ) there exists the other set (a_1',a_2',φ') $(a_1'\neq a_1,\ a_2'\neq a_2,\ \varphi'\neq\varphi)$, such as:

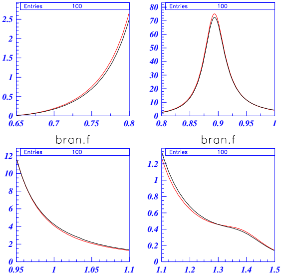
$$|A|^2(s|a_1',a_2',\varphi') = |A|^2(s|a_1,a_2,\varphi) \text{ for all values of } s$$

$$a_1' = f(a_1,a_2,\varphi),\ a_2' = g(a_1,a_2,\varphi),\ \varphi' = h(a_1,a_2,\varphi)$$

For example, for Breit-Wigner(BW) functions with the following parameters: $m_1 = 0.878\ \mathrm{GeV/c^2},\ \Gamma_1 = 0.499\ \mathrm{GeV}$, $m_2 = 1.412\ \mathrm{GeV/c^2},\ \Gamma_2 = 0.294\ \mathrm{GeV}$,
$\mathbf{a_1 = 1.270,\ a_2 = 0.954,\ \varphi = 0.62}$;
the second solution is:
$\mathbf{a_1' = 3.268,\ a_2' = 1.481,\ \varphi' = 4.19}$.

Multiple solutions (two Breit-Wigner amplitides) (II)

In the case of s-dependent widths $\Gamma_{1,2}(s)$ the s-spectrum degeneration disappears and spectra for (a_1, a_2, φ) and (a_1', a_2', φ') sets become different:



But if the experimental errors are large enough, the χ^2 for both solutions will be almost the same, so we have to take into account both solutions, just like we have for $F_S(s)$, approximated by BW($K_0^*(800)$)+BW($K_0^*(1430)$).

In our case, the vector form factor, $F_V(s)$, is also described by a sum of two BWs (BW($K^*(892)$)+BW($K^*(1410)$)), but the statistics around $K^*(892)$ meson is so big that we can choose the best solution, for the second solution the χ^2 becomes notably higher.

In general, if the total amplitude is parametrized by sum of N BW functions (determined by $2N-1$ parameter set $(a_1, ..., a_N, \varphi_1, ..., \varphi_{N-1})$), there are 2^{N-1} solutions to check.

LASS parametrization of F_S

P. Estabrooks, Phys. Rev. D **19**, 2678 (1979).
D. Aston *et al.* (LASS), Nucl. Phys. B **296**, 493 (1988).

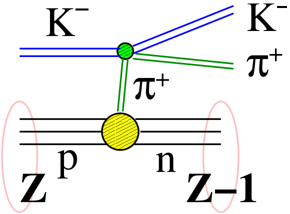
$$F_S = \lambda \frac{M_{K\pi}}{P} (\sin\delta_{\rm B} e^{i\delta_{\rm B}} + e^{2i\delta_{\rm B}} BW_{K_0^*(1430)}(M_{K\pi}))$$

$$\cot\delta_{\rm B} = \frac{1}{aP} + \frac{bP}{2}$$

$$a = (2.07 \pm 0.10)\ ({\rm GeV/c})^{-1}$$

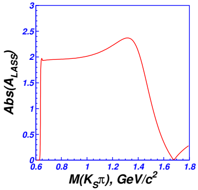
$$b = (3.32 \pm 0.34)\ ({\rm GeV/c})^{-1}$$

$$P = \frac{\sqrt{(M^2_{K\pi} - (M_{\rm K}+M_\pi)^2)(M^2_{K\pi} - (M_{\rm K}-M_\pi)^2)}}{2M_{K\pi}}$$

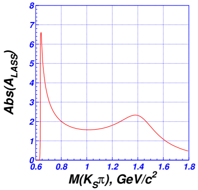


	$K^*(892)$+LASS a, b-fixed	$K^*(892)$+LASS a, b-free
M_{K^*}, ${\rm MeV/c^2}$	895.42 ± 0.19	895.38 ± 0.23
Γ_{K^*}, MeV	46.46 ± 0.47	46.53 ± 0.50
λ	0.282 ± 0.011	0.298 ± 0.012
a, $({\rm GeV/c})^{-1}$	2.13 ± 0.10	$10.9 + 7.4 - 3.0$
b, $({\rm GeV/c})^{-1}$	3.96 ± 0.31	$19.0 + 4.5 - 3.6$
$\chi^2/{\rm n.d.f.}$	$196.9/86$	$97.3/83$
$P(\chi^2)$, %	10^{-8}	13

LASS



Belle



Careful study of the $\tau^- \to K^0_S\pi^-\nu_\tau$ near the $K^0_S\pi$ production threshold is needed

$K^*(892)^\pm$ mass and width (I)

Model uncertainties in $K^*(892)^\pm$ mass and width are evaluated from approximations with the following models:
$K_0^*(800) + K^*(892) + K^*(1410)$, $K_0^*(800) + K^*(892) + K_0^*(1430)$, $K_0^*(800) + K^*(892) + K^*(1680)$, $K^*(892)$+LASS.

	$M(K^*(892))$, MeV/c^2	$\Gamma(K^*(892))$, MeV
This work	$895.47 \pm 0.20_{\rm stat} \pm 0.44_{\rm syst} \pm 0.59_{\rm mod}$	$46.2 \pm 0.6_{\rm stat} \pm 1.0_{\rm syst} \pm 0.7_{\rm mod}$
PDG-2017	891.76 ± 0.25	50.3 ± 0.8
Difference	3.71 ± 0.80	-4.1 ± 1.7

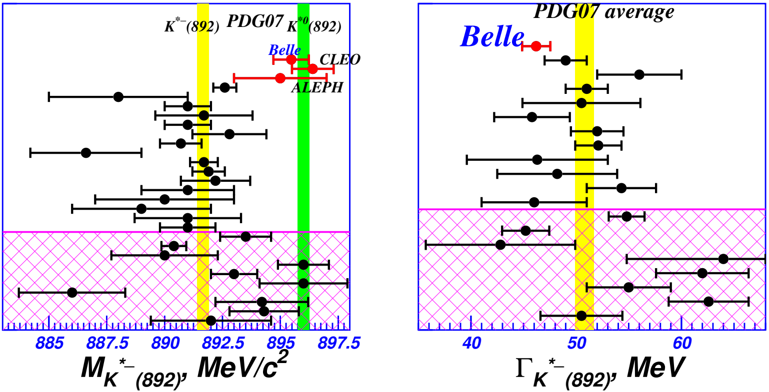
PDG average is based on the results from the fixed target experiments

894.3 $\pm$1.5	1150	[2,3] CLARK	73	HBC	$-$	3.3 $K^- p \to \overline{K}^0 \pi^- p$
892.0 $\pm$2.6	341	[2] SCHWEING...68		HBC	$-$	5.5 $K^- p \to \overline{K}^0 \pi^- p$

CHARGED ONLY, PRODUCED IN τ LEPTON DECAYS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
895.47$\pm$0.20$\pm$0.74	53k	[6] EPIFANOV	07	BELL	$\tau^- \to K_S^0 \pi^- \nu_\tau$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
895.3 $\pm$0.2		[7,8] JAMIN	08	RVUE	$\tau^- \to K_S^0 \pi^- \nu_\tau$
896.4 $\pm$0.9	11970	[9] BONVICINI	02	CLEO	$\tau^- \to K^- \pi^0 \nu_\tau$
895 $\pm$2		[10] BARATE	99R	ALEP	$\tau^- \to K^- \pi^0 \nu_\tau$

$K^*(892)^{\pm}$ mass and width (II)



The $K^*(892)^-$ width is compatible with the previous measurements within experimental errors, however the $K^*(892)^-$ mass value obtained in $\tau^- \to K^0_S\pi^-\nu_\tau$ is systematically higher than those before and is consistent with the world average value of the neutral $K^*(892)^0$ mass.

None of the previous measurements in PDG, all of which were performed more than 30 years ago, present the systematic uncertainties for their measurements !

Further studies at Belle II (I)

- **To elucidate the nature of the $K^*(892)^- - K^*(892)^0$ mass difference is fundamental task in the low energy hadron spectroscopy.**
- It is suggested to study simulataneously: $\tau^- \to K^0_S\pi^-\nu_\tau$, $\tau^- \to K^0_S\pi^-\pi^0\nu_\tau$, $\tau^- \to K^0_SK^-\nu_\tau$, $\tau^- \to K^0_SK^-\pi^0\nu_\tau$ for the modes with K^0_S. The modes with K^-/π^-: $\tau^- \to K^-\pi^0\nu_\tau$, $\tau^- \to K^-\pi^0\pi^0\nu_\tau$, $\tau^- \to \pi^-\pi^0\nu_\tau$, $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$; $\tau^- \to K^-\pi^+\pi^-\nu_\tau$, $\tau^- \to \pi^-K^+K^-\nu_\tau$, $\tau^- \to \pi^-\pi^+\pi^-\nu_\tau$.
- $K^*(892)^-$ mass and width can be measured in the clean experimental conditions without disturbance from the final state interactions in the $\tau^- \to K^0_S\pi^-\nu_\tau$ and $\tau^- \to K^-\pi^0\nu_\tau$ decays.
- Study of the $\tau^- \to K^0_S\pi^-\pi^0\nu_\tau$, $\tau^- \to K^-\pi^0\pi^0\nu_\tau$ and $\tau^- \to K^-\pi^+\pi^-\nu_\tau$ modes allows one to measure:
  1) simultaneously, in one mode the $K^*(892)^-(K^0_S\pi^-)$ and the $K^*(892)^0(K^0_S\pi^0)$ masses in the case of one accompanying pion. The effect of the pure hadronic interaction of the $K^*(892)^-(K^*(892)^0)$ and $\pi^0(\pi^-)$ on the $K^*(892)^-(K^*(892)^0)$ mass can be precisely measured.
  2) Cross check the impact of the hadronic (π^0) interactions on the $K^*(892)^-$ mass with $\tau^- \to K^-\pi^0\pi^0\nu_\tau$, cross check the impact of the hadronic (π^-) interactions on the $K^*(892)^0$ mass with $\tau^- \to K^-\pi^+\pi^-\nu_\tau$.
  3) It is possible to investigate precisely an effect of the pure hadronic interaction of the $K^*(892)^-(K^*(892)^0)$ and $K^0_S(K^-)$ on the $K^*(892)^-(K^*(892)^0)$ mass in the $\tau^- \to K^0_SK^-\pi^0\nu_\tau$ decay.
  4) Cross check the impact of the hadronic (K^-) interactions on the $K^*(892)^0$ mass with $\tau^- \to \pi^-K^+K^-\nu_\tau$.
- Hadronic τ decays with kaons provide unique laboratory to study K^*-family precisely.

CPV in hadronic τ decays at B factories

- CPV has not been observed in lepton decays
- It is strongly suppressed in the SM ($A^{\rm CP}_{\rm SM} \lesssim 10^{-12}$) and observation of large CPV in lepton sector would be clean sign of New Physics
- τ lepton provides unique possibility to search for CPV effects, as it is the only lepton decaying to hadrons, so that the associated strong phases allows us to visualize CPV in hadronic τ decays.

I. CPV in $\tau^- \to \pi^- K_S(\geq 0\pi^0)\nu_\tau$ at BaBar (Phys. Rev. D 85, 031102 (2012))

Data sample of $\int Ldt = 476$ fb^{-1} was analyzed

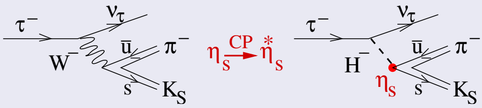
$$A_{\rm CP} = \frac{\Gamma(\tau^+\to\pi^+K^0_S(\geq 0\pi^0)\bar{\nu}_\tau)-\Gamma(\tau^-\to\pi^-K^0_S(\geq 0\pi^0)\nu_\tau)}{\Gamma(\tau^+\to\pi^+K^0_S(\geq 0\pi^0)\bar{\nu}_\tau)+\Gamma(\tau^-\to\pi^-K^0_S(\geq 0\pi^0)\nu_\tau)} = (-0.36 \pm 0.23 \pm 0.11)\%$$

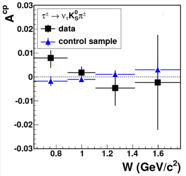
2.8σ deviation from the SM expectation: $A^{K^0}_{\rm CP} = (+0.36 \pm 0.01)\%$

II. CPV in $\tau^- \to K^0_S\pi^-\nu_\tau$ at Belle (Phys. Rev. Lett. 107, 131801 (2011)) $\int$Ldt=699 fb^{-1}

Angular distributions were analyzed, $A_{\rm CP}(W=\sqrt{s})$ was measured ($d\omega = d\cos\beta d\cos\theta$):

$$A_{\rm CP}(W) = \frac{\int \cos\beta\cos\psi\left(\frac{d\Gamma_{\tau^-}}{d\omega} - \frac{d\Gamma_{\tau^+}}{d\omega}\right)d\omega}{\frac{1}{2}\int\left(\frac{d\Gamma_{\tau^-}}{d\omega} + \frac{d\Gamma_{\tau^+}}{d\omega}\right)d\omega} \simeq \langle\cos\beta\cos\psi\rangle_{\tau^-} - \langle\cos\beta\cos\psi\rangle_{\tau^+}$$





CPV in $\tau^\pm \to K_S \pi^\pm \nu_\tau$ at Belle (I)

The $K_S^0\pi^-$ hadronic current is parametrized by vector ($F_V(s)$) and scalar ($F_S(s)$) form factor:

$$J^\mu = \langle K_S(q_1)\pi^-(q_2)|\bar{s}\gamma^\mu u|0\rangle = F_V(s)\left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{s}\right)(q_1 - q_2)_\nu + F_S(s)Q^\mu$$

Effect of CP violating scalar boson exchange diagram can be introduced by replacing the SM scalar form factor:

$$F_S(s) \to \overline{F}_S(s) = F_S(s) + \frac{\eta_S}{m_\tau}F_H(s),\ F_H = \langle K_S(q_1)\pi^-(q_2)|\bar{s}u|0\rangle = \frac{s}{m_s - m_u}F_S(s)$$

$$d\Gamma_{\tau^-}(\eta_S) \overset{CP}{\to} d\Gamma_{\tau^+}(\overset{*}{\eta}_S)$$

$\tau^- \to K_S\pi^-\nu_\tau$ differential decay width:

$$\frac{d\Gamma}{ds d\cos\beta d\cos\theta} = (A(s) - B(s)(3\cos^2\beta - 1)(3\cos^2\psi - 1))|F_V(s)|^2 + M_\tau^2|F_S|^2 +$$
$$+C(s)\cos\beta\cos\psi \mathrm{Re}(F_V \overset{*}{\overline{F}}_S(\eta_S))$$

To extract CPV term the following observable is defined in bin i-th of s ($d\omega = ds\, d\cos\theta\, d\cos\beta$):

$$A_i^{\rm CP} = \frac{\int_i \cos\beta\cos\psi\left(\frac{d\Gamma_{\tau^-}}{d\omega} - \frac{d\Gamma_{\tau^+}}{d\omega}\right)d\omega}{\frac{1}{2}\int_i\left(\frac{d\Gamma_{\tau^-}}{d\omega} + \frac{d\Gamma_{\tau^+}}{d\omega}\right)d\omega} \simeq \langle\cos\beta\cos\psi\rangle^i_{\tau^-} - \langle\cos\beta\cos\psi\rangle^i_{\tau^+}$$

CPV in $\tau^\pm \to K_S \pi^\pm \nu_\tau$ at Belle (II)

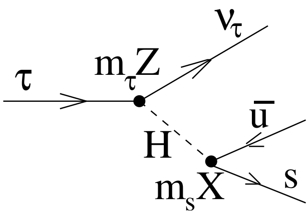
From the $A_i^{\rm CP}$ the CPV parameter $Im(\eta_S)$ can be extracted:

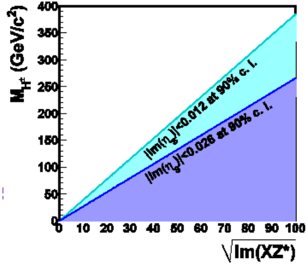
$$A_i^{\rm CP} \simeq {\rm Im}(\eta_S)\frac{N_s}{n_i}\int_i C(s)\frac{{\rm Im}(F_V F_H^*)}{m_\tau}ds \equiv c_i {\rm Im}(\eta_S)$$

Use several parametrizations of F_V and F_S from the previous Belle study of $M_{K_S\pi}$ spectrum and **floating relative phase ($\phi_S = 0^\circ ... 360^\circ$)**:

$$|{\rm Im}(\eta_S)| < (0.012 - 0.026) \text{ at } 90\% \text{ CL}$$

Theoretical predictions for ${\rm Im}(\eta_S)$ in MHDM:





$$\eta_S \simeq \frac{m_\tau m_s}{M_{H^\pm}^2}X^* Z \quad |{\rm Im}(XZ^*)| < 0.15\frac{M_{H^\pm}^2}{1\,{\rm GeV}^2/c^4} \quad (|{\rm Im}(\eta_S)| < 0.026)$$

Further studies at Belle II (II)

- In the analysis of the $\tau^- \to K_S^0 \pi^- \nu_\tau$ and $\tau^- \to K^- \pi^0 \nu_\tau$ decays, it is very desirable to measure separately vector (W_B), scalar (W_{SA}) form factors and the interference term (W_{SF}).
- $K^*(892)^-$ mass and width are measured in the vector form factor (properly taking into account the effect of the interference of the $K^*(892)^-$ amplitude with the contributions from the radial exitations, $K^*(1410)^-$ and $K^*(1680)^-$).
- The scalar form factor, W_{SA}, is important for the tests of the various fenomenological models and search for CPV.
- The interference between vector and scalar form factors, W_{SF}, is necessary in the search for CPV in $\tau^- \to K \pi \nu_\tau$ decays.

Further studies at Belle II (III)

A complete study of the hadronic τ decays into ≥ 3 hadrons can be done in the full multidimensional phase-space of the reaction:

$$\boldsymbol{e^+e^- \to (\tau^- \to \text{hadrons}^-\nu_\tau;\ \tau^+ \to \ell^+\nu_\ell\bar{\nu}_\tau)}$$

or

$$\boldsymbol{e^+e^- \to (\tau^- \to \text{hadrons}^-\nu_\tau;\ \tau^+ \to \rho^+\bar{\nu}_\tau)}$$

The parametrization of the hadronic current in the $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$ decay was established by CLEO in their unbinned analysis of the $e^+e^- \to (\tau^- \to \pi^-\pi^0\pi^0\nu_\tau,\ \tau^+ \to \ell^+\nu_\ell\bar{\nu}_\tau)$ process in the full phase space: D. M. Asner *et al.* [CLEO], Phys. Rev. D **61**, 012002 (2000).

$$J^\mu = \beta_1 j_1^\mu(\rho\pi^0)_{S-wave} + \beta_2 j_2^\mu(\rho'\pi^0)_{S-wave} + \beta_3 j_3^\mu(\rho\pi^0)_{D-wave} + \beta_4 j_4^\mu(\rho'\pi^0)_{D-wave} + \beta_5 j_5^\mu(f_2(1270)\pi)_{P-wave} + \beta_6 j_6^\mu(f_0(500)\pi)_{P-wave} + \beta_7 j_7^\mu(f_0(1370)\pi)_{P-wave}$$

- Before studying hadronic decays, leptonic decay should be analyzed (measurement of Michel parameters) to develop the fitter and polish the procedure (CLEO studied $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$ after they measured Michel parameters).
- The same procedure can be used to study dynamics of the $(\tau^\mp \to (K\pi)^\mp\nu\ ;\ \tau^\pm \to \rho^\pm\nu)$ and $(\tau^\mp \to (K\pi)^\mp\nu\ ;\ \tau^\pm \to \ell^\pm\nu\nu)$ processes and to search for CPV in $\tau^- \to (K\pi)^-\nu_\tau$ (also in the spin-dependent part of the differential decay width).

Further studies at Belle II (IV)

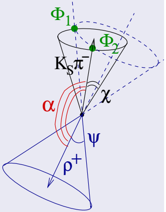
Analysis of the $(\tau^\mp \to (K\pi)^\mp\nu\,;\ \tau^\pm \to \rho^\pm\nu)$ events, search for CPV in $\tau^- \to (K\pi)^-\nu_\tau$.
The analysis of the decay products of both taus allows one to constrain direction of $\tau^- - \tau^+$ axis. Such a constraint is efficient to suppress background from $\tau^- \to (K\pi)^- K_L^0\nu_\tau$.

$$\frac{d\sigma(\vec{\zeta}^*, \vec{\zeta'}^*)}{d\Omega_\tau} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i^*\zeta_j'^*), \quad \frac{d\Gamma(\tau^\pm(\vec{\zeta'}^*) \to \rho^\pm\nu)}{dm_{\pi\pi}^2\, d\Omega_\rho^*\, d\tilde{\Omega}_\pi} = A' \mp \vec{B'}\vec{\zeta'}^*$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*) \to (K\pi)^\mp\nu)}{dm_{K\pi}^2\, d\Omega_{K\pi}^*\, d\tilde{\Omega}_\pi} = \begin{matrix} (A_0 + \eta_{CP}A_1) + (\vec{B}_0 + \eta_{CP}\vec{B}_1)\vec{\zeta}^* \\ (A_0 + \eta_{CP}^*A_1) - (\vec{B}_0 + \eta_{CP}^*\vec{B}_1)\vec{\zeta}^* \end{matrix}$$

$$\frac{d\sigma((K\pi)^\mp, \rho^\pm)}{dm_{K\pi}^2\, d\Omega_{K\pi}^*\, d\tilde{\Omega}_\pi\, dm_{\pi\pi}^2\, d\Omega_\rho^*\, d\tilde{\Omega}_\pi\, d\Omega_\tau} = \frac{\alpha^2\beta_\tau}{64E_\tau^2}\begin{pmatrix} \mathcal{F} + \eta_{CP}\mathcal{G} \\ \mathcal{F} + \eta_{CP}^*\mathcal{G} \end{pmatrix}$$

$$\mathcal{F} = D_0A_0A' - D_{ij}B_{0i}B_j', \quad \mathcal{G} = D_0A_1A' - D_{ij}B_{1i}B_j'$$



$$\frac{d\sigma((K\pi)^\mp, \rho^\pm)}{dp_{K\pi}\, d\Omega_{K\pi}\, dm_{K\pi}^2\, d\tilde{\Omega}_\pi\, dp_\rho\, d\Omega_\rho\, dm_{\pi\pi}^2\, d\tilde{\Omega}_\pi} = \sum_{\Phi_1,\Phi_2} \frac{d\sigma((K\pi)^\mp, \rho^\pm)}{dm_{K\pi}^2\, d\Omega_{K\pi}^*\, d\tilde{\Omega}_\pi\, dm_{\pi\pi}^2\, d\Omega_\rho^*\, d\tilde{\Omega}_\pi\, d\Omega_\tau}\left|\frac{\partial(\Omega_{K\pi}^*, \Omega_\rho^*, \Omega_\tau)}{\partial(p_{K\pi}, \Omega_{K\pi}, p_\rho, \Omega_\rho)}\right|$$

CPV parameter $\eta_{\rm CP}$ is extracted in the simultaneous unbinned maximum likelihood fit of the $((K\pi)^-, \rho^+)$ and $((K\pi)^+, \rho^-)$ events in the 12D phase space.

Further studies at Belle II (V)

Such kind of analysis was done only once by CLEO for the $(\tau \to \ell\nu\nu;\ \tau \to \pi\pi^0\pi^0\nu)$ events to study the dynamics of $\tau \to \pi\pi^0\pi^0\nu$ decay.

If we pretend on the $\lesssim$ 1% level in the studies of the dynamics of hadronic τ decays, the research program for any hadronic τ decay should be:

- Measure hadronic structure functions on the signal side.
- Perform the unbinned fit of the full event configuration $(\tau \to \text{signal};\ \tau \to \text{tag})$, where the dynamics of the $\tau \to \text{tag}$ is well known (for example, leptonic tag). Identify the structure of the "remnant" from the spin-spin correlation term and correct the hadronic structure functions, measured on the first step.
- Extract CPV parameter in the simulataneous approximation of the $(\tau^- \to \text{signal}^-;\ \tau^+ \to \text{tag}^+)$ and $(\tau^+ \to \text{signal}^+;\ \tau^- \to \text{tag}^-)$ events.
- The usage of the proper generator (TAUOLA), where the effects related to τ spin are implemented, is mandatory.

Summary

- The world largest statistics of τ leptons collected by Belle and *BABAR* opens new era in the precision tests of the Standard Model, search for the effects of New Physics and precision studies of low energy QCD.
  Belle II is the main player in τ studies in the nearest future.
- Belle and *BABAR* essentially improved the accuracy of the branching fractions of $\tau^- \to (K\pi)^-\nu_\tau$ decays.
- At Belle the $K^0_S\pi$ invariant mass spectrum was studied. The $K^*(892)$ alone is not sufficient to describe the $K^0_S\pi$ mass spectrum. The best description is achieved with the $K^*_0(800) + K^*(892) + K^*(1410)$ and $K^*_0(800) + K^*(892) + K^*_0(1430)$ models. There is large systematic uncertainty in the near $K^0_S\pi$ production threshold part of the spectrum due to the large background from the $\tau^- \to K^0_S\pi^-K^0_L\nu_\tau$ decay, whose dynamics is not precisely known. In the new study it will be possible to suppress this background essentially applying special kinematical constraints.
- For the first time the the $K^*(892)^-$ mass and width have been measured in τ decay at B factories. The $K^*(892)^-$ mass is significantly different from the current world average value, it agrees with the $K^*(892)^0$ mass.
  Future high precision measurements of the $\boldsymbol{K^*(892)^-}$ parameters at Belle II are necessary to clarify this discrepancy.
- Simulataneous study of the discussed τ decays with kaon at Belle II as well as the $e^+e^- \to K^0_SK^\pm\pi^\mp$, $e^+e^- \to K^+K^-\pi^0$, $e^+e^- \to K^0_SK^0_L\pi^0$ reactions at the VEPP-2000 could provide additional valuable information about the $K^*(892)^-$ mass, namely unveil an impact of the hadronic and electromagnetic interactions in the final state.
- Hadronic structure functions in $\tau^- \to (K\pi)^-\nu_\tau$ can/should be measured precisely at Belle II.
- The unbinned analysis of the reaction $e^+e^- \to (\tau^- \to \text{hadrons}^-\nu_\tau;\ \tau^+ \to \ell^+\nu_\ell\bar{\nu}_\tau)$ or $e^+e^- \to (\tau^- \to \text{hadrons}^-\nu_\tau;\ \tau^+ \to \rho^+\bar{\nu}_\tau)$ in the full miltidimensional phase space opens the fruitful possibility for the comprehensive investigation of the dynamics of hadronic τ decays. It is very acute for the improved searches for the CPV violation in hadronic τ decays.