Physics of tau lepton

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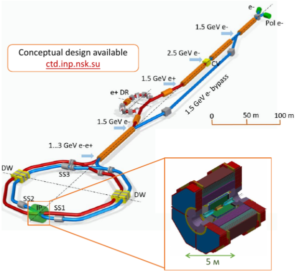
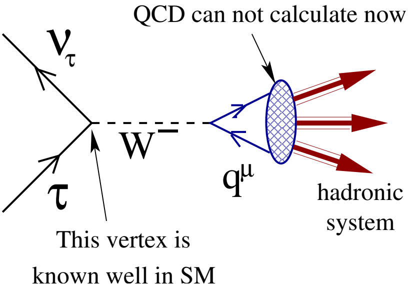
Outline:

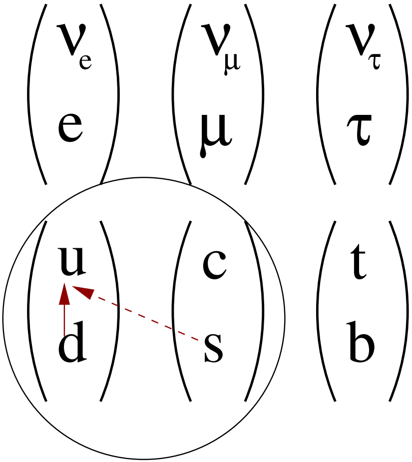
1. Introduction
2. Study of leptonic τ decays
3. Study of hadronic τ decays
4. Search for EDM, LFV, Majorana/sterile neutrinos
5. Summary



Introduction: τ physics (I)

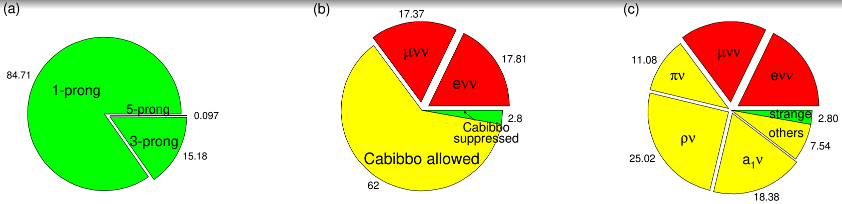
τ is the only lepton decaying to hadrons.



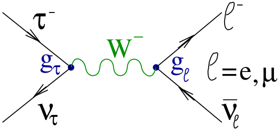


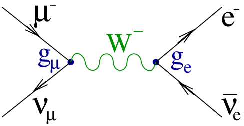
There is no strong interaction between initial and final particles.

Introduction: τ physics (II)

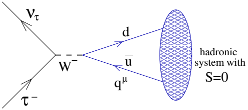


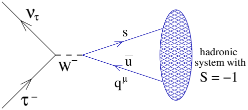
Leptonic decays





Hadronic decays

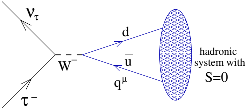




$$\Gamma_\tau \approx \frac{G_F^2 m_\tau^5}{192\pi^3}(1 + 1 + 3(\cos^2\theta_c + \sin^2\theta_c)) = 5\frac{G_F^2 m_\tau^5}{192\pi^3}$$

Cabibbo allowed decays ($\mathcal{B} \sim \cos^2\theta_c$)

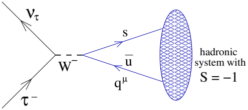
$$\tau^- \to \text{hadrons}(S=0)^- \nu_\tau$$



Mode	$\mathcal{B}$, %
$\pi^-\pi^0\nu_\tau$	25.49 ± 0.09
$\pi^-\nu_\tau$	10.82 ± 0.05
$\pi^-\pi^0\pi^0\nu_\tau$	9.26 ± 0.10
$\pi^-\pi^-\pi^+\nu_\tau$	9.31 ± 0.05
$\pi^-\pi^-\pi^+\pi^0\nu_\tau$	4.62 ± 0.05
$K^-K^0\nu_\tau$	0.148 ± 0.005
$K^0\bar{K}^0\pi^-\nu_\tau$	0.155 ± 0.024
$K^-K^0\pi^0\nu_\tau$	0.150 ± 0.007
$K^-K^+\pi^-\nu_\tau$	0.144 ± 0.003

Cabibbo suppressed decays ($\mathcal{B} \sim \sin^2\theta_c$)

$$\tau^- \to \text{hadrons}(S=-1)^- \nu_\tau$$



Mode	$\mathcal{B}$, 10^{-3}
$\bar{K}^0\pi^-\nu_\tau$	8.40 ± 0.14
$K^-\nu_\tau$	6.96 ± 0.10
$K^-\pi^0\nu_\tau$	4.33 ± 0.15
$\bar{K}^0\pi^-\pi^0\nu_\tau$	3.82 ± 0.13
$K^-\pi^0\pi^0\nu_\tau$	0.65 ± 0.22
$K^-3\pi^0\nu_\tau$	0.48 ± 0.21
$K^-\pi^+\pi^-\nu_\tau$	3.45 ± 0.07
$K^-\pi^+\pi^-\pi^0\nu_\tau$	1.31 ± 0.12
$K^-K^+K^-\nu_\tau$	0.022 ± 0.008

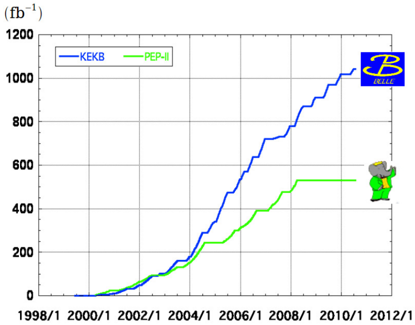
Cabibbo suppression in $\mathcal{B}$: $\tan^2\theta_c \simeq 1/20$

Introduction: τ physics

- In the SM τ decays due to the charged weak interaction described by the exchange of $W^{\pm}$ with a pure vector coupling to only left-handed fermions. There are two main classes of tau decays:
  - Decays with leptons, like: $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$, $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma$, $\tau^- \to \ell^- \ell'^+ \ell'^- \bar{\nu}_\ell \nu_\tau$; $\ell, \ell' = e, \mu$. They provide very clean laboratory to probe electroweak couplings, which is complementary/competitive to precision studies with muon (in experiments with muon beam). Plenty of New Physics models can be tested/constrained in the precision studies of the dynamics of decays with leptons.
  - Hadronic decays of τ offer unique tools for the precision study of low energy QCD.
- The world largest statistics of τ leptons collected by e^+e^- B factories (Belle and *BABAR*) opens new era in the precision tests of the Standard Model (SM).
  Still, many interesting and important studies with τ lepton will be done using Belle/*BABAR* statistics.
- **Belle II and Super Charm-Tau Factory (SCTF) are the next players in this area.**

Introduction: e^+e^- B factories

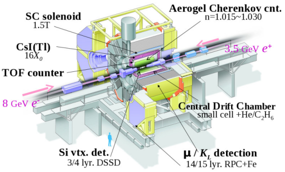
Integrated luminosity of B factories



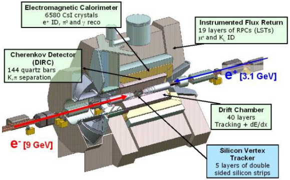
> 1 ab^{-1}
On resonance:
$\Upsilon(5S)$: 121 fb^{-1}
$\Upsilon(4S)$: 711 fb^{-1}
$\Upsilon(3S)$: 3 fb^{-1}
$\Upsilon(2S)$: 25 fb^{-1}
$\Upsilon(1S)$: 6 fb^{-1}
Off reson./scan:
$\sim$ 100 fb^{-1}

$\sim$ 550 fb^{-1}
On resonance:
$\Upsilon(4S)$: 433 fb^{-1}
$\Upsilon(3S)$: 30 fb^{-1}
$\Upsilon(2S)$: 14 fb^{-1}
Off resonance:
$\sim$ 54 fb^{-1}

Belle Detector



BABAR detector



$$\sigma(b\bar{b}) = 1.05 \text{ nb} \quad N_{b\bar{b}} = 1.2 \times 10^9$$
$$\sigma(c\bar{c}) = 1.30 \text{ nb} \quad N_{c\bar{c}} = 2.0 \times 10^9$$
$$\sigma(\tau\tau) = 0.92 \text{ nb} \quad N_{\tau\tau} = 1.4 \times 10^9$$

B-factories are also charm- and τ-factories !

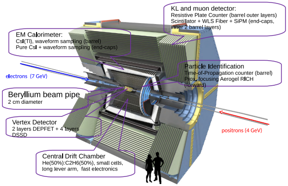
Introduction: Belle II and SCTF

Belle II with unpolarized beams

Planned integrated luminosity is 50 ab^{-1}

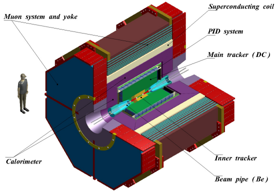
$$\sigma(b\bar{b}) = 1.05\text{ nb} \quad N_{b\bar{b}} = 53 \times 10^9$$
$$\sigma(c\bar{c}) = 1.30\text{ nb} \quad N_{c\bar{c}} = 65 \times 10^9$$
$$\boldsymbol{\sigma(\tau\tau) = 0.92\text{ nb} \quad N_{\tau\tau} = 46 \times 10^9}$$

Belle II detector



SCTF with polarized e^- beam

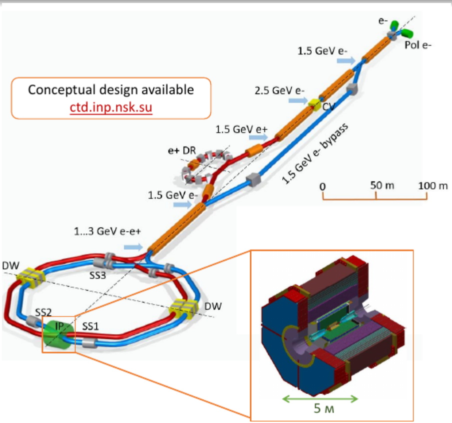
In five c.m.s. energy points
($2E = 3.554,\ 3.686,\ 3.770,\ 4.170,\ 4.650$ GeV)
it is planned to accumulate 7 ab^{-1}, which
corresponds to $\boldsymbol{N_{\tau\tau} = 21 \times 10^9}$

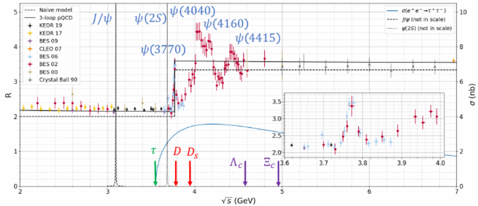


The polarized e^- beam results in the nonzero average polarization of single τ, which provide advantages in some studies with τ lepton, where the τ spin-dependent effects dominate

The SCTF in Russia

- **Electron-positron collider**
  - Center-of-mass energy range $\sqrt{s} = 2E_{\mathrm{beam}} = (3-7)\,\mathrm{GeV}$
  - Luminosity $L = 10^{35}\,1/\mathrm{cm}^2/\mathrm{s}$ at $2E = 4\,\mathrm{GeV}$
  - Longitudinal e^- beam polarization $(\mathcal{P}_e > 0.5)$
- **Universal particle detector**
  - Advanced inner and main tracker
  - Electromagnetic calorimeter
  - PID system
  - Magnet and Muon system
  - **DAQ with the trigger rate up to 300 kHz at the J/Ψ peak**





$2E$, GeV	N/year
$3.55 \div 4.3$	$2\cdot10^9\ \tau^+\tau^-$
3.1	$10^{12}\ J/\Psi$
3.69	$10^{11}\ \psi(2S)$
3.77	$10^9\ D\bar{D}$
4.17	$10^8\ D_s\bar{D}_s$
4.65	$10^8\ \Lambda_c^+\Lambda_c^-$

Introduction: τ properties at B factories

- **Tau mass:**
  BES3: $m_\tau = (1776.91 \pm 0.12(\mathrm{stat}) \pm {}^{0.10}_{0.13}(\mathrm{syst}))$ MeV/c^2; PRD 90, 012001 (2014)
  KEDR: $m_\tau = (1776.81 \pm {}^{0.25}_{0.23}(\mathrm{stat}) \pm 0.15(\mathrm{syst}))$ MeV/c^2; JETPL 85, 347 (2007)
  Belle: $m_\tau = (1776.61 \pm 0.13(\mathrm{stat}) \pm 0.35(\mathrm{syst}))$ MeV/c^2; PRL 99, 011801 (2007)
  BABAR: $m_\tau = (1776.68 \pm 0.12(\mathrm{stat}) \pm 0.41(\mathrm{syst}))$ MeV/c^2; PRD 80, 092005 (2009)
- **Tau lifetime:**
  Belle: $\tau_\tau = (290.17 \pm 0.53(\mathrm{stat}) \pm 0.33(\mathrm{syst}))$ fs; PRL 112, 031801 (2014)
  BABAR(prelim.): $\tau_\tau = (289.40 \pm 0.91(\mathrm{stat}) \pm 0.90(\mathrm{syst}))$ fs; Nucl. Phys. B 144, 105 (2005)
- **Michel parameters in $\tau \to \ell\nu\nu$ (ρ, η, ξ, δ):**
  Belle: Systematic uncertainties are about $(1 \div 3)$%; arXiv:1409.4969
- **Study of the radiative leptonic decays $\tau \to \ell\nu\nu\gamma$:**
  BABAR: Measurement of $\mathcal{B}(\tau \to \ell\nu\nu\gamma)$; PRD 91, 051103(R) (2015)
  Belle: $\bar{\eta} = -1.3 \pm 1.5 \pm 0.8$, $\xi\kappa = 0.5 \pm 0.4 \pm 0.2$; PTEP **2018** no.2, 023C01
- **Study of the 5-lepton decays $\tau \to \ell\ell'^+\ell'^-\nu\nu$:**
  CLEO: $\mathcal{B}(\tau \to ee e\nu\nu) = (2.8 \pm 1.5) \times 10^{-5}$,
  $\mathcal{B}(\tau \to \mu ee\nu\nu) < 3.6 \times 10^{-5} (CL = 90\%)$; PRL 76, 2637 (1996)
  Belle: statistical uncertainties are about $(3 \div 5)$%; J. Phys. Conf. Ser. **912** (2017) no.1, 012002.
- **Lepton universality with $\tau \to \ell\nu\nu$ and $\tau \to h\nu$ (h=π,K):**
  BABAR : $\left(\frac{g_\mu}{g_e}\right)_\tau = 1.0036 \pm 0.0020$, $\left(\frac{g_\tau}{g_\mu}\right)_\mathrm{h} = 0.9850 \pm 0.0054$; PRL 105, 051602 (2010)
- **Tau electric dipole moment (EDM):**
  Belle: $\mathrm{Re}(d_\tau) = (-0.62 \pm 0.63) \times 10^{-17}$ e·cm, $\mathrm{Im}(d_\tau) = (-0.40 \pm 0.32) \times 10^{-17}$ e·cm; submitted to JHEP in 2021 ($\int Ldt = 833$ fb^{-1})
- **Hadronic contribution to a_μ ($\tau^- \to \pi^-\pi^0\nu_\tau$):**
  Belle: $a_\mu^{\pi\pi} = (523.5 \pm 1.1(\mathrm{stat}) \pm 3.7(\mathrm{syst})) \times 10^{-10}$; PRD 78, 072006 (2008)

Michel parameters in τ decays

In the SM, charged weak interaction is described by the exchange of $W^{\pm}$ with a pure vector coupling to only left-handed fermions ("V-A" Lorentz structure). Deviations from "V-A" indicate New Physics. $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ ($\ell = e, \mu$) decays provide clean laboratory to probe electroweak couplings.
The most general, Lorentz invariant four-lepton interaction matrix element:

$$\mathcal{M} = \frac{4G}{\sqrt{2}} \sum_{\substack{N=S,V,T \\ i,j=L,R}} g_{ij}^N \left[\bar{u}_i(l^-)\Gamma^N v_n(\bar{\nu}_l)\right] \left[\bar{u}_m(\nu_\tau)\Gamma_N u_j(\tau^-)\right],$$

$$\Gamma^S = 1,\ \Gamma^V = \gamma^\mu,\ \Gamma^T = \frac{i}{2\sqrt{2}}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

Ten couplings g_{ij}^N, in the SM the only non-zero constant is $g_{LL}^V = 1$

Four bilinear combinations of g_{ij}^N, which are called as Michel parameters (MP): $\boldsymbol{\rho}$, $\boldsymbol{\eta}$, $\boldsymbol{\xi}$ and $\boldsymbol{\delta}$ appear in the energy spectrum of the outgoing lepton:

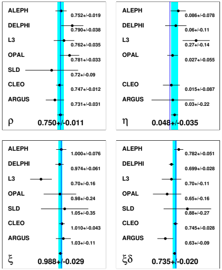
$$\frac{d\Gamma(\tau^\mp)}{d\Omega dx} = \frac{4G_F^2 M_\tau E_{\max}^4}{(2\pi)^4}\sqrt{x^2 - x_0^2}\left(x(1-x) + \frac{2}{9}\boldsymbol{\rho}(4x^2 - 3x - x_0^2) + \boldsymbol{\eta} x_0(1-x)\right.$$

$$\left.\mp\frac{1}{3}P_\tau cos\theta_\ell \boldsymbol{\xi}\sqrt{x^2 - x_0^2}\left[1 - x + \frac{2}{3}\boldsymbol{\delta}\left(4x - 4 + \sqrt{1 - x_0^2}\right)\right]\right),\ x = \frac{E_\ell}{E_{\max}},\ x_0 = \frac{m_\ell}{E_{\max}}$$

In the SM: $\boldsymbol{\rho} = \frac{3}{4}$, $\boldsymbol{\eta} = 0$, $\boldsymbol{\xi} = 1$, $\boldsymbol{\delta} = \frac{3}{4}$

Michel parameters of τ, current status, NP

Michel par.	Measured value	Experiment	SM value
ρ (e or μ)	$0.747 \pm 0.010 \pm 0.006$ 1.2%	CLEO-97	3/4
η (e or μ)	$0.012 \pm 0.026 \pm 0.004$ 2.6%	ALEPH-01	0
ξ (e or μ)	$1.007 \pm 0.040 \pm 0.015$ 4.3%	CLEO-97	1
$\xi\delta$ (e or μ)	$0.745 \pm 0.026 \pm 0.009$ 2.8%	CLEO-97	3/4
ξ_{h} (all hadr.)	$0.992 \pm 0.007 \pm 0.008$ 1.1%	ALEPH-01	1



In BSM models the couplings to τ are expected to be enhanced in comparison with μ.

- **Type II 2HDM:** $\eta_\mu(\tau) = \frac{m_\mu M_\tau}{2}\left(\frac{tan^2\beta}{M^2_{H\pm}}\right)^2$; $\frac{\eta_\mu(\tau)}{\eta_e(\mu)} = \frac{M_\tau}{m_e} \approx 3500$
- **Tensor interaction:** $\mathcal{L} = \frac{g}{2\sqrt{2}} W^\mu \left\{\bar{\nu}\gamma_\mu(1-\gamma^5)\tau + \frac{\kappa_\tau^W}{2m_\tau}\partial^\nu\left(\bar{\nu}\sigma_{\mu\nu}(1-\gamma^5)\tau\right)\right\}$,
  $-0.096 < \kappa_\tau^W < 0.037$: DELPHI Abreu EPJ C16 (2000) 229.
- **Unparticles:** Moyotl PRD 84 (2011) 073010, Choudhury PLB 658 (2008) 148.
- **Lorentz and CPTV:** Hollenberg PLB **701** (2011) 89
- **Heavy Majorana neutrino:** M. Doi *et al.*, Prog. Theor. Phys. **118** (2007) 1069.

Effect of the e^- beam polarization at the SCTF

At the SCTF with polarized electron beam the average polarization of single τ is nonzero, hence the differential decay probability will contain both, τ spin-dependent and spin-independent parts.

$$\frac{d\sigma(\vec{\zeta}^-,\vec{\zeta}^+)}{d\Omega_\tau} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i^-\zeta_j^+ + \mathcal{P}_e(F_i^-\zeta_i^- + F_j^+\zeta_j^+))$$

$$D_0 = 1 + \cos^2\theta + \frac{1}{\gamma_\tau^2}\sin^2\theta,\ \mathcal{P}_e = \frac{N_e(+) - N_e(-)}{N_e(+) + N_e(-)}$$

$$D_{ij} = \begin{pmatrix} (1+\frac{1}{\gamma_\tau^2})\sin^2\theta & 0 & \frac{1}{\gamma_\tau}\sin 2\theta \\ 0 & -\beta_\tau^2\sin^2\theta & 0 \\ \frac{1}{\gamma_\tau}\sin 2\theta & 0 & 1+\cos^2\theta - \frac{1}{\gamma_\tau^2}\sin^2\theta \end{pmatrix}$$

Single τ studies at the SCTF:

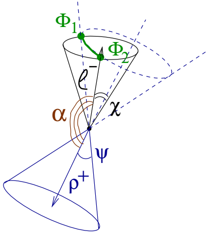
$$\frac{d\sigma(\vec{\zeta}^-)}{d\Omega_\tau} = \frac{\alpha^2}{32E_\tau^2}\beta_\tau(D_0 + \mathcal{P}_e F_i^-\zeta_i^-)$$

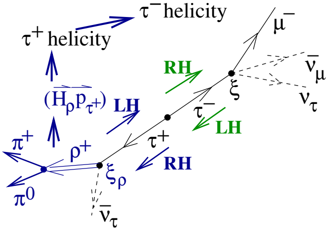
As a result, there are two methods to measure MP:

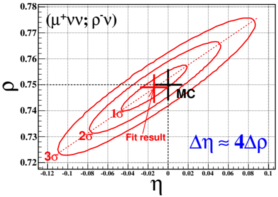
- **(I) Unbinned fit of the $(\ell,\ \rho)$ events in 9D phase space (spin-spin correlations + polarized e^- beam)**
- **(II) Unbinned fit of the $(\ell,\ \mathrm{all})$ events in 3D lepton phase space (only polarized e^- beam)**

Method at e^+e^- factory with unpolarized beams

Effect of τ spin-spin correlation is used to measure ξ and δ MP.
Events of the $(\tau^\mp \to \ell^\mp\nu\nu;\, \tau^\pm \to \rho^\pm\nu)$ topology are used to measure: ρ, η, $\xi_\rho\xi$ and $\xi_\rho\xi\delta$, while $(\tau^\mp \to \rho^\mp\nu;\, \tau^\pm \to \rho^\pm\nu)$ events are used to extract ξ_ρ^2.







$$\frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dE_\ell^*\,d\Omega_\ell^*\,d\Omega_\rho^*\,dm_{\pi\pi}^2\,d\tilde{\Omega}_\pi\,d\Omega_\tau} = A_0 + \rho A_1 + \eta A_2 + \xi_\rho\xi A_3 + \xi_\rho\xi\delta A_4 = \sum_{i=0}^{4} A_i\Theta_i$$

$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dp_\ell\,d\Omega_\ell\,dp_\rho\,d\Omega_\rho\,dm_{\pi\pi}^2\,d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dE_\ell^*\,d\Omega_\ell^*\,d\Omega_\rho^*\,dm_{\pi\pi}^2\,d\tilde{\Omega}_\pi\,d\Omega_\tau} \left|\frac{\partial(E_\ell^*,\Omega_\ell^*,\Omega_\rho^*,\Omega_\tau)}{\partial(p_\ell,\Omega_\ell,p_\rho,\Omega_\rho,\Phi_\tau)}\right| d\Phi_\tau$$

$$L = \prod_{k=1}^{N}\mathcal{P}^{(k)},\ \mathcal{P}^{(k)} = \mathcal{F}(\vec{z}^{(k)})/\mathcal{N}(\vec{\Theta}),\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z},\ \vec{\Theta} = (1,\rho,\eta,\xi_\rho\xi_\ell,\xi_\rho\xi_\ell\delta_\ell)$$

$$\mathcal{P}_{\text{total}} = (1-\sum_{i=1}^{4}\lambda_i)\mathcal{P}_{signal}^{\ell-\rho} + \lambda_1\mathcal{P}_{bg}^{\ell-3\pi} + \lambda_2\mathcal{P}_{bg}^{\pi-\rho} + \lambda_3\mathcal{P}_{bg}^{\rho-\rho} + \lambda_4\mathcal{P}_{bg}^{other}\,\text{(MC)}$$

MP are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu;\ \rho\nu)$ events in the 9D phase space $\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m_{\pi\pi}^2,\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$ in CMS.

Analysis of $(\ell,\ \mathrm{all})$ events in 3D

$$\frac{d\sigma(\vec{\zeta})}{d\Omega_\tau} = \frac{\alpha^2}{32E_\tau^2}\beta_\tau(D_0 + \mathcal{P}_e F_i \zeta_i)$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*) \to \ell^\mp\nu\nu)}{dx^* d\Omega_\ell^*} = \kappa_\ell(A(x^*) \mp \xi_\ell \vec{n}_\ell^* \vec{\zeta}^* B(x^*)),\ x^* = E_\ell^*/E_{\ell max}^*$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*),\ B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\sigma(\ell^\mp)}{dE_\ell^* d\Omega_\ell^* d\Omega_\tau} = \kappa_\ell \frac{\alpha^2\beta_\tau}{32E_\tau^2}(D_0 A(E_\ell^*) \mp \mathcal{P}_e \xi_\ell F_i n_{\ell i}^* B(E_\ell^*))$$

$$\frac{d\sigma(\ell^\mp)}{dp_\ell d\Omega_\ell} = \int\limits_{\Omega_\tau - \mathrm{sector}} \frac{d\sigma(\ell^\mp)}{dE_\ell^* d\Omega_\ell^* d\Omega_\tau}\left|\frac{\partial(E_\ell^*, \Omega_\ell^*)}{\partial(p_\ell, \Omega_\ell)}\right| d\Omega_\tau$$

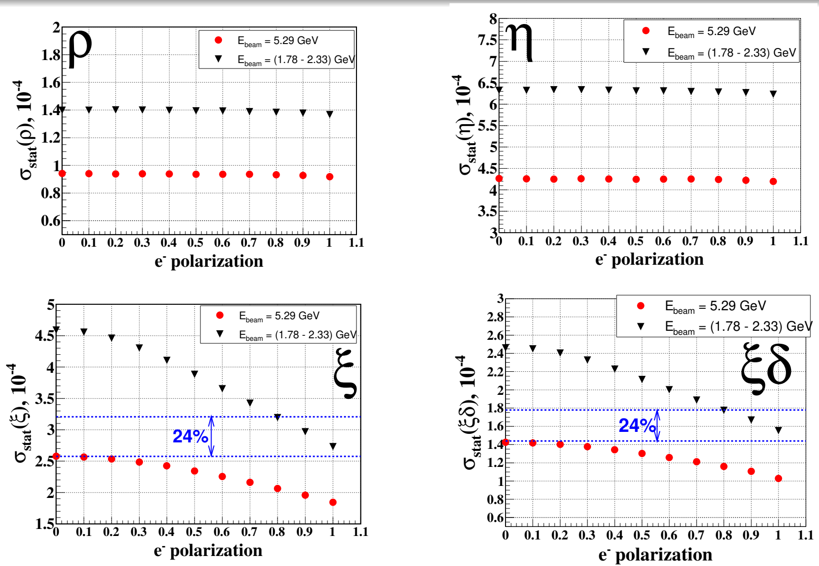
Ω_τ-sector is determined by the kinematical constraint $m_{\nu\nu} > 0$

- All Michel parameters (ρ, η, $\mathcal{P}_e\xi$, $\mathcal{P}_e\xi\delta$) are measured in the unbinned maximum likelihood fit of ($\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau;\ \tau^+ \to \mathrm{all}$) events in the **3D** phase space.
- The reduced 3D phase space allows one to tabulate various EXP/MC corrections to the detection efficiency more precisely.
- **The crucial point in this method is to have high-efficiency 1-track trigger.**

Toy MC studies of the effect of polarized e^- beam

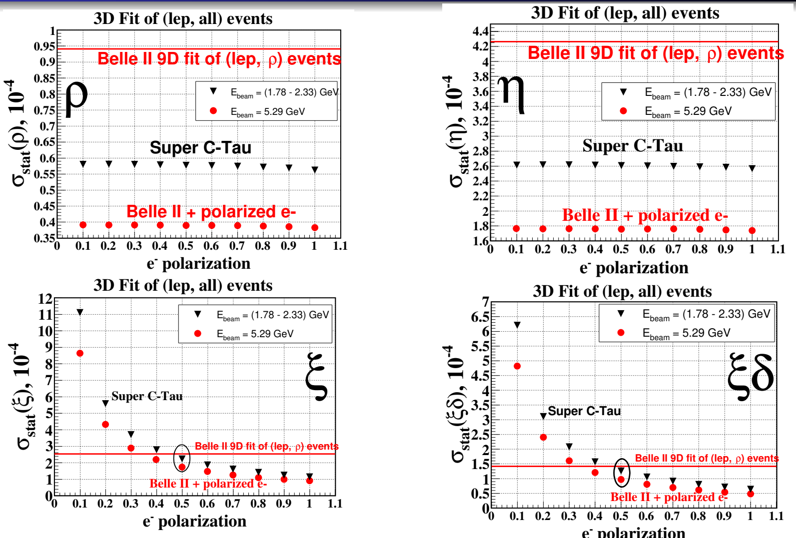
- 66 10M $(\mu,\ \rho)$ samples, at 6 center-of-mass (c.m.s.) energies (according to Table 1.1 in SCTF CDR part I) : $2E = 3.554$ GeV ($\tau^+\tau^-$ production threshold), $2E = 3.686$ GeV ($\psi(2S)$), $2E = 3.770$ GeV ($\psi(3770)$), $2E = 4.170$ GeV ($\psi(4160)$), $2E = 4.650$ GeV (maximum of the $\sigma(e^+e^- \to \Lambda_c^+\Lambda_c^-)$), $2E = 10.58$ GeV (Belle II), for 11 values of e^- beam polarization: 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, were generated for the calculation of the normalizations. 66 statistically independent 1M samples at the same energies and polarizations were generated for the fit.
- To evaluate MP sensitivities (rescaling the sensitivities obtained in the fits of 1M samples) we took the detection efficiency of $(\mu,\ \rho)$ events to be 20% (to be compared with 12% efficiency obtained at Belle, where the π^0 rec. efficiency is only 40%). The detection efficiency of (μ, all) events was taken to be 30%.
- To measure ρ, ξ and $\xi\delta$ MP, samples with $\ell = e,\ \mu$ were taken into account, while η MP is measured in samples with $\ell = \mu$ only.

Fit of (ℓ, ρ) in 9D at Belle II and SCTF



Sensitivities to Michel par. at Belle II (with unpolarized e^- beam) are slightly better (by a factor of 1.2–1.5) than those at the SCTF.

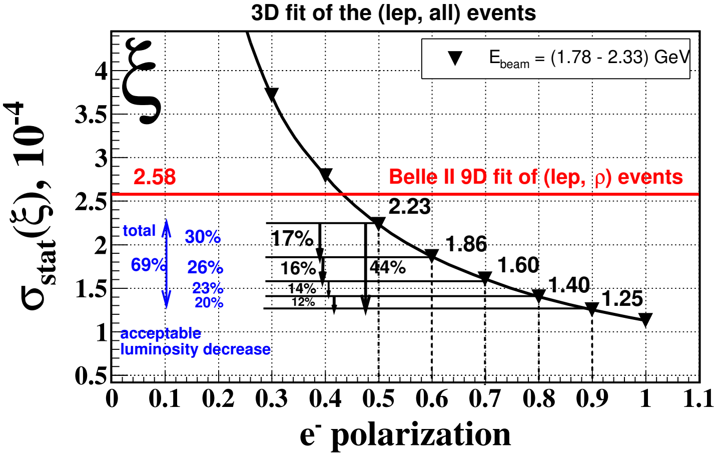
Fit of $(\ell,\ \mathrm{all})$ in 3D at Belle II and SCTF



The sensitivities to all Michel par. at the SCTF become slightly better than those at Belle II (with unpolarized e^- beam) for $\mathcal{P}_e > 0.5$.

Expected MP stat. uncertainties are $\sim 10^{-4}$, to reach the same level systematic uncertainty, the NNLO corrections ($\mathcal{O}(\alpha^4)$) to the differential $e^+e^- \to \tau^+\tau^-$ cross section are mandatory.

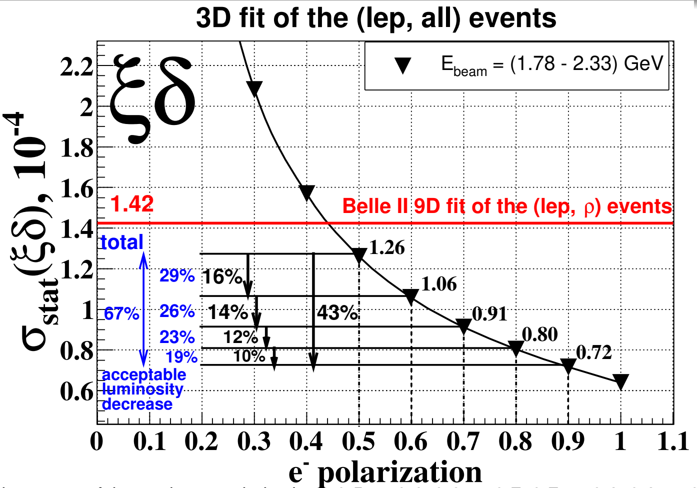
ξ from the fit of $(\ell,\ \text{all})$ in 3D at Super C-Tau



For the increases of the e^- beam polarizations, 0.5 $\to$ 0.6, 0.6 $\to$ 0.7, 0.7 $\to$ 0.8, 0.8 $\to$ 0.9, the corresponding improvements in the sensitivities to the ξ parameter, 17%, 16%, 14%, 12%, respectively. If we move from the polarization of 0.5 to the higher polarizations: 0.5 $\to$ 0.6, 0.5 $\to$ 0.7, 0.5 $\to$ 0.8, 0.5 $\to$ 0.9, the acceptable luminosity decrease factors (to keep the sensitivity at the level of that we have for polarization 0.5) are:
$(1.86/2.23)^2 = 0.70$, $(1.60/2.23)^2 = 0.51$, $(1.40/2.23)^2 = 0.39$, $(1.25/2.23)^2 = 0.31$,

respectively.

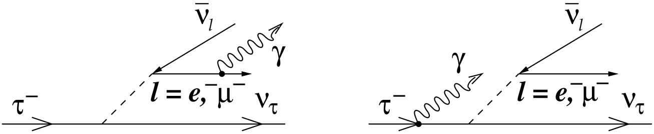
$\xi\delta$ from the fit of $(\ell,\ \mathrm{all})$ in 3D at Super C-Tau



For the increases of the e^- beam polarizations, 0.5 $\to$ 0.6, 0.6 $\to$ 0.7, 0.7 $\to$ 0.8, 0.8 $\to$ 0.9, the corresponding improvements in the sensitivities to the $\xi\delta$ parameter, 16%, 14%, 12%, 10%, respectively. If we move from the polarization of 0.5 to the higher polarizations: 0.5 $\to$ 0.6, 0.5 $\to$ 0.7, 0.5 $\to$ 0.8, 0.5 $\to$ 0.9, the acceptable luminosity decrease factors (to keep the sensitivity at the level of that we have for polarization 0.5) are: $(1.06/1.26)^2 = 0.71$, $(0.91/1.26)^2 = 0.52$, $(0.80/1.26)^2 = 0.40$, $(0.72/1.26)^2 = 0.33$, respectively.

Michel parameters in $\tau \to \ell\nu\nu\gamma$ at Belle (I)

A. B. Arbuzov and T. V. Kopylova, JHEP **1609** (2016) 109. ($m_\ell \neq 0$)



Photon carries information about spin state of outgoing lepton, as a result two additional parameters, $\bar{\eta}$ and $\xi\kappa$, can be extracted.
These parameters were measured in τ decays at Belle for the first time.

$$\frac{d\Gamma(\tau^\mp \to \ell^\mp \nu_\ell \nu_\tau \gamma)}{dx\, dy\, d\Omega_\ell\, d\Omega_\gamma} = \Gamma_0 \frac{\alpha}{64\pi^3} \frac{\beta_\ell}{y} \left[F(x,y,d) \pm P_\tau \left(\beta_\ell \cos\theta_\ell G(x,y,d) + \cos\theta_\gamma H(x,y,d) \right) \right],$$

$$\Gamma_0 = G_F^2 m_\tau^5 / 192\pi^3, \;\; \beta_\ell = \sqrt{1 - m_\ell^2/E_\ell^2}, \;\; x = 2E_\ell/m_\tau, \;\; y = 2E_\gamma/m_\tau, \;\; d = 1 - \beta_\ell \cos\theta_{\ell\gamma}$$

$$F = F_0 + \bar{\eta} F_1, \;\; G = G_0 + \xi\kappa G_1, \;\; H = H_0 + \xi\kappa H_1, \;\; \frac{d\sigma(\ell^\mp \nu\nu\gamma, \rho^\pm \nu)}{dE_\ell^*\, d\Omega_\ell^*\, dE_\gamma^*\, d\Omega_\gamma^*\, d\Omega_\rho^*\, dm_{\pi\pi}^2\, d\tilde{\Omega}_\pi\, d\Omega_\tau} = A_0 + \bar{\eta} A_1 + \xi\kappa A_2$$

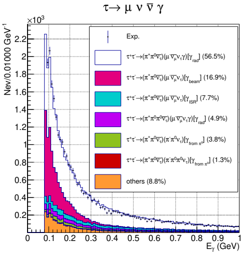
$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp \nu\nu\gamma, \rho^\pm \nu)}{dp_\ell\, d\Omega_\ell\, dp_\gamma\, d\Omega_\gamma\, dp_\rho\, d\Omega_\rho\, dm_{\pi\pi}^2\, d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp \nu\nu\gamma, \rho^\pm \nu)}{dE_\ell^*\, d\Omega_\ell^*\, dE_\gamma^*\, d\Omega_\gamma^*\, d\Omega_\rho^*\, dm_{\pi\pi}^2\, d\tilde{\Omega}_\pi\, d\Omega_\tau} |\mathrm{JACOBIAN}|\, d\Phi_\tau$$

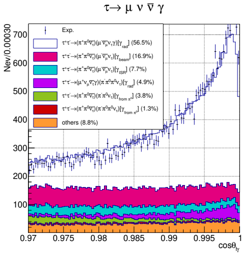
$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)}, \;\; \mathcal{P}^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})} = \frac{\mathcal{F}_0 + \mathcal{F}_1 \bar{\eta} + \mathcal{F}_2 \xi\kappa}{\mathcal{N}_0 + \mathcal{N}_1 \bar{\eta} + \mathcal{N}_2 \xi\kappa}, \;\; \mathcal{N}_k = \int \mathcal{F}_k(\vec{z}) d\vec{z}, \;\; (k = 0, 1, 2)$$

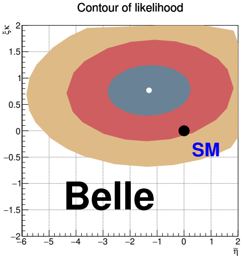
$\bar{\eta}$ and $\xi\kappa$ are extracted in the unbinned maximum likelihood fit of ($\ell\nu\nu\gamma;\ \rho\nu$) events in the 12D phase space in CMS.

Michel parameters in $\tau \to \ell\nu\nu\gamma$ at Belle (II)

$N_{\tau\tau} = 646 \times 10^6$, **selected: 71171** $(\mu\nu\nu\gamma;\ \rho\nu)$ **and 776834** $(e\nu\nu\gamma;\ \rho\nu)$ **events**







Source	$\sigma^e_{\bar\eta}$	$\sigma^e_{\xi\kappa}$	$\sigma^\mu_{\bar\eta}$	$\sigma^\mu_{\xi\kappa}$
Normalization	4.3	0.94	0.15	0.04
Background PDF	2.5	0.24	0.67	0.22
Branching ratios	3.8	0.05	0.25	0.01
Cluster merge in ECL	2.2	0.46	0.02	0.06
Detector resolution	0.74	0.20	0.22	0.02
Data/MC eff. corr.	1.9	0.14	0.04	0.04
Total	7.0	1.1	0.76	0.24

Belle result

$\bar\eta = -\mathbf{1.3} \pm \mathbf{1.5} \pm \mathbf{0.8}$

$\xi\kappa = \mathbf{0.5} \pm \mathbf{0.4} \pm \mathbf{0.2}$

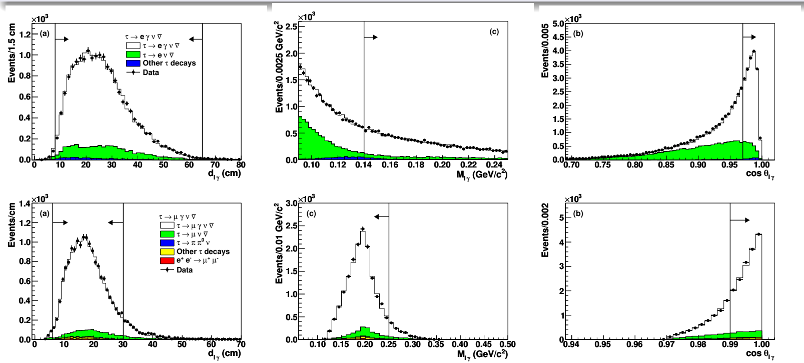
N. Shimizu *et al.* [Belle Collab.], PTEP **2018** (2018) no.2, 023C01.

Measurement of $\mathcal{B}(\tau \to \ell\nu\nu\gamma)$ at *BABAR* (I)

$$\int \mathit{Ldt} = 431\ \mathrm{fb}^{-1}$$

Selections:

- 2-track events with zero net charge and 1 photon with $E_\gamma > 50$ MeV;
- 0.9<thrust<0.995, signal hemisphere: $\ell + \gamma$, tag hemisphere: track+neutrals;
- reject $\ell^\mp - \ell^\pm$ events, $E_{\mathrm{tot}} < 9$ GeV, distance between track and photon clusters $d_{\ell\gamma} < 100$ cm.



$e\nu\nu\gamma$	$0.22 \le E_\gamma \le 2.0$ GeV, $M_{e\gamma} \ge 0.14$ GeV/c^2, $\cos\theta_{e\gamma} \ge 0.97$, $8 \le d_{e\gamma} \le 65$ cm
$\mu\nu\nu\gamma$	$0.10 \le E_\gamma \le 2.5$ GeV, $M_{\mu\gamma} \le 0.25$ GeV/c^2, $\cos\theta_{\mu\gamma} \ge 0.99$, $6 \le d_{\mu\gamma} \le 30$ cm

$$N_{\mathrm{sel}}(\mu\nu\nu\gamma) = 15688 \pm 125 \quad N_{\mathrm{sel}}(e\nu\nu\gamma) = 18149 \pm 135$$

Measurement of $\mathcal{B}(\tau \to \ell\nu\nu\gamma)$ at *BABAR* (II)

$$\mathcal{B} = \frac{N_{\rm sel}(1 - f_{\rm bg})}{2\sigma_{\tau\tau}\mathcal{L}\varepsilon}$$

	$\mu\nu\nu\gamma$	$e\nu\nu\gamma$
ε (%)	0.480 ± 0.010	0.105 ± 0.003
$f_{\rm bg}$	0.102 ± 0.002	0.156 ± 0.003

	$\tau \to \mu\nu\nu\gamma$	$\tau \to e\nu\nu\gamma$
Photon efficiency	1.8	1.8
Particle identification	1.5	1.5
Background evaluation	0.9	0.7
BF	0.7	0.7
Luminosity and cross section	0.6	0.6
MC statistics	0.5	0.6
Selection criteria	0.5	0.5
Trigger selection	0.5	0.6
Track reconstruction	0.3	0.3
Total	2.8	2.8

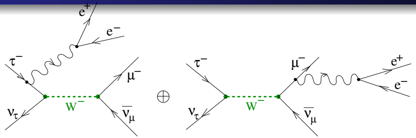
$$\mathcal{B}(\tau \to \mu\nu\nu\gamma)[E^*_\gamma > 10\,\mathrm{MeV}] = (3.69 \pm 0.03 \pm 0.10) \times 10^{-3}$$

$$\mathcal{B}(\tau \to e\nu\nu\gamma)[E^*_\gamma > 10\,\mathrm{MeV}] = (1.847 \pm 0.015 \pm 0.052) \times 10^{-2}$$

Measured branching ratios agree with the LO predictions ($\mathcal{B}(\mu\nu\nu\gamma) = 3.663 \times 10^{-3}$, $\mathcal{B}(e\nu\nu\gamma) = 1.834 \times 10^{-2}$), however the LO+NLO prediction for the $\tau \to e\nu\nu\gamma$ ($\mathcal{B}(e\nu\nu\gamma) = 1.645 \times 10^{-2}$) differs from the experimental result by **3.5σ**.
It is important to embed NLO corrections to the MC generator (TAUOLA) of the radiative leptonic decay. Also background from the doubly-radiative leptonic decays should be properly studied and subtracted.

M. Fael, L. Mercolli and M. Passera, JHEP 1507 (2015) 153.

Tau decays into 5 leptons



D. A. Dicus and R. Vega, Phys. Lett. B **338** (1994) 341.

M. S. Alam *et al.* [CLEO Collaboration], Phys. Rev. Lett. **76** (1996) 2637.

A. Flores-Tlalpa, G. Lopez Castro and P. Roig, JHEP 1604 (2016) 185.

Mode	$\mathcal{B}_{\text{theory}}$	$\mathcal{B}_{\text{CLEO}}$
$e^\mp e^+ e^- 2\nu$	$(4.21 \pm 0.01) \times 10^{-5}$	$(2.7^{+1.6}_{-1.2}) \times 10^{-5}$
$\mu^\mp e^+ e^- 2\nu$	$(1.984 \pm 0.004) \times 10^{-5}$	$< 3.2 \times 10^{-5}$(90% CL)
$e^\mp \mu^+ \mu^- 2\nu$	$(1.247 \pm 0.001) \times 10^{-7}$	
$\mu^\mp \mu^+ \mu^- 2\nu$	$(1.183 \pm 0.001) \times 10^{-7}$	

A. Kersch, N. Kraus and R. Engfer [SINDRUM], Nucl. Phys. A **485** (1988) 606.

$$\frac{d\Gamma(\tau)}{d\mathcal{PS}} = Q_{LL} d_1 + Q_{LR} d_2 + Q_{RL} d_3 + Q_{RR} d_4 + B_{RL} d_5 + B_{LR} d_6$$

Up to now Q_{LL}, Q_{LR}, Q_{RL}, Q_{RR}, B_{RL}, B_{LR} were measured only in muon decays ($\mu^- \to e^- e^- e^+ \nu_\mu \bar{\nu}_e$) with the accuracy of about $10 \div 20\%$. Michel parameters can be measured in two ways: in the study of the dynamics and from the measurement of the branching fraction:

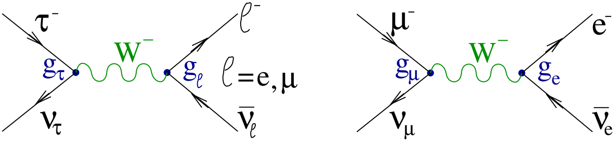
$$\mathcal{B}_{\text{exp}}/\mathcal{B}_{\text{SM}} = Q_{LL} + \alpha_{LR} Q_{LR} + \alpha_{RL} Q_{RL} + \alpha_{RR} Q_{RR} + \beta_{RL} B_{RL} + \beta_{LR} B_{LR}$$

Tau decays with leptons at Belle II and SCTF

- Precise study of the radiative leptonic decays $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma$.
  Measured branching ratios (*BABAR*) agree with the LO predictions however the LO+NLO theoretical prediction for the $\tau \to e\nu\nu\gamma$ differs from the experimental result by **3.5σ**:
  J. P. Lees *et al.*, Phys. Rev. D **91** (2015) 051103.
  M. Fael, L. Mercolli and M. Passera, JHEP **1507** (2015) 153.
  It is important to embed NLO corrections to TAUOLA to calculate the detection efficiency correctly.
- Precise study of the doubly radiative decay $\tau^- \to \ell^- \nu\nu\gamma\gamma$.
- Measurement of Michel parameters in the radiative leptonic τ decays:
  A. B. Arbuzov and T. V. Kopylova, JHEP **1609** (2016) 109. ($m_\ell \neq 0$)
  First experimental result from Belle:
  N. Shimizu et al., PTEP **2018** (2018) no.2, 023C01.
  Statistical improvement at Belle II and SCTF. At the SCTF the polarized τ allows one further to inrease sensitivity to $\xi\kappa$ parameter.
- Decay-in-flight muon polarization measurement ($\tau \to \mu\nu\nu$, $\mu \to e\nu\nu$) to measure ξ' Michel parameter:
  D. Bodrov, Physics of Atomic Nuclei **84**(2) (2021) 212.
  This analysis is now ongoing at Belle, it allows one to measure $\xi\kappa$ parameter with much better sensitivity than in the radiative leptonic decay. At Belle II and SCTF with the improved track rec. algorithm in the drift chamber of the detector and higher statistics it will be possible to improve the accuracy.
- Precise study of the five-body leptonic τ decays $\tau \to \ell\ell'^+\ell'^-\nu\nu$:
  A. Flores-Tlalpa, G. Lopez Castro and P. Roig, JHEP **1604** (2016) 185.
  While $\tau^- \to e^-e^+e^-2\nu$ and $\tau^- \to \mu^-e^+e^-2\nu$ modes can be studies already at *B* factories, the $\tau^- \to e^-\mu^+\mu^-2\nu$ and $\tau^- \to \mu^-\mu^+\mu^-2\nu$ modes can be discovered at Belle II and SCTF.
  Possibility to search for T-odd correlations of the form $\vec{s}_\tau(\vec{p}_i \times \vec{p}_j)$.

Lepton universality in the SM (I)

$g_e = g_\mu = g_\tau$



$$\Gamma(L^- \to \ell^- \bar{\nu}_\ell \nu_L(\gamma)) = \frac{\mathcal{B}(L^- \to \ell^- \bar{\nu}_\ell \nu_L(\gamma))}{\tau_L} = \frac{g_L^2 g_\ell^2}{32 M_W^4} \frac{m_L^5}{192\pi^3} F_{\rm corr}(m_L, m_\ell)$$

$$F_{\rm corr}(m_L, m_\ell) = f(x)\left(1 + \frac{3}{5}\frac{m_L^2}{M_W^2}\right)\left(1 + \frac{\alpha(m_L)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right)$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x, \;\; x = m_\ell / m_L$$

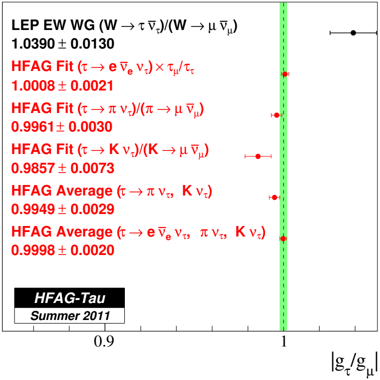
$$\mathcal{B}(\mu^- \to e^- \bar{\nu}_e \nu_\mu(\gamma)) = 1$$

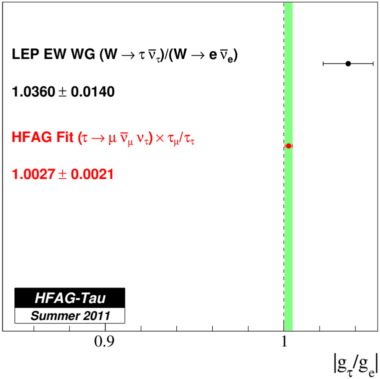
$$\frac{g_\tau}{g_e} = \sqrt{\mathcal{B}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau(\gamma)) \frac{\tau_\mu}{\tau_\tau} \frac{m_\mu^5}{m_\tau^5} \frac{F_{\rm corr}(m_\mu, m_e)}{F_{\rm corr}(m_\tau, m_\mu)}}, \quad \frac{\mathbf{g}_\tau}{\mathbf{g_e}} = \mathbf{1.0027 \pm 0.0014} \text{ (HFLAV2022)}$$

$$\frac{g_\tau}{g_\mu} = \sqrt{\mathcal{B}(\tau^- \to e^- \bar{\nu}_e \nu_\tau(\gamma)) \frac{\tau_\mu}{\tau_\tau} \frac{m_\mu^5}{m_\tau^5} \frac{F_{\rm corr}(m_\mu, m_e)}{F_{\rm corr}(m_\tau, m_e)}}, \quad \frac{\mathbf{g}_\tau}{\mathbf{g}_\mu} = \mathbf{1.0009 \pm 0.0014} \text{ (HFLAV2022)}$$

$$\frac{g_\mu}{g_e} = \sqrt{\frac{\mathcal{B}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau(\gamma))}{\mathcal{B}(\tau^- \to e^- \bar{\nu}_e \nu_\tau(\gamma))} \frac{F_{\rm corr}(m_\tau, m_e)}{F_{\rm corr}(m_\tau, m_\mu)}}, \quad \frac{\mathbf{g}_\mu}{\mathbf{g_e}} = \mathbf{1.0019 \pm 0.0014} \text{ (HFLAV2022)}$$

Lepton universality in the SM (II)





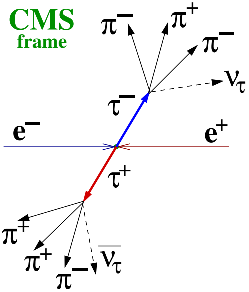
S. Schael *et al.* [ALEPH, DELPHI, L3, OPAL, LEP EWG]
Phys. Rep. 532, 119 (2013)

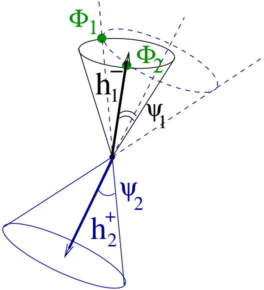
$$\frac{2\mathcal{B}(W \to \tau\nu_\tau)}{\mathcal{B}(W \to \mu\nu_\mu) + \mathcal{B}(W \to e\nu_e)} = 1.066 \pm 0.025$$

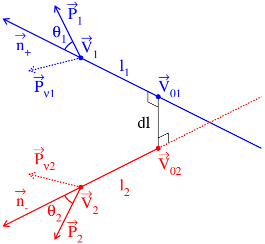
2.6σ deviation from the Standard Model

Measurement of τ_τ at Belle, method

We analyze $e^+e^- \to \tau^+\tau^- \to (\pi^+\pi^+\pi^-\bar{\nu}_\tau,\ \pi^+\pi^-\pi^-\nu_\tau)$ events.







$$\cos\psi_{1,2} = \frac{2E_\tau E_{h_{1,2}} - M_\tau^2 - m_{h_{1,2}}^2}{2p_\tau p_{h_{1,2}}}$$

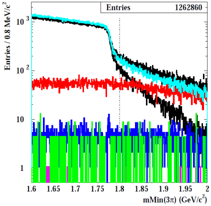
$$x = \frac{\ell}{\beta_\tau \gamma_\tau}$$

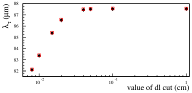
- τ momentum direction is determined with two-fold ambiguity in CMS, for the analysis we use the average axis.
- Asymmetric-energy layout of experiment allows us to determine $\tau^+\tau^-$ production point in LAB independently from the position of beam IP.
- Possibility to test CPT conservation measuring τ^- and τ^+ lifetimes separately.

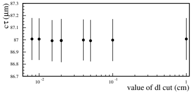
Use the data sample of $\int Ldt = 711$ fb^{-1} with $N_{\tau\tau} = 650 \times 10^6$

Selection criteria:

- Event is separated into two hemispheres in CMS, Thrust>0.9.
- Each hemisphere contains 3 charge pions with the ± 1 net charge.
- There are no additional K^0_S, Λ, π^0 candidates. Number of additional photons $N_\gamma < 6$ with $E_\gamma^{\rm TOT} < 0.7$ GeV.
- $P_\perp(6\pi) > 0.5$ GeV/c, 4 GeV/$c^2 < M_{\rm inv}(6\pi) < 10.25$ GeV/c^2.
- Pseudomass $\sqrt{M_h^2 + 2(E_{\rm beam} - E_h)(E_h - P_h)} < 1.8$ GeV/c^2, $h = (3\pi)^-,\ (3\pi)^+$.
- Cuts on the quality parameters of the vertex fits and tau axis reconstruction.
- Minimal distance between τ^- and τ^+ axes in LAB $dl < 0.02$ cm.







1148360 events were selected with $\sim$2% background contamination, the main background comes from $e^+e^- \to q\bar{q}$ ($q =$u, d, s).

Measurement of τ_τ at Belle, fit of decay length

Decay length PDF

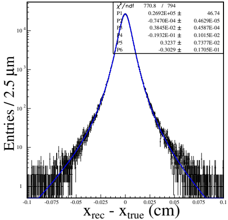
$$\mathcal{P}(x) = \mathcal{N} \int e^{-x'/\lambda_\tau} R(x - x'; \vec{P})dx' + \mathcal{N}_{uds} R(x; \vec{P}) + \mathcal{P}_{cb}(x),$$

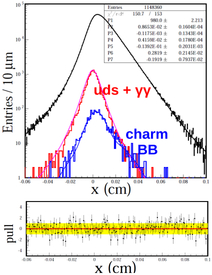
$$R(x; \vec{P}) = (1 - 2.5x) \cdot \exp\left(-\frac{(x - P_1)^2}{2\sigma^2}\right),$$

$$\sigma = P_2 + P_3|x - P_1|^{1/2} + P_4|x - P_1| + P_5|x - P_1|^{3/2}$$

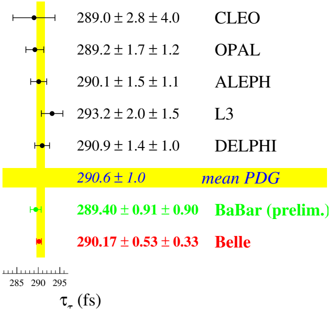
- Free parameters of the fit: λ_τ, $\mathcal{N}$, $\vec{P} = (P_1, ..., P_5)$
- λ_τ - estimator of $c\tau_\tau$, $c\tau_\tau = \lambda_\tau + \Delta_{\rm corr}$, $\Delta_{\rm corr}$ is determined from MC;
- $R(x; \vec{P})$ - detector resolution function;
- $\mathcal{N}_{uds}$ - contribution of background from $e^+e^- \to q\bar{q}$ ($q = u,\ d,\ s$) (predicted by MC)
- $\mathcal{P}_{cb}(x)$ - PDF for background from $e^+e^- \to q\bar{q}$ ($q = c,\ b$) (fixed from MC)

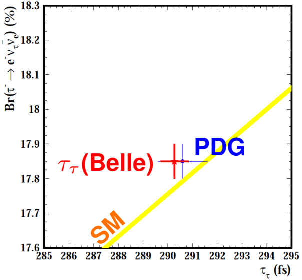
From the fit of experimental data
$\lambda_\tau = (86.53 \pm 0.16)$ μm, applying correction
$\Delta_{\rm corr} = 0.46$ μm we got: $c\tau_\tau = (86.99 \pm 0.16)$ μm





Measurement of τ_τ at Belle, result





Systematic uncertainties

Source	**$\Delta c\tau$ (μm)**
Silicon vertex detector alignment	0.090
Asymmetry fixing	0.030
Fit range	0.020
Beam energy, ISR, FSR	0.024
Background contribution	0.010
τ-lepton mass	0.009
Total	**0.101**

$$c\tau_\tau = (86.99 \pm 0.16(\text{stat}) \pm 0.10(\text{syst}))\ \mu\text{m}.$$

$$\tau_\tau = (290.17 \pm 0.53(\text{stat}) \pm 0.33(\text{syst}))\ \text{fs}.$$

$$|\tau_{\tau^+} - \tau_{\tau^-}|/\tau_{\text{average}} < 7.0 \times 10^{-3}\ \text{at 90\% CL}.$$

Lepton universality

$$g_\tau/g_e = 1.0024 \pm 0.0021\ (\text{HFAG2012})$$

$$\mathbf{g_\tau/g_e = 1.0031 \pm 0.0016}\ (\text{new Belle } \tau_\tau)$$

$$g_\tau/g_\mu = 1.0006 \pm 0.0021\ (\text{HFAG2012})$$

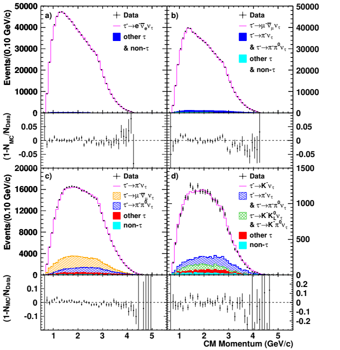
$$\mathbf{g_\tau/g_\mu = 1.0013 \pm 0.0016}\ (\text{new Belle } \tau_\tau)$$

Test of lepton universality at *BABAR*

$\int Ldt = 467\ \text{fb}^{-1}$

Selections:

- 4-track events with zero net charge;
- $0.1\sqrt{s} < E_{\text{miss}}^{\text{CMS}} < 0.7\sqrt{s}$, $|\cos(\theta_{\text{miss}}^{\text{CMS}})| < 0.7$
- thrust> 0.9, signal hemisphere: $\ell/h(\ell = e, \mu;\ h = \pi, K)$, tag hemisphere: $\tau \to \pi\pi\pi\nu$;
- signal hemisphere: $E_{\text{extra}\gamma}^{LAB} < \{1.0,\ 0.5,\ 0.2,\ 0.2\}$ GeV for $\{e,\ \mu,\ \pi,\ K\}$, respectively



	μ	π	K
$\mathbf{N}^{\text{D}}$	731102	369091	25123
Purity	97.3%	78.7%	76.6%
Total Efficiency	0.485%	0.324%	0.330%
Particle ID Efficiency	74.5%	74.6%	84.6%
Systematic uncertainties:			
Particle ID	0.32	0.51	0.94
Detector response	0.08	0.64	0.54
Backgrounds	0.08	0.44	0.85
Trigger	0.10	0.10	0.10
$\pi^-\pi^-\pi^+$ modelling	0.01	0.07	0.27
Radiation	0.04	0.10	0.04
$\mathcal{B}(\tau^- \to \pi^-\pi^-\pi^+\nu_\tau)$	0.05	0.15	0.40
$\mathcal{L}\sigma_{\tau\tau}$	0.02	0.39	0.20
Total [%]	**0.36**	**1.0**	**1.5**

$\boldsymbol{\tau \to e\nu\nu}$: $N_{\text{sel}} = 884426$, $\varepsilon = (0.589 \pm 0.010)\%$, purity is $(99.69 \pm 0.06)\%$

Test of lepton universality

$$R_\mu = \frac{\mathcal{B}(\tau \to \mu\nu\nu)}{\mathcal{B}(\tau \to e\nu\nu)} = 0.9796 \pm 0.0016 \pm 0.0036$$

$$R_\pi = \frac{\mathcal{B}(\tau \to \pi\nu)}{\mathcal{B}(\tau \to e\nu\nu)} = 0.5945 \pm 0.0014 \pm 0.0061$$

$$R_K = \frac{\mathcal{B}(\tau \to K\nu)}{\mathcal{B}(\tau \to e\nu\nu)} = 0.03882 \pm 0.00032 \pm 0.00057$$

$$(g_\mu/g_e)_\tau = \sqrt{R_\mu \frac{F_{\rm corr}(m_\tau, m_e)}{F_{\rm corr}(m_\tau, m_\mu)}} = 1.0036 \pm 0.0020$$

$$(g_\tau/g_\mu)^2_P = \frac{\mathcal{B}(\tau \to P\nu_\tau)}{\mathcal{B}(P \to \mu\nu_\mu)} \frac{2m_P m_\mu^2 \tau_P}{(1+\delta_P)m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_P^2}{1 - m_P^2/m_\tau^2}\right)^2$$

$$(g_\tau/g_\mu)_\pi = 0.9856 \pm 0.0057, \; (g_\tau/g_\mu)_K = 0.9827 \pm 0.0086$$

$(g_\tau/g_\mu)_{\pi\&K} = 0.9850 \pm 0.0054$ (2.8σ away from SM)

$(g_\tau/g_\mu)_{\tau+\pi+K} = 1.0003 \pm 0.0014$ (HFLAV2022)

Test of LFU at Belle II and SCTF

- The correct determination of δ_P is not enough. The proper description of $\tau \to P\nu\gamma$ and virt. corrections to $\tau \to P\nu$ are important to embed in TAUOLA for the reliable estimation of the detection efficiency. As a result, to improve LFU test at Belle II and SCTF, a high statistics study of $\tau \to P\nu\gamma$ is needed:
  Z. H. Guo and P. Roig, Phys. Rev. D **82** (2010) 113016.
  The structure of $W - P - \gamma$ vertex can be studied also in the $\tau^- \to P^-\nu_\tau\ell^+\ell^-$ decays:
  A. Guevara, G. Lopez Castro and P. Roig, Phys. Rev. D **88** (2013) no.3, 033007.
  The first experimental study of $\tau^- \to \pi^-\nu_\tau\ell^+\ell^-$ at Belle:
  Y. Jin et al., Phys. Rev. D **100** (2019) no.7, 071101.
  At Belle II and SCTF the $\tau \to P\nu\gamma$ and $\tau^- \to P^-\nu_\tau\ell^+\ell^-$ decays will be studied with better accuracy, the decay mechanism will be established and LFU test can be improved.
- At the SCTF at the $\tau^+\tau^-$ production threshold (τ is at rest) the pion/kaon from $\tau \to \pi/K\nu$ can be easily separated via their momentum difference (of about 63 MeV). The radiative decay $\tau \to P\nu\gamma$ is easier to study near the $\tau^+\tau^-$ production threshold due to the lack of ISR background.

Introduction: hadronic τ decays

Cabibbo-allowed decays ($\mathcal{B} \sim \cos^2\theta_c$)

$\mathcal{B}(S=0) = (61.85 \pm 0.11)\%$ (PDG)

Cabibbo-suppressed decays ($\mathcal{B} \sim \sin^2\theta_c$)

$\mathcal{B}(S=-1) = (2.88 \pm 0.05)\%$ (PDG)

$$iM_{\rm fi}\left\{\begin{array}{c} S=0 \\ S=-1 \end{array}\right\} = \frac{G_F}{\sqrt{2}}\bar{u}_{\nu_\tau}\gamma^\mu(1-\gamma^5)u_\tau \cdot \left\{\begin{array}{c} \cos\theta_c \cdot \langle \text{hadrons}(q^\mu)|\hat{J}^{S=0}_\mu(q^2)|0\rangle \\ \sin\theta_c \cdot \langle \text{hadrons}(q^\mu)|\hat{J}^{S=-1}_\mu(q^2)|0\rangle \end{array}\right\},\ q^2 \le M^2_\tau$$

The main tasks

- Measurement of branching fractions with highest possible accuracy
- Measurement of low-energy hadronic spectral functions
  - Determination of the decay mechanism (what are intermediate mesons and their contributions)
  - Precise measurement of masses and widths of the intermediate mesons
- Search for CP violation (CPV)
- Comparison with hadronic formfactors from e^+e^- experiments to check CVC theorem
- Measurement of $\Gamma_{\rm inclusive}(S=0)$ to determine α_s
- Measurement of $\Gamma_{\rm inclusive}(S=-1)$ to determine s-quark mass and V_{us}:

$$|V_{us}| = \sqrt{\frac{R_{\rm strange}}{\frac{R_{\rm non-strange}}{|V_{ud}|^2} - \delta R_{\rm theory}}}$$

- $R_{\rm strange} = \mathcal{B}_{\rm strange}/\mathcal{B}_e$
- $R_{\rm non-strange} = \mathcal{B}_{\rm non-strange}/\mathcal{B}_e$
- $\delta R_{\rm theory}$ - SU(3)-breaking contribution

Study of the $\tau^- \to K_S^0 \pi^- \nu_\tau,\ K^- \pi^0 \nu_\tau$ decays

- **Measurement of $\mathcal{B}(\tau \to K_S^0 \pi \nu_\tau)$ and $\mathcal{B}(\tau \to K^- \pi^0 \nu_\tau)$:** $\tau \to K \pi \nu_\tau$ has the largest $\mathcal{B}$ among decays with one kaon, so, it provides the dominant contribution to the s-quark mass sensitive total strange hadronic spectral function.
- **Study of the $K\pi$ dynamics (mass spectrum):**
  M. FINKEMEIER, E. MIRKES, Z. PHYS. C **72**, 619 (1996).
  The hadronic current in the case of two pseudoscalar hadrons with $q^\mu_{1,2}$:

$$J^\mu = F_V(q^2)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)(q_1 - q_2)_\nu + F_S(q^2) q^\mu,\ q^\mu = q_1^\mu + q_2^\mu$$

  - F_V: $K^*(892)^\pm$, $K^*(1410)^\pm$, $K^*(1680)^\pm$;
  - F_S: $K^*(800)^\pm(\kappa)$, $K^*(1430)^\pm$;
  - Precision measurement of $M(K^*(892)^\pm)$ and $\Gamma(K^*(892)^\pm)$.
- **CPV in $\tau \to K_S^0 \pi \nu_\tau,\ K^- \pi^0 \nu_\tau$**
  - in the mode with the K_S^0 there is additional CPV asymmetry related to the known CPV in the system of neutral kaons
  - J.KUHN, E.MIRKES,PHYS. LETT. **B398**, 407 (1997).
  - Y. GROSSMAN AND Y. NIR, JHEP **1204**, 002 (2012).
  - J. P. LEES *et al.* [BABAR], PHYS. REV. D **85**, 031102 (2012).
  - M. BISCHOFBERGER *et al.* [BELLE], PHYS. REV. LETT. **107**, 131801 (2011).

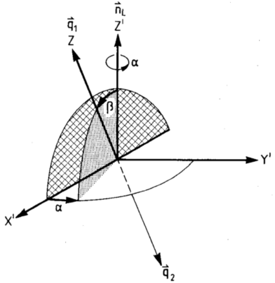
$\tau \to K(q_1)\pi(q_2)\nu_\tau$ hadronic spectral functions (I)

$$d\Gamma = \frac{G_F^2}{256\pi^3 m_\tau}\sin^2\theta_c\{L_{\mu\nu}H^{\mu\nu}\}\left(1-\frac{q^2}{m_\tau^2}\right)|\vec{q}_1|\frac{dq^2}{\sqrt{q^2}}\frac{d\alpha}{2\pi}\frac{d\cos\beta}{2}\frac{d\cos\theta}{2}$$

$$L_{\mu\nu}H^{\mu\nu} = 2m_\tau^2\left(1-\frac{q^2}{m_\tau^2}\right)(\overline{L}_B W_B + \overline{L}_{SA}W_{SA} + \overline{L}_{SF}W_{SF}),\ q = q_1 + q_2,$$

$$W_B = 4|\vec{q}_1|^2|F_V|^2,\ W_{SA} = q^2|F_S|^2,\ W_{SF} = 4\sqrt{q^2}|\vec{q}_1|\mathrm{Re}[F_V F_S^*]$$

$$\overline{L}_B = \frac{1}{3}\left(2+\frac{m_\tau^2}{q^2}\right)-\frac{1}{6}\left(1-\frac{m_\tau^2}{q^2}\right)(3\cos^2\psi-1)(3\cos^2\beta-1),\ \overline{L}_{SA} = \frac{m_\tau^2}{q^2},\ \overline{L}_{SF} = -\frac{m_\tau^2}{q^2}\cos\psi\cos\beta$$



$$\cos\beta = -\vec{n}_q\cdot\frac{\vec{q}_1}{|\vec{q}_1|}$$

$$\cos\theta = \frac{(2\frac{E_{K\pi}}{E_\tau}-1-\frac{q^2}{m_\tau^2})}{(1-\frac{q^2}{m_\tau^2})\sqrt{1-m_\tau^2/E_\tau^2}}$$

$$\cos\psi = \frac{\frac{E_{K\pi}}{E_\tau}(m_\tau^2+q^2)-2q^2}{(m_\tau^2-q^2)\sqrt{(E_{K\pi}^2-q^2)/E_\tau^2}}$$

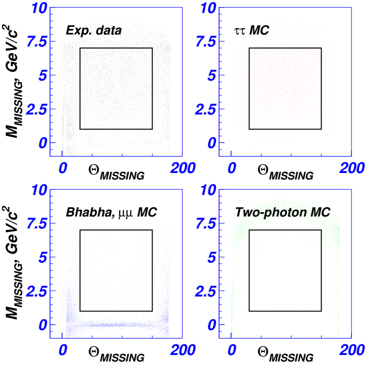
$\tau \to K\pi\nu_\tau$ hadronic spectral functions (II)

- β - angle between $\vec{q}_1$ (kaon) and direction to CMS frame in the $K\pi$ rest frame
- ψ - angle between $\vec{p}_\tau$ and direction to CMS frame in the $K\pi$ rest frame
- θ - angle between $\vec{p}_\tau$ in CMS and momentum of $K\pi$ in τ rest frame (correlated with ψ)

The form factors (or hadronic spectral functions) can be extracted by averaging particular functions of β, ψ and θ angles in bins of q^2. In this case the detection efficiency dependence on α, $\cos\beta$ and $\cos\theta$ should be taken into account.

Selection of $\tau^+\tau^-$ events at B factories

- General preselection of low-multiplicity events
- Selection on the 2D plot $\theta^{\rm CMS}_{\rm missing}$ vs. $M_{\rm missing}$
- Tag one τ by 1-prong or leptonic mode, and reconstruct the decay products (except neutrino(s)) of the signal tau. At B factories the decay products of the oppositely charged taus almost don't overlap (they are located in the opposite hemispheres).



Background from $B\bar{B}$, $q\bar{q}$ $(q = u,\ d,\ s,\ c)$, two-photon, Bhabha, $\mu\mu(\gamma)$ is small.

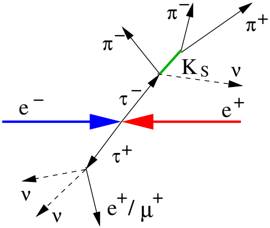
Measurement of $\mathcal{B}(\tau^- \to K_S^0 \pi^- \nu_\tau)$

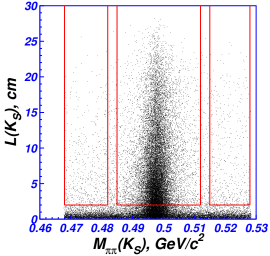
D. Epifanov *et al.* [Belle], Phys. Lett. B **654**, 65 (2007).

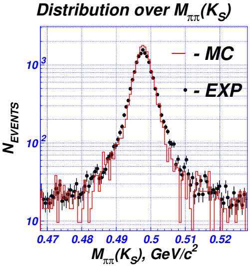
Statistics: $\int Ldt = 351$ fb^{-1}, $N_{\tau\tau} = 323 \times 10^6$

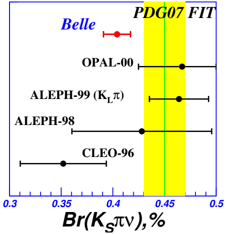
53110 signal events with efficiency $\varepsilon_{\rm det} \simeq 6\%$.

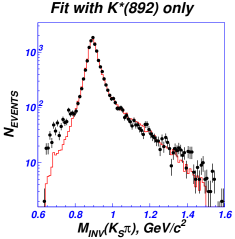
Two-lepton ($\tau \to e\nu\nu, \tau \to \mu\nu\nu$) events are used for normalization.











Mode	Contents, %
$K_S\pi\nu$	79
$K_S\pi K_L\nu$	9
$K_S\pi\pi^0\nu$	4
$K_S K\nu$	2
$3\pi\nu$	5
non-$\tau\tau$	1

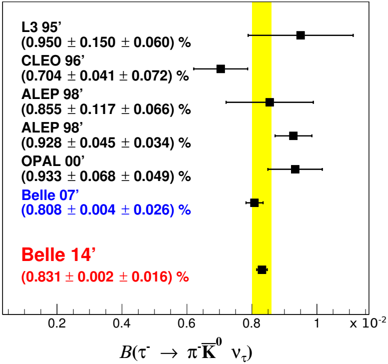
$$\mathcal{B}(\tau^- \to K_S\pi^-\nu_\tau) = (0.404 \pm 0.002(\text{stat.}) \pm 0.013(\text{syst.}))\%$$

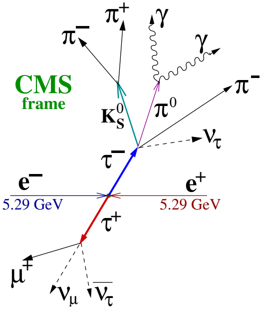
Study of $\tau^- \to K^0_S X^- \nu_\tau$ decays at Belle

S. Ryu *et al.* [Belle], Phys. Rev. D **89**, 072009 (2014)

Data sample of $\int Ldt = 669$ fb^{-1} with $N_{\tau\tau} = 616 \times 10^6$ was used to study inclusive decay $\tau^- \to K^0_S X^- \nu_\tau$ as well as 6 exclusive modes:

$$\pi^- K^0_S \nu_\tau \qquad K^- K^0_S \nu_\tau \qquad \pi^- K^0_S K^0_S \nu_\tau$$
$$\pi^- K^0_S \pi^0 \nu_\tau \qquad K^- K^0_S \pi^0 \nu_\tau \qquad \pi^- K^0_S K^0_S \pi^0 \nu_\tau$$



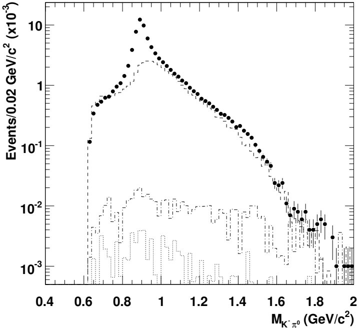


$$\mathcal{B}(\tau^- \to K^0_S \pi^- \nu_\tau) = (4.16 \pm 0.01 \pm 0.08) \times 10^{-3}$$
$$\mathcal{B}(\tau^- \to K^0_S X^- \nu_\tau) = (9.14 \pm 0.01 \pm 0.22) \times 10^{-3}$$

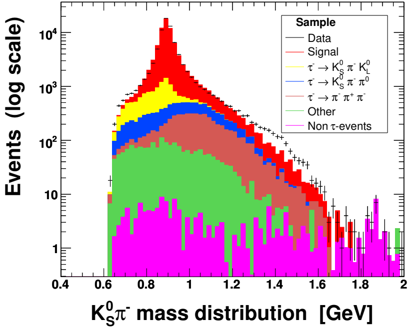
Study of $\tau \to K\pi\nu$ at *BABAR*

B. Aubert *et al.* [BaBar], Phys. Rev. D **76**, 051104 (2007).
B. Aubert *et al.* [BaBar], Nucl. Phys. Proc. Suppl. **189**, 193 (2009).

$\tau^- \to K^-\pi^0\nu_\tau$



$\tau^- \to K^0_S\pi^-\nu_\tau$



$$\mathcal{B}(\tau^- \to K^-\pi^0\nu_\tau) = (0.416 \pm 0.003(\text{stat.}) \pm 0.018(\text{syst.}))\%$$
$$\mathcal{B}(\tau^- \to K^0_S\pi^-\nu_\tau) = (0.420 \pm 0.002(\text{stat.}) \pm 0.012(\text{syst.}))\% \text{ (preliminary)}$$

Study of the $K_S^0\pi$ mass spectrum at Belle (I)

$$\frac{d\Gamma}{d\sqrt{s}} \sim \frac{1}{s}\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+2\frac{s}{M_\tau^2}\right)P\left\{P^2|F_V|^2+\frac{3(M_K^2-M_\pi^2)^2}{4s(1+2\frac{s}{M_\tau^2})}|F_S|^2\right\}$$

$$s = q^2 = M^2_{\mathrm{inv}}(K_S^0\pi)$$

$$\mathrm{F_V} = \frac{\mathrm{BW_{K^*(892)}} + \mathrm{a(K^*(1410))\cdot BW_{K^*(1410)}} + \mathrm{a(K^*(1680))\cdot BW_{K^*(1680)}}}{1+\mathrm{a(K^*(1410))}+\mathrm{a(K^*(1680))}}$$

$$\mathrm{F_S} = \mathrm{a(K_0^*(800))\cdot BW_{K_0^*(800)}} + \mathrm{a(K_0^*(1430))\cdot BW_{K_0^*(1430)}}$$

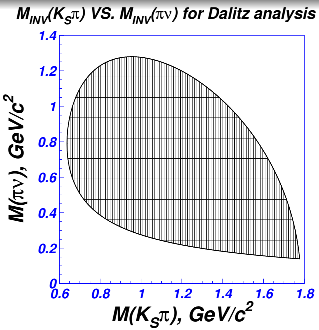
$$\mathrm{BW_X} = \frac{\mathrm{M_X^2}}{\mathrm{M_X^2 - s - i\sqrt{s}\Gamma_X(s)}}$$

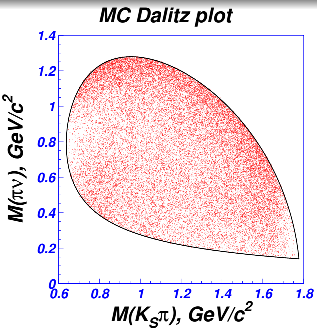
$$\Gamma_X(s) = \Gamma_X\frac{\mathrm{M_X^2}}{\mathrm{s}}\left(\frac{\mathrm{P(s)}}{\mathrm{P(M_X^2)}}\right)^{2\ell+1}\cdot \mathrm{F_R^{\ell 2}}$$

$$\mathrm{P(s)} = \frac{\sqrt{\mathrm{(s-(M_K+M_\pi)^2)(s-(M_K-M_\pi)^2)}}}{2\sqrt{\mathrm{s}}}$$

Spin ℓ	Blatt-Weisskopf factor $\mathrm{F_R^\ell}$
0	1
1	$\sqrt{\frac{1+R^2P^2(M_X^2)}{1+R^2P^2(s)}}$
2	$\sqrt{\frac{9+3R^2P^2(M_X^2)+R^4P^4(M_X^2)}{9+3R^2P^2(s)+R^4P^4(s)}}$

Study of the $K_S^0\pi$ mass spectrum at Belle (II)





To take into account the detector apparatus function we introduce 100×1000 efficiency matrix:

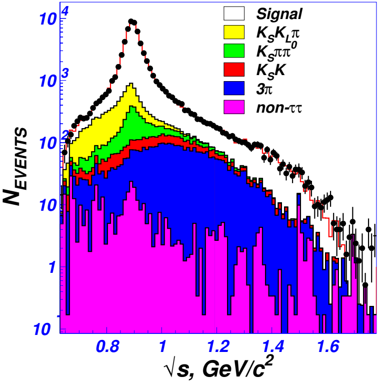
$$\varepsilon_{ij}^{\mathrm{MC}} = \frac{N_i^{\mathrm{MC}}(\mathrm{sel})}{N_j^{\mathrm{MC}}(\mathrm{gen})},\ i = 1 \div 100,\ j = 1 \div 1000$$

$$\chi^2 = \sum_{\mathrm{bins}} \frac{(N_i^{\mathrm{EXP}} - \varepsilon_{ij}^{\mathrm{MC}} N_j^{\mathrm{THEORY}})^2}{N_i^{\mathrm{EXP}} + \sigma_{\varepsilon N}^2}$$

$$N_j^{\mathrm{THEORY}} = \int_j \frac{d\Gamma}{dm_{12}dm_{23}} dm_{12}dm_{23},\ m_{12} = M(K_S^0\pi) = \sqrt{s},\ m_{23} = M(\pi\nu)$$

$K_0^*(800) + K^*(892) + K^*(1410)$

The $K^*(892)$ alone is not sufficient to describe the $K_S^0\pi$ spectrum



$M_{K^*(892)} = 895.47 \pm 0.20\ \mathrm{MeV/c^2}$

$\Gamma_{K^*(892)} = 46.19 \pm 0.57\ \mathrm{MeV}$

$|\mathrm{a(K^*(1410))}| = (75 \pm 6) \times 10^{-3}$

$\arg(\mathrm{a(K^*(1410))}) = 1.44 \pm 0.15$

$|\mathrm{a(K_0^*(800))}| = 1.57 \pm 0.23$

$\chi^2/\mathrm{Ndf} = 90.2/84,\ P(\chi^2) = 30\%$

We take $K_0^*(800)$ parameters:
$M_{K_0^*(800)} = (878 \pm 23 \pm 60)\ \mathrm{MeV/c^2},\ \Gamma_{K_0^*(800)} = (499 \pm 52 \pm 71)\ \mathrm{MeV/c^2}$ from:
M. Ablikim *et al.*, [BES Collaboration], Phys. Lett. B **633**, 681 (2006).
There is large systematic uncertainty in the near $K_S^0\pi$ production threshold part of the spectrum due to the large background from the $\tau^- \to K_S^0\pi^-K_L^0\nu_\tau$ decay, whose dynamics is not precisely known.

$K_0^*(800) + K^*(892) + K_0^*(1430)$

	solution 1	solution 2
$M_{K^*(892)}$, MeV/c^2	895.42 ± 0.19	895.50 ± 0.22
$\Gamma_{K^*(892)}$, MeV	46.14 ± 0.55	46.20 ± 0.69
$\|a(K_0^*(1430))\|$	0.954 ± 0.081	1.92 ± 0.20
$arg(a(K_0^*(1430)))$	0.62 ± 0.34	4.03 ± 0.09
$a(K_0^*(800))$	1.27 ± 0.22	2.28 ± 0.47
χ^2/ndf	$86.5/84$	$95.1/84$
$P(\chi^2), \%$	41	19
$\mathcal{B}(K_0^*(1430) \to K_S\pi)$	$1/3$	$1/3$
$\mathcal{B}(\tau \to K_0^*(1430)\nu_\tau)$	$(13 \pm ^{3}_{2}) \times 10^{-5}$	$(54 \pm ^{18}_{9}) \times 10^{-5}$

M. Z. Yang, "Testing the structure of the scalar meson $K_0^*(1430)$ in $\tau \to K_0^*(1430)\nu_\tau$ decay", Mod. Phys. Lett. A **21**, 1625 (2006) [arXiv:hep-ph/0509102]:

$$\mathcal{B}(\tau \to K_0^*(1430)\nu_\tau) = (7.9 \pm 3.1) \times 10^{-5}$$

From the $M_{\rm inv}(K_S^0\pi)$ fit only it is not possible to extract precisely the $K_0^*(1430)$ component due to the multiple solutions for the $K_0^*(800)$ and $K_0^*(1430)$ amplitudes in the scalar form factor F_S.

Multiple solutions (two Breit-Wigner amplitides) (I)

$$|A|^2(s|a_1,a_2,\varphi) = \left| a_1 \frac{m_1^2}{s - m_1^2 + im_1\Gamma_1} + a_2 e^{i\varphi} \frac{m_2^2}{s - m_2^2 + im_2\Gamma_2} \right|^2, \ s = m^2.$$

In the case of constant widths for each set of parameters (a_1, a_2, φ) there exists the other set (a_1', a_2', φ') $(a_1' \neq a_1,\ a_2' \neq a_2,\ \varphi' \neq \varphi)$, such as:

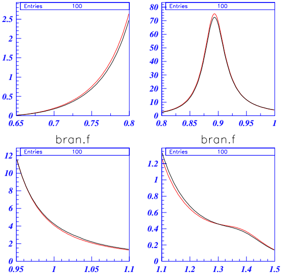
$$|A|^2(s|a_1', a_2', \varphi') = |A|^2(s|a_1, a_2, \varphi) \text{ for all values of } s$$

$$a_1' = f(a_1, a_2, \varphi),\ a_2' = g(a_1, a_2, \varphi),\ \varphi' = h(a_1, a_2, \varphi)$$

For example, for Breit-Wigner(BW) functions with the following parameters: $m_1 = 0.878\ \mathrm{GeV/c^2},\ \Gamma_1 = 0.499\ \mathrm{GeV}$, $m_2 = 1.412\ \mathrm{GeV/c^2},\ \Gamma_2 = 0.294\ \mathrm{GeV}$,
$\mathbf{a_1 = 1.270},\ \mathbf{a_2 = 0.954},\ \varphi = \mathbf{0.62}$;
the second solution is:
$\mathbf{a_1' = 3.268},\ \mathbf{a_2' = 1.481},\ \varphi' = \mathbf{4.19}$.

Multiple solutions (two Breit-Wigner amplitides) (II)

In the case of s-dependent widths $\Gamma_{1,2}(s)$ the s-spectrum degeneration disappears and spectra for (a_1, a_2, φ) and (a'_1, a'_2, φ') sets become different:



But if the experimental errors are large enough, the χ^2 for both solutions will be almost the same, so we have to take into account both solutions, just like we have for $F_S(s)$, approximated by BW($K_0^*(800)$)+BW($K_0^*(1430)$).

In our case, the vector form factor, $F_V(s)$, is also described by a sum of two BWs (BW($K^*(892)$)+BW($K^*(1410)$)), but the statistics around $K^*(892)$ meson is so big that we can choose the best solution, for the second solution the χ^2 becomes notably higher.

In general, if the total amplitude is parametrized by sum of N BW functions (determined by $2N-1$ parameter set $(a_1, ..., a_N, \varphi_1, ..., \varphi_{N-1})$), there are 2^{N-1} solutions to check.

LASS parametrization of F_S

P. Estabrooks, Phys. Rev. D **19**, 2678 (1979).
D. Aston *et al.* (LASS), Nucl. Phys. B **296**, 493 (1988).

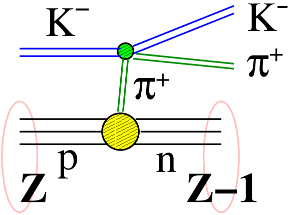
$$F_S = \lambda \frac{M_{K\pi}}{P}(\sin\delta_\mathrm{B} e^{i\delta_\mathrm{B}} + e^{2i\delta_\mathrm{B}} BW_{K_0^*(1430)}(M_{K\pi}))$$

$$\cot\delta_\mathrm{B} = \frac{1}{aP} + \frac{bP}{2}$$

$$a = (2.07 \pm 0.10)\ (\mathrm{GeV/c})^{-1}$$

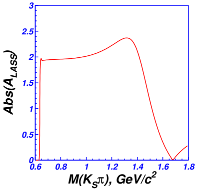
$$b = (3.32 \pm 0.34)\ (\mathrm{GeV/c})^{-1}$$

$$P = \frac{\sqrt{(M^2_{K\pi} - (M_\mathrm{K}+M_\pi)^2)(M^2_{K\pi} - (M_\mathrm{K}-M_\pi)^2)}}{2M_{K\pi}}$$

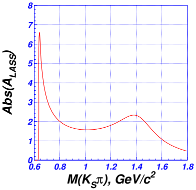


	$K^*(892)$+LASS a, b-fixed	$K^*(892)$+LASS a, b-free
M_{K^*}, $\mathrm{MeV/c^2}$	895.42 ± 0.19	895.38 ± 0.23
Γ_{K^*}, MeV	46.46 ± 0.47	46.53 ± 0.50
λ	0.282 ± 0.011	0.298 ± 0.012
a, $(\mathrm{GeV/c})^{-1}$	2.13 ± 0.10	$10.9 + 7.4 - 3.0$
b, $(\mathrm{GeV/c})^{-1}$	3.96 ± 0.31	$19.0 + 4.5 - 3.6$
χ^2/n.d.f.	196.9/86	97.3/83
$P(\chi^2)$, %	10^{-8}	13

LASS



Belle



Careful study of the $\tau^- \to K^0_S\pi^-\nu_\tau$ near the $K^0_S\pi$ production threshold is needed

$K^*(892)^\pm$ mass and width (I)

Model uncertainties in $K^*(892)^\pm$ mass and width are evaluated from approximations with the following models:
$K_0^*(800) + K^*(892) + K^*(1410)$, $K_0^*(800) + K^*(892) + K_0^*(1430)$, $K_0^*(800) + K^*(892) + K^*(1680)$, $K^*(892)$+LASS.

	$M(K^*(892)),\ \mathrm{MeV}/c^2$	$\Gamma(K^*(892)),\ \mathrm{MeV}$
This work	$895.47 \pm 0.20_{\rm stat} \pm 0.44_{\rm syst} \pm 0.59_{\rm mod}$	$46.2 \pm 0.6_{\rm stat} \pm 1.0_{\rm syst} \pm 0.7_{\rm mod}$
PDG-2017	891.76 ± 0.25	50.3 ± 0.8
Difference	3.71 ± 0.80	-4.1 ± 1.7

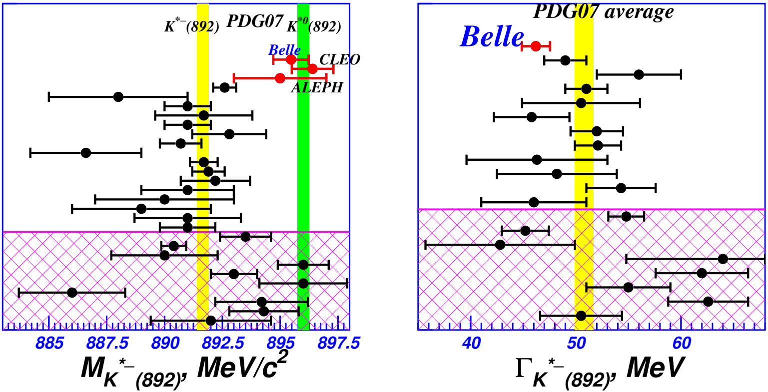
PDG average is based on the results from the fixed target experiments

894.3 $\pm$1.5	1150	[2,3] CLARK	73	HBC	−	3.3 $K^- p \to \overline{K}^0 \pi^- p$
892.0 $\pm$2.6	341	[2] SCHWEING...68		HBC	−	5.5 $K^- p \to \overline{K}^0 \pi^- p$

CHARGED ONLY, PRODUCED IN τ LEPTON DECAYS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
895.47$\pm$0.20$\pm$0.74	53k	[6] EPIFANOV	07	BELL	$\tau^- \to K^0_S \pi^- \nu_\tau$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
895.3 $\pm$0.2		[7,8] JAMIN	08	RVUE	$\tau^- \to K^0_S \pi^- \nu_\tau$
896.4 $\pm$0.9	11970	[9] BONVICINI	02	CLEO	$\tau^- \to K^- \pi^0 \nu_\tau$
895 $\pm$2		[10] BARATE	99R	ALEP	$\tau^- \to K^- \pi^0 \nu_\tau$

$K^*(892)^\pm$ mass and width (II)



The $K^*(892)^-$ width is compatible with the previous measurements within experimental errors, however the $K^*(892)^-$ mass value obtained in $\tau^- \to K^0_S\pi^-\nu_\tau$ is systematically higher than those before and is consistent with the world average value of the neutral $K^*(892)^0$ mass.
None of the previous measurements in PDG, all of which were performed more than 30 years ago, present the systematic uncertainties for their measurements !

Further studies at Belle II and SCTF (I)

- **To elucidate the nature of the $K^*(892)^- - K^*(892)^0$ mass difference is fundamental task in the low energy hadron spectroscopy.**
- It is suggested to study simulataneously: $\tau^- \to K^0_S\pi^-\nu_\tau$, $\tau^- \to K^0_S\pi^-\pi^0\nu_\tau$, $\tau^- \to K^0_SK^-\nu_\tau$, $\tau^- \to K^0_SK^-\pi^0\nu_\tau$ for the modes with K^0_S. The modes with K^-/π^-: $\tau^- \to K^-\pi^0\nu_\tau$, $\tau^- \to K^-\pi^0\pi^0\nu_\tau$, $\tau^- \to \pi^-\pi^0\nu_\tau$, $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$; $\tau^- \to K^-\pi^+\pi^-\nu_\tau$, $\tau^- \to \pi^-K^+K^-\nu_\tau$, $\tau^- \to \pi^-\pi^+\pi^-\nu_\tau$.
- $K^*(892)^-$ mass and width can be measured in the clean experimental conditions without disturbance from the final state interactions in the $\tau^- \to K^0_S\pi^-\nu_\tau$ and $\tau^- \to K^-\pi^0\nu_\tau$ decays.
- Study of the $\tau^- \to K^0_S\pi^-\pi^0\nu_\tau$, $\tau^- \to K^-\pi^0\pi^0\nu_\tau$ and $\tau^- \to K^-\pi^+\pi^-\nu_\tau$ modes allows one to measure:
  1) simultaneously, in one mode the $K^*(892)^-(K^0_S\pi^-)$ and the $K^*(892)^0(K^0_S\pi^0)$ masses in the case of one accompanying pion. The effect of the pure hadronic interaction of the $K^*(892)^-(K^*(892)^0)$ and $\pi^0(\pi^-)$ on the $K^*(892)^-(K^*(892)^0)$ mass can be precisely measured.
  2) Cross check the impact of the hadronic (π^0) interactions on the $K^*(892)^-$ mass with $\tau^- \to K^-\pi^0\pi^0\nu_\tau$, cross check the impact of the hadronic (π^-) interactions on the $K^*(892)^0$ mass with $\tau^- \to K^-\pi^+\pi^-\nu_\tau$.
  3) It is possible to investigate precisely an effect of the pure hadronic interaction of the $K^*(892)^-(K^*(892)^0)$ and $K^0_S(K^-)$ on the $K^*(892)^-(K^*(892)^0)$ mass in the $\tau^- \to K^0_SK^-\pi^0\nu_\tau$ decay.
  4) Cross check the impact of the hadronic (K^-) interactions on the $K^*(892)^0$ mass with $\tau^- \to \pi^-K^+K^-\nu_\tau$.
- Hadronic τ decays with kaons provide unique laboratory to study K^*-family precisely.

CPV in hadronic τ decays at B factories

- CPV has not been observed in lepton decays
- It is strongly suppressed in the SM ($A^{\rm CP}_{\rm SM} \lesssim 10^{-12}$) and observation of large CPV in lepton sector would be clean sign of New Physics
- τ lepton provides unique possibility to search for CPV effects, as it is the only lepton decaying to hadrons, so that the associated strong phases allows us to visualize CPV in hadronic τ decays.

I. CPV in $\tau^- \to \pi^- K_S(\geq 0\pi^0)\nu_\tau$ at BaBar (Phys. Rev. D 85, 031102 (2012))

Data sample of $\int Ldt = 476$ fb^{-1} was analyzed

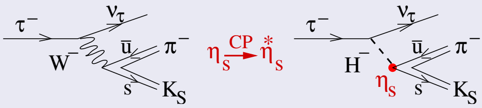
$$A_{\rm CP} = \frac{\Gamma(\tau^+\to\pi^+K^0_S(\geq 0\pi^0)\bar{\nu}_\tau)-\Gamma(\tau^-\to\pi^-K^0_S(\geq 0\pi^0)\nu_\tau)}{\Gamma(\tau^+\to\pi^+K^0_S(\geq 0\pi^0)\bar{\nu}_\tau)+\Gamma(\tau^-\to\pi^-K^0_S(\geq 0\pi^0)\nu_\tau)} = (-0.36 \pm 0.23 \pm 0.11)\%$$

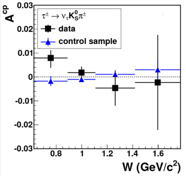
2.8σ deviation from the SM expectation: $A^{K^0}_{\rm CP} = (+0.36 \pm 0.01)\%$

II. CPV in $\tau^- \to K^0_S\pi^-\nu_\tau$ at Belle (Phys. Rev. Lett. 107, 131801 (2011)) $\int$Ldt=699 fb^{-1}

Angular distributions were analyzed, $A_{\rm CP}(W=\sqrt{s})$ was measured ($d\omega = d\cos\beta\, d\cos\theta$):

$$A_{\rm CP}(W) = \frac{\int \cos\beta\cos\psi\left(\frac{d\Gamma_{\tau^-}}{d\omega} - \frac{d\Gamma_{\tau^+}}{d\omega}\right)d\omega}{\frac{1}{2}\int\left(\frac{d\Gamma_{\tau^-}}{d\omega} + \frac{d\Gamma_{\tau^+}}{d\omega}\right)d\omega} \simeq \langle\cos\beta\cos\psi\rangle_{\tau^-} - \langle\cos\beta\cos\psi\rangle_{\tau^+}$$





CPV in $\tau^\pm \to K_S\pi^\pm\nu_\tau$ at Belle (I)

The $K_S^0\pi^-$ hadronic current is parametrized by vector ($F_V(s)$) and scalar ($F_S(s)$) form factor:

$$J^\mu = \langle K_S(q_1)\pi^-(q_2)|\bar{s}\gamma^\mu u|0\rangle = F_V(s)\left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{s}\right)(q_1 - q_2)_\nu + F_S(s)Q^\mu$$

Effect of CP violating scalar boson exchange diagram can be introduced by replacing the SM scalar form factor:

$$F_S(s) \to \overline{F}_S(s) = F_S(s) + \frac{\eta_S}{m_\tau}F_H(s),\ F_H = \langle K_S(q_1)\pi^-(q_2)|\bar{s}u|0\rangle = \frac{s}{m_s - m_u}F_S(s)$$

$$d\Gamma_{\tau^-}(\eta_S) \stackrel{CP}{\to} d\Gamma_{\tau^+}(\overset{*}{\eta}_S)$$

$\tau^- \to K_S\pi^-\nu_\tau$ differential decay width:

$$\frac{d\Gamma}{ds d\cos\beta d\cos\theta} = (A(s) - B(s)(3\cos^2\beta - 1)(3\cos^2\psi - 1))|F_V(s)|^2 + M_\tau^2|F_S|^2 +$$
$$+C(s)\cos\beta\cos\psi\,\mathrm{Re}(F_V\overset{*}{\overline{F}}_S(\eta_S))$$

To extract CPV term the following observable is defined in bin i-th of s ($d\omega = ds\,d\cos\theta\,d\cos\beta$):

$$A_i^{\rm CP} = \frac{\int\limits_i \cos\beta\cos\psi\left(\frac{d\Gamma_{\tau^-}}{d\omega} - \frac{d\Gamma_{\tau^+}}{d\omega}\right)d\omega}{\frac{1}{2}\int\limits_i\left(\frac{d\Gamma_{\tau^-}}{d\omega} + \frac{d\Gamma_{\tau^+}}{d\omega}\right)d\omega} \simeq \langle\cos\beta\cos\psi\rangle^i_{\tau^-} - \langle\cos\beta\cos\psi\rangle^i_{\tau^+}$$

CPV in $\tau^\pm \to K_S \pi^\pm \nu_\tau$ at Belle (II)

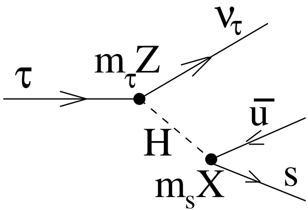
From the $A_i^{\rm CP}$ the CPV parameter $Im(\eta_S)$ can be extracted:

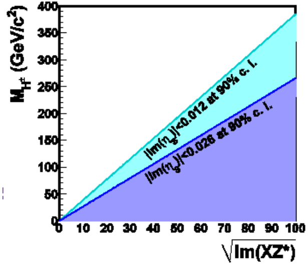
$$A_i^{\rm CP} \simeq {\rm Im}(\eta_S) \frac{N_s}{n_i} \int_i C(s) \frac{{\rm Im}(F_V F_H^*)}{m_\tau} ds \equiv c_i {\rm Im}(\eta_S)$$

Use several parametrizations of F_V and F_S from the previous Belle study of $M_{K_S\pi}$ spectrum and **floating relative phase ($\phi_{\mathbf{S}} = \mathbf{0}^\circ ... \mathbf{360}^\circ$)**:

$$|{\rm Im}(\eta_S)| < (0.012 - 0.026) \text{ at } 90\% \text{ CL}$$

Theoretical predictions for ${\rm Im}(\eta_S)$ in MHDM:





$$\eta_S \simeq \frac{m_\tau m_s}{M_{H^\pm}^2} X^* Z \quad |{\rm Im}(XZ^*)| < 0.15 \frac{M_{H^\pm}^2}{1\,{\rm GeV}^2/c^4} \quad (|{\rm Im}(\eta_S)| < 0.026)$$

Further studies at Belle II and SCTF (II)

$$L_{\mu\nu}H^{\mu\nu} = 2m_\tau^2\left(1-\frac{q^2}{m_\tau^2}\right)(\overline{L}_B W_B + \overline{L}_{SA} W_{SA} + \overline{L}_{SF} W_{SF}),\ q = q_1 + q_2,$$

$$W_B = 4|\vec{q}_1|^2|F_V|^2,\ W_{SA} = q^2|F_S|^2,\ W_{SF} = 4\sqrt{q^2}|\vec{q}_1|\mathrm{Re}[F_V F_S^*]$$

- In the analysis of the $\tau^- \to K_S^0\pi^-\nu_\tau$ and $\tau^- \to K^-\pi^0\nu_\tau$ decays, it is very desirable to measure separately vector (W_B), scalar (W_{SA}) form factors and the interference term (W_{SF}).
- $K^*(892)^-$ mass and width are measured in the vector form factor (properly taking into account the effect of the interference of the $K^*(892)^-$ amplitude with the contributions from the radial exitations, $K^*(1410)^-$ and $K^*(1680)^-$).
- The scalar form factor, W_{SA}, is important for the tests of the various fenomenological models and search for CPV.
- The interference between vector and scalar form factors, W_{SF}, is necessary in the search for CPV in $\tau^- \to K\pi\nu_\tau$ decays.

Further studies at Belle II and SCTF (III)

A complete study of the hadronic τ decays into ≥ 3 hadrons can be done in the full multidimensional phase-space of the reaction:

$\boldsymbol{e^+e^- \to (\tau^- \to \textbf{hadrons}^-\nu_\tau;\ \tau^+ \to \ell^+\nu_\ell\bar{\nu}_\tau)}$

or

$\boldsymbol{e^+e^- \to (\tau^- \to \textbf{hadrons}^-\nu_\tau;\ \tau^+ \to \rho^+\bar{\nu}_\tau)}$

The parametrization of the hadronic current in the $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$ decay was established by CLEO in their unbinned analysis of the $e^+e^- \to (\tau^- \to \pi^-\pi^0\pi^0\nu_\tau,\ \tau^+ \to \ell^+\nu_\ell\bar{\nu}_\tau)$ process in the full phase space: D. M. ASNER *et al.* [CLEO], PHYS. REV. D **61**, 012002 (2000).

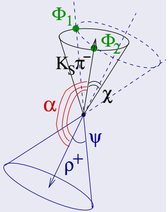
$$J^\mu = \beta_1 j_1^\mu(\rho\pi^0)_{S-wave} + \beta_2 j_2^\mu(\rho'\pi^0)_{S-wave} + \beta_3 j_3^\mu(\rho\pi^0)_{D-wave} + \beta_4 j_4^\mu(\rho'\pi^0)_{D-wave} + \beta_5 j_5^\mu(f_2(1270)\pi)_{P-wave} + \beta_6 j_6^\mu(f_0(500)\pi)_{P-wave} + \beta_7 j_7^\mu(f_0(1370)\pi)_{P-wave}$$

- Before studying hadronic decays, leptonic decay should be analyzed (measurement of Michel parameters) to develop the fitter and polish the procedure (CLEO studied $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$ after they measured Michel parameters).
- The same procedure can be used to study dynamics of the $(\tau^\mp \to (K\pi)^\mp\nu\ ;\ \tau^\pm \to \rho^\pm\nu)$ and $(\tau^\mp \to (K\pi)^\mp\nu\ ;\ \tau^\pm \to \ell^\pm\nu\nu)$ processes and to search for CPV in $\tau^- \to (K\pi)^-\nu_\tau$ (also in the spin-dependent part of the differential decay width).

Further studies at Belle II and SCTF (IV)

Analysis of the $(\tau^\mp \to (K\pi)^\mp \nu\ ;\ \tau^\pm \to \rho^\pm \nu)$ events, search for CPV in $\tau^- \to (K\pi)^- \nu_\tau$.
The analysis of the decay products of both taus allows one to constrain direction of $\tau^- - \tau^+$ axis. Such a constraint is efficient to suppress background from $\tau^- \to (K\pi)^- K^0_L \nu_\tau$.

$$\frac{d\sigma(\vec{\zeta}^*, \vec{\zeta'}^*)}{d\Omega_\tau} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau (D_0 + D_{ij}\zeta_i^*\zeta_j'^*), \quad \frac{d\Gamma(\tau^\pm(\vec{\zeta'}^*) \to \rho^\pm \nu)}{dm^2_{\pi\pi}\, d\Omega^*_\rho\, d\tilde{\Omega}_\pi} = A' \mp \vec{B'}\vec{\zeta'}^*$$



$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*) \to (K\pi)^\mp \nu)}{dm^2_{K\pi}\, d\Omega^*_{K\pi}\, d\tilde{\Omega}_\pi} = \begin{matrix} (A_0 + \eta_{CP} A_1) + (\vec{B}_0 + \eta_{CP}\vec{B}_1)\vec{\zeta}^* \\ (A_0 + \eta^*_{CP} A_1) - (\vec{B}_0 + \eta^*_{CP}\vec{B}_1)\vec{\zeta}^* \end{matrix}$$

$$\frac{d\sigma((K\pi)^\mp, \rho^\pm)}{dm^2_{K\pi}\, d\Omega^*_{K\pi}\, d\tilde{\Omega}_\pi\, dm^2_{\pi\pi}\, d\Omega^*_\rho\, d\tilde{\Omega}_\pi\, d\Omega_\tau} = \frac{\alpha^2\beta_\tau}{64E_\tau^2}\begin{pmatrix} \mathcal{F} + \eta_{CP}\mathcal{G} \\ \mathcal{F} + \eta^*_{CP}\mathcal{G} \end{pmatrix}$$

$$\mathcal{F} = D_0 A_0 A' - D_{ij} B_{0i} B'_j, \quad \mathcal{G} = D_0 A_1 A' - D_{ij} B_{1i} B'_j$$

$$\frac{d\sigma((K\pi)^\mp, \rho^\pm)}{dp_{K\pi}\, d\Omega_{K\pi}\, dm^2_{K\pi}\, d\tilde{\Omega}_\pi\, dp_\rho\, d\Omega_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi} = \sum_{\Phi_1, \Phi_2} \frac{d\sigma((K\pi)^\mp, \rho^\pm)}{dm^2_{K\pi}\, d\Omega^*_{K\pi}\, d\tilde{\Omega}_\pi\, dm^2_{\pi\pi}\, d\Omega^*_\rho\, d\tilde{\Omega}_\pi\, d\Omega_\tau}\left|\frac{\partial(\Omega^*_{K\pi}, \Omega^*_\rho, \Omega_\tau)}{\partial(p_{K\pi}, \Omega_{K\pi}, p_\rho, \Omega_\rho)}\right|$$

CPV parameter η_{CP} is extracted in the simultaneous unbinned maximum likelihood fit of the $((K\pi)^-, \rho^+)$ and $((K\pi)^+, \rho^-)$ events in the 12D phase space.

Further studies at Belle II and SCTF (V)

Such kind of analysis was done only once by CLEO for the $(\tau \to \ell\nu\nu;\ \tau \to \pi\pi^0\pi^0\nu)$ events to study the dynamics of $\tau \to \pi\pi^0\pi^0\nu$ decay.

If we pretend on the $\lesssim$ 1% level in the studies of the dynamics of hadronic τ decays, the research program for any hadronic τ decay should be:

- Measure hadronic structure functions on the signal side.
- Perform the unbinned fit of the full event configuration $(\tau \to \mathrm{signal};\ \tau \to \mathrm{tag})$, where the dynamics of the $\tau \to \mathrm{tag}$ is well known (for example, leptonic tag). Identify the structure of the "remnant" from the spin-spin correlation term and correct the hadronic structure functions, measured on the first step.
- Extract CPV parameter in the simulataneous approximation of the $(\tau^- \to \mathrm{signal}^-;\ \tau^+ \to \mathrm{tag}^+)$ and $(\tau^+ \to \mathrm{signal}^+;\ \tau^- \to \mathrm{tag}^-)$ events.
- The usage of the proper generator (TAUOLA), where the effects related to τ spin are implemented, is mandatory.

CPV in $\tau^\pm \to K_S\pi^\pm\nu_\tau$ with polarized τ lepton (I)

S. Y. Choi *et al.*, PLB **437**, 191 (1998).

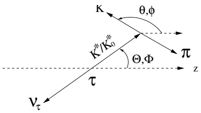
$$M_\sigma = \frac{G_F}{\sqrt{2}}\sin\theta_c\left[(1+\chi)\bar{u}_\nu(k,-)\gamma^\mu(1-\gamma^5)u(p,\sigma)J_\mu + \eta\bar{u}_\nu(k,-)(1+\gamma^5)u(p,\sigma)J_S\right]$$

$$J_\mu = \langle(K\pi)^-|\bar{s}\gamma_\mu u|0\rangle = F_V(q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)(q_1-q_2)^\nu + F_S(q^2)q^\mu$$

$$J_S = \langle(K\pi)^-|\bar{s}u|0\rangle = \frac{q^2}{m_s - m_u}F_S(q^2)$$

- $\sigma = \pm1$ – helicity of τ^-;
- χ, η parametrize BSM contribution, $\xi = \frac{m^2_{K_0^*}}{(m_s-m_u)m_\tau}\left(\frac{\eta}{1+\chi}\right)$
- τ^-: $M_\pm(\chi,\xi)$, τ^+: $\overline{M}_\pm(\chi^*,\xi^*)$

If χ and η are real: $M_\pm(\Theta; q^2; \theta, \phi) = \mp\overline{M}_\mp(\Theta; q^2; \theta, -\phi)$
τ is polarized in the $(\theta_p,\ \phi_p)$ direction:
$|\theta_p,\ \phi_p\rangle = \cos\frac{\theta_p}{2}|+\rangle + \sin\frac{\theta_p}{2}|-\rangle$
$\langle\Theta,\ \Phi|\theta_p,\ \phi_p\rangle = \cos\frac{\theta_p}{2}M_+ + \sin\frac{\theta_p}{2}M_-$



$$d\Gamma = \frac{1}{2}d(\Gamma_{++}+\Gamma_{--}) + P_\tau\left(\frac{1}{2}\cos_p d(\Gamma_{++}-\Gamma_{--}) + \sin_p\cos(\phi_p-\Phi)\,d\mathrm{Re}\Gamma_{+-} - \sin_p\sin(\phi_p-\Phi)\,d\mathrm{Im}\Gamma_{+-}\right)$$

$$d\Gamma_{\sigma\sigma'} = \frac{1}{(2\pi)^3}\frac{1}{32m_\tau}\left(1-\frac{q^2}{m_\tau^2}\right)M_\sigma M^*_{\sigma'}P_K\,d\Phi_3\,d\Phi, \quad d\Phi_3 = d\sqrt{q^2}\,d\cos\,d\phi\,d\cos$$

CPV in $\tau^\pm \to K_S \pi^\pm \nu_\tau$ with polarized τ lepton (II)

After integration on Φ (and $P_\tau = 1$):

$$\frac{d\Gamma_1}{d\Phi_3} = \frac{d(\Gamma_{++} + \Gamma_{--})}{d\Phi_3},\ \frac{d\Gamma_2}{d\Phi_3} = \frac{d(\Gamma_{++} - \Gamma_{--})}{d\Phi_3},\ \frac{d\Gamma_3}{d\Phi_3} = 2\mathrm{Re}\left(\frac{d\Gamma_{+-}}{d\Phi_3}\right),\ \frac{d\Gamma_4}{d\Phi_3} = 2\mathrm{Im}\left(\frac{d\Gamma_{+-}}{d\Phi_3}\right)$$

$$\frac{d\Gamma_i}{d\Phi_3} = \frac{1}{2}(\Sigma_i + \Delta_i),\quad \Sigma_i/\Delta_i - \text{CP even/odd part},\quad i = 1 \div 4$$

$$\Sigma_1 = \frac{d(\Gamma_1 + \bar{\Gamma}_1)}{d\Phi_3},\ \Sigma_2 = \frac{d(\Gamma_2 - \bar{\Gamma}_2)}{d\Phi_3},\ \Sigma_3 = \frac{d(\Gamma_3 - \bar{\Gamma}_3)}{d\Phi_3},\ \Sigma_4 = \frac{d(\Gamma_4 + \bar{\Gamma}_4)}{d\Phi_3},$$

$$\Delta_1 = \frac{d(\Gamma_1 - \bar{\Gamma}_1)}{d\Phi_3},\ \Delta_2 = \frac{d(\Gamma_2 + \bar{\Gamma}_2)}{d\Phi_3},\ \Delta_3 = \frac{d(\Gamma_3 + \bar{\Gamma}_3)}{d\Phi_3},\ \Delta_4 = \frac{d(\Gamma_4 - \bar{\Gamma}_4)}{d\Phi_3}$$

CP even: $\Sigma_1 \gg \Sigma_2, \Sigma_3, \Sigma_4$,
CP odd: Δ_1 – P_τ-independent part, $\Delta_{2,3,4}$ – P_τ-dependent part.
Four optimal variables to search for CPV are: $w_i^{\rm opt} = \Delta_i/\Sigma_1$.

P_τ-independent $\boldsymbol{w_1^{\rm opt}}$ was used at Belle, while 3 P_τ-dependent $\boldsymbol{w_{2\div4}^{\rm opt}}$ can be additionally measured at the Super Charm-Tau factory:

$$w_1^{\rm opt} = A_1(q^2;\Theta,\theta,\phi)\mathrm{Im}(\xi)\mathrm{Im}(F_V F_S^*),$$

$$w_{2\div4}^{\rm opt} = A_{2\div4}(q^2;\Theta,\theta,\phi)\mathrm{Im}(\xi)\mathrm{Im}(F_V F_S^*) + B_{2\div4}(q^2;\Theta,\theta,\phi)\ \mathrm{Im}(\xi)\mathrm{Re}(F_V F_S^*)$$

At the Super Charm-Tau factory CPV search doesn't depend on $F_V F_S^*$ phase.

CPV in $\tau^\pm \to K_S\pi^\pm\nu_\tau$ with polarized τ lepton (III)

At the center-of-mass energies close to the $\tau^+\tau^-$ production threshold the τ lepton is produced with the polarization $|\vec{P}_\tau| = P_e \frac{2E_{\rm beam}\sqrt{p^2_{\rm beam}\cos^2\theta + M^2_\tau}}{E^2_{\rm beam}+M^2_\tau+p^2_{\rm beam}\cos^2\theta} \approx P_e$ along electron beam polarization ($(P_\tau)_Z = P_e \frac{E_{\rm beam}\cos^2\theta + M_\tau \sin^2\theta}{\sqrt{p^2_{\rm beam}\cos^2\theta+M^2_\tau}} \approx P_e$).

In case of New Physics contribution, the amplitudes for the decays $\tau^- \to (K\pi)^-\nu_\tau$ and $\tau^+ \to (K\pi)^+\bar{\nu}_\tau$ are:

$$\mathcal{A} = A_1 + A_2 e^{i\phi}e^{i\delta}, \;\; \overline{\mathcal{A}} = A_1 + A_2 e^{-i\phi}e^{i\delta}$$

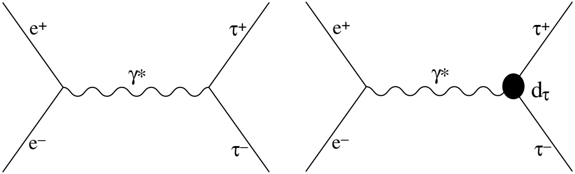
where ϕ and δ are relative weak (CP-odd) and strong (CP-even) phases. CPV is studied comparing $|\mathcal{A}|^2$ and $|\overline{\mathcal{A}}|^2$, there are three possibilities to construct CPV asymmetry:

- decay rate asymmetry $\sim \sin\delta \sin\phi$
- weighted rate asymmetry $\sim \sin\delta \sin\phi$
- asymmetry based on $\vec{P}_\tau(\vec{p}_K \times \vec{p}_\pi)$ triple product $\sim \cos\delta \sin\phi$

At the Super Charm-Tau factory, with nonzero single τ polarization, nonzero strong-phase difference, δ, is not needed to measure CPV.

Electric dipole moment of τ, introduction

Electric dipole moment (EDM) of τ is strongly suppressed in the Standard Model ($\mathcal{O}(10^{-37})$ e·cm), EDM$\neq 0$ indicates the nonconservation of **T**(**CP**) and **P** symmetries. EDM provides powerful tool to search for New Physics in lepton sector.



$$\mathcal{L} = \bar{\tau}((i\partial_\mu - eA_\mu)\gamma^\mu - m)\tau - id_\tau\bar{\tau}\sigma^{\mu\nu}\gamma^5\tau\partial_\mu A_\nu$$

$$\mathcal{M}^2_{tot} = \mathcal{M}^2_{SM} + Re(d_\tau)\mathcal{M}^2_{Re} + Im(d_\tau)\mathcal{M}^2_{Im} + |d_\tau|^2\mathcal{M}^2_{d^2}$$

$$\frac{d\Gamma(\tau^\mp \to h^\mp\nu)}{d\mathcal{PS}} = F(1 \pm \vec{\zeta}_{\tau^\mp}\vec{H}_{h^\mp}),\ \vec{H}_{\pi^\mp} = \vec{p}_{\pi^\mp}/|\vec{p}_{\pi^\mp}|$$

$\vec{\zeta}_{\tau^\mp}$ - unitary $\tau^\mp$ polarization vector; $\vec{H}_{h^\mp}$ - $h^\mp$ polarimeter vector.

$$\mathcal{M}^2_{Re} \sim (\vec{H}_{h_1^+}\times\vec{H}_{h_2^-})\vec{p}_e,\ (\vec{H}_{h_1^+}\times\vec{H}_{h_2^-})\vec{p}_\tau : \mathrm{CP-odd,\ T-odd\ (CPT-cons.)}$$

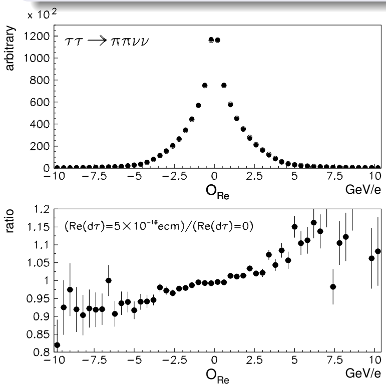
$$\mathcal{M}^2_{Im} \sim (\vec{H}_{h_1^+}-\vec{H}_{h_2^-})\vec{p}_e,\ (\vec{H}_{h_1^+}-\vec{H}_{h_2^-})\vec{p}_\tau : \mathrm{CP-odd,\ T-even\ (CPT-viol.)}$$

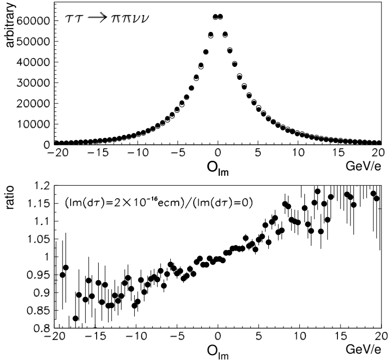
Method of optimal variable is used to measure $\mathrm{Re}(d_\tau)$ and $\mathrm{Im}(d_\tau)$.

$$\mathcal{O}_{\mathrm{Re}} = \frac{\mathcal{M}^2_{\mathrm{Re}}}{\mathcal{M}^2_{\mathrm{SM}}},\ \mathcal{O}_{\mathrm{Im}} = \frac{\mathcal{M}^2_{\mathrm{Im}}}{\mathcal{M}^2_{\mathrm{SM}}},\ \langle \mathcal{O}_{\mathrm{Re,Im}} \rangle \sim \int \mathcal{O}_{\mathrm{Re,Im}} \mathcal{M}^2_{\mathrm{tot}} d\mathcal{PS}$$

$$\langle \mathcal{O}_{\mathrm{Re}} \rangle = \mathbf{a}_{\mathrm{Re}} \mathrm{Re}(d_\tau) + \mathbf{b}_{\mathrm{Re}},\ \langle \mathcal{O}_{\mathrm{Im}} \rangle = \mathbf{a}_{\mathrm{Im}} \mathrm{Im}(d_\tau) + \mathbf{b}_{\mathrm{Im}}$$

$$\mathbf{a}_{\mathrm{Re,Im}} = \langle \mathcal{O}^2_{\mathrm{Re,Im}} \rangle = \int \frac{(\mathcal{M}^2_{\mathrm{Re,Im}})^2}{\mathcal{M}^2_{\mathrm{SM}}} d\mathcal{PS},\ \mathbf{b}_{\mathrm{Re,Im}} = \int \mathcal{M}^2_{\mathrm{Re,Im}} d\mathcal{PS}$$





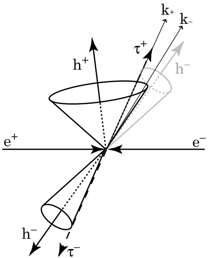
Tau EDM at Belle, data/selections

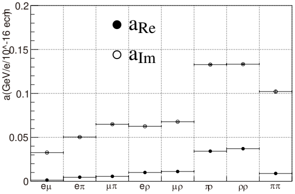
Statistics with $\int Ldt = 825$ fb^{-1} ($N_{\tau\tau} = 758 \times 10^6$) is used.
In total about 35M events are selected with the purity of 88%.

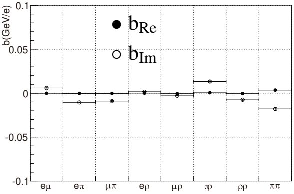
Select 8 configurations: $(e\nu\nu;\ \mu\nu\nu)$, $(e\nu\nu;\ \pi\nu)$, $(e\nu\nu;\ \rho\nu)$, $(\mu\nu\nu;\ \pi\nu)$, $(\mu\nu\nu;\ \rho\nu)$, $(\pi\nu;\ \pi\nu)$, $(\pi\nu;\ \rho\nu)$, $(\rho\nu;\ \rho\nu)$.

In the calculation of $\mathcal{O}_{\rm Re,Im}$ the average allowed τ direction is used. Coefficients $\mathbf{a}_{\rm Re,Im}$ and $\mathbf{b}_{\rm Re,Im}$ are determined from MC.

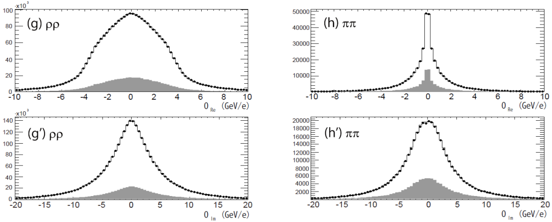
mode	yield	purity(%)	background (%)
$e\mu$	6434k	95.8	$2\gamma \to \mu\mu(2.5)$, $\tau\tau \to e\pi(1.3)$
$e\pi$	2645k	85.7	$\tau\tau \to e\rho(6.5)\ e\mu(5.1)\ eK^*(1.3)$
$e\rho$	7219k	91.7	$\tau\tau \to e\pi 2\pi^0(4.6)\ eK^*(1.7)$
$\mu\pi$	2504k	80.5	$\tau\tau \to \mu\rho(6.4)\ \mu\mu(4.9)\ \mu K^*(1.3)$, $2\gamma \to \mu\mu(3.1)$
$\mu\rho$	6203k	91.0	$\tau\tau \to \mu\pi 2\pi^0(4.3)\ \mu K^*(1.6)\ \pi\rho(1.1)$
$\pi\pi$	921k	71.9	$\tau\tau \to \pi\rho(11.3)\ \pi\mu(8.8)\ \pi K^*(2.5)$
$\pi\rho$	2656k	77.0	$\tau\tau \to \rho\rho(6.7)\ \pi\pi 2\pi^0(3.9)\ \mu\rho(5.1)$ $\rho K^*(1.4)\ \pi K^*(1.4)$
$\rho\rho$	6554k	82.4	$\tau\tau \to \rho\pi 2\pi^0(9.4)\ \rho K^*(3.1)$







Tau EDM at Belle, result



$\mathrm{Re}(d_\tau)$	$e\mu$	$e\pi$	$\mu\pi$	$e\rho$	$\mu\rho$	$\pi\rho$	$\rho\rho$	$\pi\pi$
Mismatch of distribution	0.30	0.47	0.35	0.08	0.17	0.08	0.08	0.34
Charge asymmetry	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
Background variation	0.16	0.03	0.16	0.04	0.02	0.02	0.02	0.33
Momentum reconstruction	0.01	0.06	0.05	0.00	0.02	0.02	0.01	0.14
Detector alignment	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.03
Radiative effects	0.07	0.05	0.05	0.02	0.02	0.00	0.00	0.09
Total	0.35	0.47	0.39	0.09	0.17	0.08	0.08	0.50

$\mathrm{Im}(d_\tau)$	$e\mu$	$e\pi$	$\mu\pi$	$e\rho$	$\mu\rho$	$\pi\rho$	$\rho\rho$	$\pi\pi$
Mismatch of distribution	0.09	0.09	0.05	0.05	0.07	0.04	0.04	0.12
Charge asymmetry	0.02	0.19	0.23	0.01	0.01	0.11	0.00	0.00
Background variation	0.14	0.01	0.07	0.03	0.01	0.01	0.01	0.01
Momentum reconstruction	0.02	0.05	0.04	0.00	0.01	0.01	0.00	0.01
Detector alignment	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Radiative effects	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00
Total	0.17	0.22	0.24	0.06	0.07	0.11	0.04	0.12

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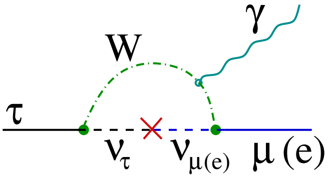
$(-1.85\times10^{-17} < \mathrm{Re}(d_\tau) < 0.61\times10^{-17})$ $e\cdot$**cm**, $(-1.03\times10^{-17} < \mathrm{Im}(d_\tau) < 0.23\times10^{-17})$ $e\cdot$**cm (CL=95%)**

Tau EDM at Belle II and SCTF

X. Chen and Y. Wu, JHEP **10** (2019) 089.
Feasibility study showed that the full statistics of Belle II experiment ($50\,\mathrm{ab}^{-1}$) allows one to get the statistical EDM sensitivity of the order of 10^{-19} e·cm.
Polarized electron beam at the SCTF essentially improves the sensitivity to the EDM (especially its real part).
B. Ananthanarayan and S. D. Rindani, Phys. Rev. D **51** (1995) 5996.
It should be also mentioned here that the absence of NNLO corrections to the $e^+e^- \to \tau^+\tau^-$ cross section doesn't limit the sensitivity to the EDM, because the EDM-dependent efficient Lagrangian is CP-odd, while the SM Lagrangian is CP-even function. Higher order NNLO SM corrections don't impact on the EDM-dependent interaction.

Lepton-flavor-violating (LFV) τ decays

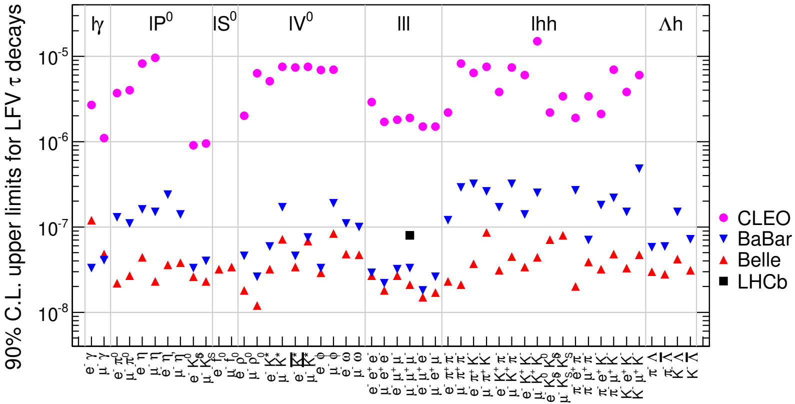


Model	$\mathcal{B}(\tau \to \mu\gamma)$	$\mathcal{B}(\tau \to \ell\ell\ell)$
mSUGRA+seesaw	10^{-8}	10^{-9}
SUSY+SO(10)	10^{-8}	10^{-10}
SM+seesaw	10^{-9}	10^{-10}
Non-universal Z'	10^{-9}	10^{-8}
SUSY+Higgs	10^{-10}	10^{-8}

- Probabililty of LFV decays of charged leptons is extremely small in the Standard Model, $\mathcal{B}(\tau \to \ell\nu) \sim \left(\frac{\Delta m_\nu^2}{m_W^2}\right)^2 < 10^{-54}$
- Many models beyond the SM predict LFV decays with the branching fractions up to $\lesssim 10^{-8}$. As a result observation of LFV is a clear signature of New Physics (NP).
- τ lepton is an excellent laboratory to search for the LFV decays due to the enhanced couplings to the new particles as well as large number of LFV decay modes
- Study of the different τ LFV decay modes allows us to test various NP models.

Results on LFV decays of τ

48 different LFV modes were studied at *B* factories



$\mathcal{B}(\tau^- \to \mu^-\gamma) < 4.2 \times 10^{-8}$, $\mathcal{B}(\tau^- \to e^-\gamma) < 5.6 \times 10^{-8}$ (CL=90%),

$\mathcal{B}(\tau^- \to pe^-e^-) < 3.0 \times 10^{-8}$, $\mathcal{B}(\tau^- \to p\mu^-\mu^-) < 4.0 \times 10^{-8}$ (CL=90%),
$\mathcal{B}(\tau^- \to \bar{p}e^+e^-) < 3.0 \times 10^{-8}$, $\mathcal{B}(\tau^- \to \bar{p}e^+\mu^-) < 2.0 \times 10^{-8}$ (CL=90%),
$\mathcal{B}(\tau^- \to \bar{p}e^-\mu^+) < 1.8 \times 10^{-8}$, $\mathcal{B}(\tau^- \to \bar{p}\mu^+\mu^-) < 1.8 \times 10^{-8}$ (CL=90%).

Recent Belle results: JHEP 10 019 (2021), PRD 102 2020, 111101(R).

$\tau \to \mu\gamma$ and $\tau^- \to \ell^- + \text{invisible}$ LFV decays

At B factories an upper limit on the probability of $\tau \to \mu\gamma$ is determined by the background from the process $e^+e^- \to \tau^+\tau^-\gamma$.
At the SCTF (near $\tau + \tau^-$ production threshold) this background is negligible, hence, in spite of less statistics, the upper limit on the branching ratio of the $\tau \to \mu\gamma$ at the SCTF can be better (to the level of 10^{-9} and below) than at the Belle II.
A. V. Bobrov and A. E. Bondar, Nucl. Phys. Proc. Suppl. **225-227** (2012) 195; Nucl. Phys. Proc. Suppl. **253-255** (2014) 199.
Of particular importance are LFV τ lepton decays to invisible BSM particles produced in various models containing axion-like particles or new Z' gauge bosons. One possibility of such processes is $\tau \to \ell\alpha$ ($\ell = e,\ \mu$), and α is a massive particle that escapes to detection. These light particles could serve also as dark matter candidates. For the massless α ARGUS obtained:

$$\mathcal{B}(\tau \to e\alpha) < 2.7 \times 10^{-3}\ (95\%\ \mathrm{CL}),\ \mathcal{B}(\tau \to \mu\alpha) < 4.5 \times 10^{-3}\ (95\%\ \mathrm{CL}).$$

The expected upper limit on the $\mathcal{B}(\tau \to \mu\alpha)$ for the massless α at Belle is $\mathcal{B}(\tau \to \mu\alpha) < 1.1 \times 10^{-4}$ (90% CL). At Belle II with the expected statistics of 46 billion τ pairs, the upper limit on the $\mathcal{B}(\tau \to \mu\alpha)$ will reach the order of 10^{-5}.
At the SCTF near $\tau^+\tau^-$ production threshold, charged lepton from the $\tau \to \ell\alpha$ decay is already monochromatic. More over this the effect of the ISR is also absent, so the signal lepton momentum peak will be much narrower than at the B factories resulting in much lower background from ordinary leptonic decay. Also, possibility of the inclusive reconstruction of the tag τ in this study at the SCTF provides notable compensation of smaller statistics of τ leptons at the SCTF in comparison with Belle II. Lack of ISR at the SCTF near $\tau^+\tau^-$ production threshold also simplifies a search for radiative decays $\tau \to \ell\alpha\gamma$, which provide additional constraints on various BSM models.

Search for heavy Majorana neutrino in τ decays (I)

C. Greub, D. Wyler and W. Fetscher, Phys. Lett. B **324** (1994) 109 [Erratum-ibid. B **329** (1994) 526]

In the case of nonzero neutrino mass additional Michel parameters, λ and σ, appear in the differential decay width of $\tau \to \ell\nu\nu$ with additional suppression factor of m_ν/m_τ. But even ordinary Michel parameters can be used to search for the effect of Majorana neutrino: M. Doi, T. Kotani and H. Nishiura, Prog. Theor. Phys. **118** (2007) 1069 [Erratum-ibid. **122** (2009) 805].

$$\Delta\rho \sim |g^V_{LR}|^2(\overline{v_\mu}^2 + |\overline{w_{e\mu}}|^2) + |g^V_{RL}|^2(\overline{v_e}^2 + |\overline{w_{e\mu h}}|^2), \;\; \Delta\eta \sim g^V_{RR}\mathrm{Re}(\overline{w_{e\mu}}^*\overline{w_{e\mu h}})$$

,

$$\Delta\xi \sim -|g^V_{RR}|^2\overline{v_e}^2\overline{v_\mu}^2, \;\; \Delta\delta \sim |g^V_{LR}|^2(\overline{v_\mu}^2 + |\overline{w_{e\mu}}|^2) - |g^V_{RL}|^2(\overline{v_e}^2 + |\overline{w_{e\mu h}}|^2)$$

$$. \sum_j |U_{\ell j}|^2 = 1 - \overline{u_\ell}^2, \;\; \sum_j |V_{\ell j}|^2 = \overline{v_\ell}^2, \;\; \ell = e, \; \mu,$$

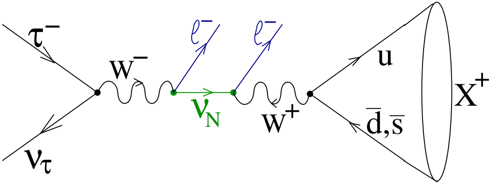
$$\sum_j U_{ej}V_{\mu j} = \overline{w_{e\mu}}, \;\; \sum_k V_{ek}U_{\mu k} = \overline{w_{e\mu h}},$$

$$\overline{u_\ell}^2 \sim \overline{v_\ell}^2 \sim \mathcal{O}((m_{\nu D}/m_{\nu R})^2), \;\; \overline{w_{e\mu}} \sim \overline{w_{e\mu h}} \sim \mathcal{O}(m_{\nu D}/m_{\nu R}),$$

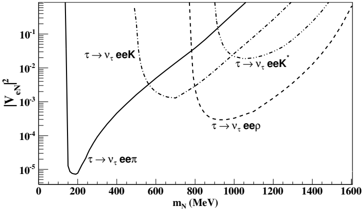
where: $U_{\ell j}/V_{\ell j}$ - left/right-chirality lepton mixing matrix, $m_{\nu D}$ and $m_{\nu R}$ are Dirac-type and right-chirality Majorana-type elements of the neutrino mass matrix.

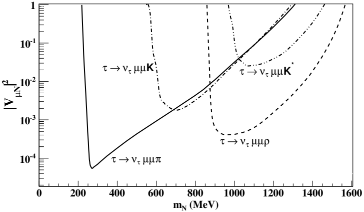
Search for heavy Majorana neutrino in τ decays (II)

$\tau^- \to \ell^- \ell^- X^+ \nu_\tau$ ($\ell = e,\ \nu$; $X = \pi,\ K,\ \rho,\ K^*$) decays with $|\Delta L| = 2$ can be induced by the exchange of Majorana neutrinos.



G. Lopez Castro and N. Quintero, Phys. Rev. D **85** (2012) 076006.





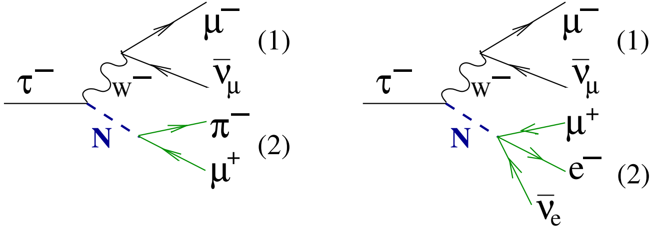
In the case of the resonant mechanism where the contribution of one heavy Majorana neutrino dominates and $\mathcal{B}$ upper limits of $\mathcal{O}(10^{-7})$ (which can be reached at Belle II) the constraints on the $|V_{\ell N}|^2$ vs m_N plane can be obtained (competitive to the constraints from B and D decays).

Search for sterile neutrino in τ decays

C. Dib *et al.*, Phys. Rev. D **85** (2012) 011301.

To clarify MiniBooNE and LSND anomalies it was suggested to search for the long-living sterile neutrino.

$400\ \text{MeV} \lesssim m_N \lesssim 600\ \text{MeV}$, $1 \times 10^{-3} \lesssim |U_{\mu N}|^2 \lesssim 4 \times 10^{-3}$, $\tau_N \lesssim 1 \times 10^{-9}$ s.



To explain anomaly the branching fractions must be:

$$\mathcal{B}(\tau^- \to \{\mu^-\bar{\nu}_\mu\}_1\{\mu^+\pi^-\}_2) = 2.0 \times 10^{-9} \div 1.3 \times 10^{-5},$$
$$\mathcal{B}(\tau^- \to \{\mu^-\bar{\nu}_\mu\}_1\{\mu^+e^-\bar{\nu}_e\}_2) = 2.1 \times 10^{-8} \div 8.2 \times 10^{-5}.$$

The main signature is the displaced vertex (2) with $L = 0.6 \div 30$ cm.

Sterile neutrino can be also searched for in radiative leptonic decays: $\tau^- \to \ell^-\bar{\nu}_\ell N,\ N \to \gamma\nu$, i.e. constraints can be obtained from Michel parameters.

Summary

- The world largest statistics of τ leptons collected by Belle and *BABAR* opens new era in the precision tests of the Standard Model, search for the effects of New Physics and precision studies of low energy QCD.
  Belle II is the main player in τ studies in the nearest future.
- Nonzero average polarization of single τ at the SCTF make it to be competitive to Belle II player in τ lepton studies regardless smaller expected statistics (by a factor of 2.2).
- Vast research program of the proper precise study of hadronic τ decays taking into account spin-spin correlation term requires great effort and competition between at least two e^+e^- experiments (Belle II and SCTF).
- The physics program of the SCTF should be further developed to unveil rich potential of the e^- beam polarization option.

Backup slides

Michel parameters

$$\rho = \frac{3}{4} - \frac{3}{4}\left(|g^V_{LR}|^2 + |g^V_{RL}|^2 + 2|g^T_{LR}|^2 + 2|g^T_{RL}|^2 + \Re(g^S_{LR}g^{T*}_{LR} + g^S_{RL}g^{T*}_{RL})\right)$$

$$\eta = \frac{1}{2}\Re\left(6g^V_{RL}g^{T*}_{LR} + 6g^V_{LR}g^{T*}_{RL} + g^S_{RR}g^{V*}_{LL} + g^S_{RL}g^{V*}_{LR} + g^S_{LR}g^{V*}_{RL} + g^S_{LL}g^{V*}_{RR}\right)$$

$$\xi = 4\Re(g^S_{LR}g^{T*}_{LR}) - 4\Re(g^S_{RL}g^{T*}_{RL}) + |g^V_{LL}|^2 + 3|g^V_{LR}|^2 - 3|g^V_{RL}|^2 - |g^V_{RR}|^2 +$$

$$+5|g^T_{LR}|^2 - 5|g^T_{RL}|^2 + \frac{1}{4}|g^S_{LL}|^2 - \frac{1}{4}|g^S_{LR}|^2 + \frac{1}{4}|g^S_{RL}|^2 - \frac{1}{4}|g^S_{RR}|^2$$

$$\xi\delta = \frac{3}{16}|g^S_{LL}|^2 - \frac{3}{16}|g^S_{LR}|^2 + \frac{3}{16}|g^S_{RL}|^2 - \frac{3}{16}|g^S_{RR}|^2 - \frac{3}{4}|g^T_{LR}|^2 + \frac{3}{4}|g^T_{RL}|^2 +$$

$$+\frac{3}{4}|g^V_{LL}|^2 - \frac{3}{4}|g^V_{RR}|^2 + \frac{3}{4}\Re(g^S_{LR}g^{T*}_{LR}) - \frac{3}{4}\Re(g^S_{RL}g^{T*}_{RL})$$

$$\bar{\eta} = \left|g^V_{RL}\right|^2 + \left|g^V_{LR}\right|^2 + \frac{1}{8}\left(\left|g^S_{RL} + 2g^T_{RL}\right|^2 + \left|g^S_{LR} + 2g^T_{LR}\right|^2\right) + 2\left(\left|g^T_{RL}\right|^2 + \left|g^T_{LR}\right|^2\right)$$

$$\xi\kappa = \left|g^V_{RL}\right|^2 - \left|g^V_{LR}\right|^2 + \frac{1}{8}\left(\left|g^S_{RL} + 2g^T_{RL}\right|^2 - \left|g^S_{LR} + 2g^T_{LR}\right|^2\right) + 2\left(\left|g^T_{RL}\right|^2 - \left|g^T_{LR}\right|^2\right)$$

Method: theoretical framework

W. Fetscher, Phys. Rev. D **42** (1990) 1544. K. Tamai, Nucl. Phys. B **668** (2003) 385.

$$\frac{d\sigma(\vec{\zeta},\vec{\zeta}')}{d\Omega} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i\zeta_j')$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*) \to \ell^\mp\nu\nu)}{dx^* d\Omega_\ell^*} = \kappa_\ell(A(x^*) \mp \xi\vec{n}_\ell^*\vec{\zeta}^* B(x^*)),\ x^* = E_\ell^*/E_{\ell max}^*$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*),\ B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\Gamma(\tau^\pm(\vec{\zeta'}^*) \to \rho^\pm\nu)}{dm_{\pi\pi}^2 d\Omega_\rho^* d\tilde{\Omega}_\pi} = \kappa_\rho(A' \mp \xi_\rho\vec{B'}\vec{\zeta'}^*)W(m_{\pi\pi}^2) = \kappa_\rho A'(1 \mp \xi_\rho\vec{H}_\rho\vec{\zeta'}^*)W(m_{\pi\pi}^2)$$

$$\vec{H}_\rho = \frac{\vec{B'}}{A'} - \text{polarimeter vector},\ \xi_\rho = -\frac{2Re(c_V^* c_A)}{|c_V|^2 + |c_A|^2} = -h_{\nu_\tau}\ (h_{\nu_\tau} = -1 \text{ in the SM})$$

$$A' = 2(q,Q)Q_0^* - Q^2 q_0^*,\ \vec{B'} = Q^2\vec{K}^* + 2(q,Q)\vec{Q}^*,\ W = |F_\pi(m_{\pi\pi}^2)|^2\frac{p_\rho(m_{\pi\pi}^2)\tilde{p}_\pi(m_{\pi\pi}^2)}{M_\tau m_{\pi\pi}}$$

$$\frac{d\sigma(\ell^\mp,\rho^\pm)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = \kappa_\ell\kappa_\rho\frac{\alpha^2\beta_\tau}{64E_\tau^2}\left(D_0 A' A(E_\ell^*) + \xi_\rho\xi_\ell D_{ij} n_{\ell i}^* B_j' B(E_\ell^*)\right)W(m_{\pi\pi}^2)$$

$$\frac{d\sigma(\ell^\mp,\rho^\pm)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2}\frac{d\sigma(\ell^\mp,\rho^\pm)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau}\left|\frac{\partial(E_\ell^*,\Omega_\ell^*,\Omega_\rho^*,\Omega_\tau)}{\partial(p_\ell,\Omega_\ell,p_\rho,\Omega_\rho,\Phi_\tau)}\right|d\Phi_\tau$$

Multidimensional unbinned maximum likelihood fit

4 Michel parameters ($\vec{\Theta} = (1,\ \rho,\ \eta,\ \xi_\rho\xi_\ell,\ \xi_\rho\xi_\ell\delta_\ell)$) are extracted in the unbinned maximum likelihood fit of ($\ell\nu\nu;\ \rho\nu$) events in the 9D phase space in CMS,
$\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$. The PDF for individual k-th event is written in the form:

$$\mathcal{P}^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})},\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z}$$

Likelihood function for N events:

$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)},\ \mathcal{L} = -\ln L = N\ln\mathcal{N}(\vec{\Theta}) - \sum_{k=1}^{N} \ln\mathcal{F}^{(k)},\ \mathcal{F}^{(k)} = \mathcal{F}(\vec{z}^{(k)})$$

$$\mathcal{F}^{(k)} = A_0^{(k)}\Theta_0 + A_1^{(k)}\Theta_1 + A_2^{(k)}\Theta_2 + A_3^{(k)}\Theta_3 + A_4^{(k)}\Theta_4 = \sum_{i=0}^{4} A_i^{(k)}\Theta_i$$

$$\mathcal{N} = C_0\Theta_0 + C_1\Theta_1 + C_2\Theta_2 + C_3\Theta_3 + C_4\Theta_4,\ C_j = \frac{1}{N}\sum_{k=1}^{N} C_j^{(k)},\ C_j^{(k)} = \frac{A_j^{(k)}}{\sum_{i=0}^{4} A_i^{(k)}\Theta_i^{MC}}$$

$$\vec{\Theta}^{MC} = (1,\ 0.75,\ 0,\ 1,\ 0.75),\ \mathcal{L} = N\ln\left(\sum_{j=0}^{4} C_j\Theta_j\right) - \sum_{k=1}^{N}\ln\left(\sum_{i=0}^{4} A_i^{(k)}\Theta_i\right)$$

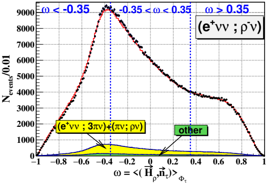
As a result fitted statistics is represented by a set of $5\times N$ values of $A_i^{(k)}$ ($k = 1 \div N,\ i = 0 \div 4$), which is calculated only once.
C_i ($i = 0 \div 4$) are calculated using MC simulation.
In ideal case (no rad. corr., $\varepsilon = 100\%$): $C_0 = 1$, $C_2 = 4m_\ell/m_\tau$, $C_{1,3,4} = 0$

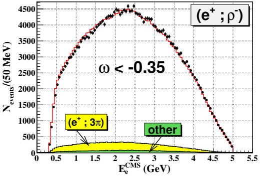
Method: helicity sensitive variable

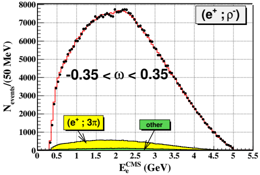
M. Davier *et. al* Phys. Lett. B **306** (1993) 411.

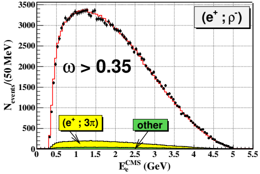
Helicity sensitive variable ω is introduced as:

$$\omega = \frac{1}{\Phi_2 - \Phi_1} \int_{\Phi_1}^{\Phi_2} (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) d\Phi = < (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) >_{\Phi_\tau}$$









Spin-spin correlation manifests itself through momentum-momentum correlations of final lepton and pions.

Method: corrections, detector effects, background

Physical corrections:

- All $\mathcal{O}(\alpha^3)$ QED and electroweak higher order corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$ are included
- Radiative leptonic decays $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau\gamma$
- Radiative decay $\tau^- \to \pi^-\pi^0\nu_\tau\gamma$

Detector effects:

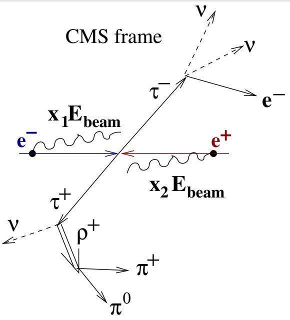
- Track momentum resolution
- γ energy and angular resolution
- Effect of external bremsstrahlung for $e-\rho$ events
- Beam energy spread
- EXP/MC efficiency corrections (trigger, track rec., π^0 rec., ℓID, πID)

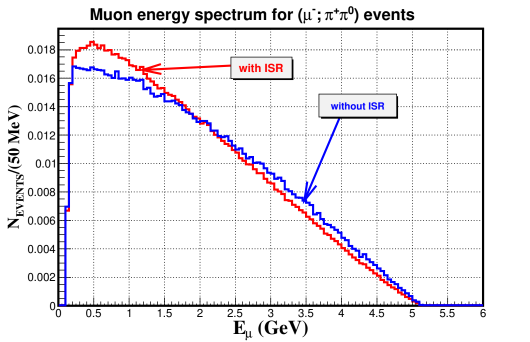
Background at Belle:

The main background comes from $(\ell\nu\nu;\ \pi2\pi^0\nu)(\sim10\%)$, $(\pi\nu;\ \pi\pi^0\nu)(\sim1.5\%)$ and $(\rho^+\nu;\ \rho^-\nu)(\sim0.5\%)$ events, it is included in PDF analytically. The remaining background($\sim$2.0%) is taken into account using MC-based approach.

Background from the non-$\tau\tau$ events is $\lesssim 0.1\%$.

Initial state radiation (ISR)



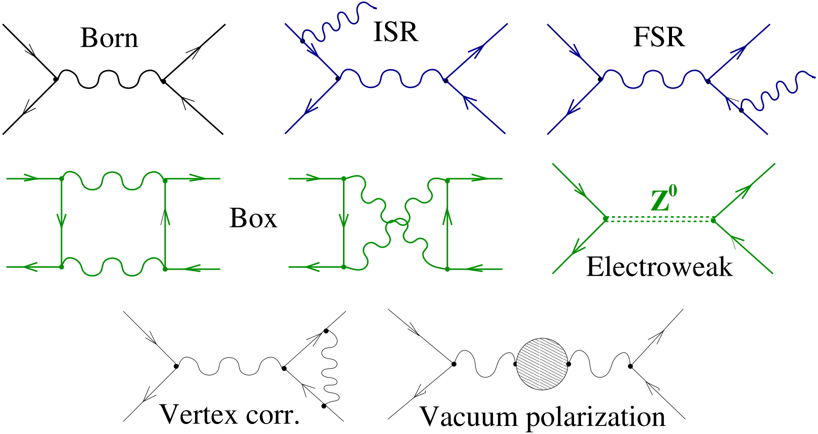


$$\frac{d\sigma_{\rm vis}(s)}{dp_\ell\, d\Omega_\ell\, dp_\rho\, d\Omega_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi} = \iint_0^1 dx_1\, dx_2\, D(x_1) D(x_2) \frac{d\sigma(s(1-x_1)(1-x_2))}{dp'_\ell\, d\Omega'_\ell\, dp'_\rho\, d\Omega'_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi} \left|\frac{\partial(p'_\ell, \Omega'_\ell)}{\partial(p_\ell, \Omega_\ell)}\right| \left|\frac{\partial(p'_\rho, \Omega'_\rho)}{\partial(p_\rho, \Omega_\rho)}\right|$$

- $D(x) = x^{\beta/2-1} h(x)$ - probability function for initial $e^{\mp}$ to emit a γ-quantum jet carrying $x_{1,2}$ part of $e^{\mp}$ energy $E_{\rm beam} = \sqrt{s}/2$. $\beta = \frac{2\alpha}{\pi}(\ln\frac{s}{m^2} - 1)$, $h(x)$ - smooth limited function.
- $\left|\frac{\partial(p'_i, \Omega'_i)}{\partial(p_i, \Omega_i)}\right|$ $(i = \ell,\ \rho)$ - Jacobian of transformation from the $\tau^+\tau^-$ rest frame to the Belle CMS.

At the Super Charm-Tau factory the impact of the ISR is expected to be essentially smaller.

$\mathcal{O}(\alpha^3)$ corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$



S. Jadach and Z. Was, Acta Phys. Polon. B **15** (1984) 1151 [Erratum-ibid. B **16** (1985) 483].
A. B. Arbuzov *et al* JHEP **9710** (1997) 001.

Charge-odd part of the cross section comes from the interference of the ISR and FSR diagrams as well as box and Born diagrams, and Z^0-exchange and Born diagrams.

Total PDF

$$\mathcal{P}(x) = \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\Big((1-\sum_i \lambda_i)\frac{S(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} S(x)dx} + \lambda_{3\pi}\frac{\tilde{B}_{3\pi}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} \tilde{B}_{3\pi}(x)dx} + \lambda_{\pi}\frac{\tilde{B}_{\pi}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} \tilde{B}_{\pi}(x)dx} + \lambda_{\rho}\frac{\tilde{B}_{\rho}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} \tilde{B}_{\rho}(x)dx} +$$

$$+(1-\sum_i \lambda_i)\frac{N^{\rm sel}_{\rm rest}(x)}{N^{\rm sel}_{\rm sig}(x)} S_{\rm SM}(x)\Big)$$

$$\tilde{B}_{3\pi}(x) = \int 2(1-\varepsilon_{\pi^0}(y))\varepsilon_{\rm add}(y) B_{3\pi}(x,y)dy,\ \tilde{B}_{\pi}(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)} B_\pi(x)$$

$$\tilde{B}_{\rho}(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)} \int (1-\varepsilon_{\pi^0}(y))\varepsilon_{\rm add}(y) B_\rho(x,y)dy,\ \overline{\varepsilon(x)} = \epsilon^{\rm EXP}_{\rm corr}(x)\varepsilon(x)$$

- $x = (p_\ell,\ \Omega_\ell,\ p_\rho,\ \Omega_\rho,\ m^2_{\pi\pi},\ \tilde{\Omega}_\pi);\ y = (p_{\pi^0},\ \Omega_{\pi^0})$;
- $S(x)$ - theoretical density of signal ($\ell^\mp\nu\nu,\ \rho^\pm\nu$) events;
- $B_{3\pi}(x,y)$ - theoretical density of background ($\ell^\mp\nu\nu,\ \pi^\pm 2\pi^0\nu$) events;
- $B_\pi(x)$ - theoretical density of background ($\pi^\mp\nu,\ \rho^\pm\nu$) events;
- $B_\rho(x)$ - theoretical density of background ($\rho^\mp\nu,\ \rho^\pm\nu$) events;
- $\varepsilon(x)$ - detection efficiency for signal events (**common multiplier**);
- $N^{\rm sel}_{\rm rest}(x)/N^{\rm sel}_{\rm sig}(x)$ - number of the selected (remaining/signal) MC events in the multidimensional cell around "x". Admixture of the remaining background is $(1 \div 2)\%$.
- λ_i - i-th background fraction (from MC)
- $\varepsilon_{\pi^0}(y)$ - π^0 detection efficiency (tabulated from MC);
- $\varepsilon_{\rm add}(y) = \varepsilon^{3\pi}_{\rm add}(y)/\varepsilon^{\rm sig}_{\rm add}$ - ratio of the $E^{\rm LAB}_{\gamma{\rm rest}}$ cut efficiencies (tabulated from MC);
- $\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)/\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)$ is tabulated from MC;
- $\epsilon^{\rm EXP}_{\rm corr}(x)$ - EXP/MC efficiency correction.

Michel par. at Belle, systematic uncertainties

Source	$\Delta(\rho)$, %	$\Delta(\eta)$, %	$\Delta(\xi_\rho \xi)$, %	$\Delta(\xi_\rho \xi \delta)$, %
Physical corrections				
ISR+$\mathcal{O}(\alpha^3)$	0.10	0.30	0.20	0.15
$\tau \to \ell\nu\nu\gamma$	0.03	0.10	0.09	0.08
$\tau \to \rho\nu\gamma$	0.06	0.16	0.11	0.02
Background	0.20	0.60	0.20	0.20
Apparatus corrections				
Resolution $\oplus$ brems.	0.10	0.33	0.11	0.19
$\sigma(E_{\rm beam})$	0.07	0.25	0.03	0.15
Normalization				
$\Delta\mathcal{N}$	0.11	0.50	0.17	0.13
without Data/MC corr.	**0.29**	**0.95**	**0.38**	**0.38**
trigger eff. corr.	**$\sim$ 1**	**$\sim$ 2**	**$\sim$ 3**	**$\sim$ 3**

At Belle, various EXP/MC (L1, L3/L4 software trigger, track and π^0 rec., πID and ℓID) efficiency corrections produce the systematic uncertainties in MP of about few percent.

$\tau^- \to \pi^-/\rho^-\nu_\tau$ decay to monitor $\mathcal{P}_e$

$$\frac{d\sigma(\vec{\zeta})}{d\Omega_\tau} = \frac{\alpha^2}{32E_\tau^2}\beta_\tau(D_0 + \mathcal{P}_e F_i \zeta_i)$$

$$\frac{d\Gamma(\tau^\mp \to \pi^\mp\nu)}{d\Omega_\pi^*} = \kappa_\pi(1 \pm \xi_\pi \vec{\zeta}\vec{n}_\pi^*), \quad \frac{d\Gamma(\tau^\mp \to \rho^\mp\nu)}{dm_{\pi\pi}^2 d\Omega_\rho^* \tilde{\Omega}_\pi} = f(\vec{k}_1, \vec{k}_2)(1 \pm \xi_\rho \vec{\zeta}\vec{H}_\rho^*)$$

$$\frac{d\sigma(\pi^\mp)}{d\Omega_\pi^* d\Omega_\tau} = \kappa_\pi \frac{\alpha^2\beta_\tau}{32E_\tau^2}(D_0 \pm \mathcal{P}_e\xi_\pi F_i n_{\pi i}^*)$$

$$\frac{d\sigma(\rho^\mp)}{d\Omega_\rho^* dm_{\pi\pi}^2 \tilde{\Omega}_\pi d\Omega_\tau} = f(\vec{k}_1, \vec{k}_2)\frac{\alpha^2\beta_\tau}{32E_\tau^2}(D_0 \pm \mathcal{P}_e\xi_\rho F_i H_{\rho i}^*)$$

$$\frac{d\sigma(\pi^\mp)}{dp_\pi d\Omega_\pi} = \int_0^{2\pi} \frac{d\sigma(\pi^\mp)}{d\Omega_\pi^* d\Omega_\tau}\left|\frac{\partial(\Omega_\pi^*, \Omega_\tau)}{\partial(p_\pi, \Omega_\pi, \Phi_\tau)}\right| d\Phi_\tau$$

$$\frac{d\sigma(\rho^\mp)}{dp_\rho d\Omega_\rho dm_{\pi\pi}^2 \tilde{\Omega}_\pi} = \int_0^{2\pi} \frac{d\sigma(\rho^\mp)}{d\Omega_\rho^* dm_{\pi\pi}^2 \tilde{\Omega}_\pi d\Omega_\tau}\left|\frac{\partial(\Omega_\rho^*, \Omega_\tau)}{\partial(p_\rho, \Omega_\rho, \Phi_\tau)}\right| d\Phi_\tau$$

Parameters ($\mathcal{P}_e\xi_\pi$, $\mathcal{P}_e\xi_\rho$) are measured in the unbinned maximum likelihood fit of the ($\tau^- \to \pi^-/\rho^-\nu_\tau$; $\tau^+ \to \text{all}$) events. These decays can be used to monitor $\mathcal{P}_e$ with high precision.