

Feasibility study of Michel parameters at the Super Charm-Tau factory with polarized e^- beam

D. Epifanov (BINP)

BINP, April 16th 2019





Introduction: Michel parameters in τ decays

In the SM, charged weak interaction is described by the exchange of W^{\pm} with a pure vector coupling to only left-handed fermions ("V-A" Lorentz structure). Deviations from "V-A" indicate New Physics. $\tau^- \rightarrow \ell^- \bar{\nu_\ell} \nu_\tau$ ($\ell = e, \mu$) decays provide clean laboratory to probe electroweak couplings.

The most general, Lorentz invariant four-lepton interaction matrix element:

$$\mathcal{M} = \frac{4G}{\sqrt{2}} \sum_{\substack{N=S,V,T\\i,i=L,R}} g_{ij}^{N} \Big[\bar{u}_i(I^-) \Gamma^N v_n(\bar{\nu}_I) \Big] \Big[\bar{u}_m(\nu_{\tau}) \Gamma_N u_j(\tau^-) \Big],$$

$$\Gamma^{S} = 1, \ \Gamma^{V} = \gamma^{\mu}, \ \Gamma^{T} = \frac{i}{2\sqrt{2}}(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu})$$

Ten couplings g_{ij}^N , in the SM the only non-zero constant is $g_{LL}^V = 1$ Four bilinear combinations of g_{ij}^N , which are called as Michel parameters (MP): ρ , η , ξ and δ appear in the energy spectrum of the outgoing lepton:

$$\frac{d\Gamma(\tau^{\mp})}{d\Omega dx} = \frac{4G_F^2 M_\tau E_{\max}^4}{(2\pi)^4} \sqrt{x^2 - x_0^2} \left(x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x) \right.$$

$$\left. \pm \frac{1}{3} P_\tau \cos\theta_\ell \xi \sqrt{x^2 - x_0^2} \left[1 - x + \frac{2}{3}\delta(4x - 4 + \sqrt{1 - x_0^2}) \right] \right), \ x = \frac{E_\ell}{E_{\max}}, \ x_0 = \frac{m_\ell}{E_{\max}}$$

In the SM: $\rho = \frac{3}{4}, \ \eta = 0, \ \xi = 1, \ \delta = \frac{3}{4}$

Introduction: Current status

without EXP/MC corr.

Michel par.	Measured value	Experiment	SM value		ALEPH	0.752+/-0.019	ALEPH	6-/-0.079
D	$0.747 \pm 0.010 \pm 0.006$	CLEO-97	3/4		DELPH	1 0.790+++0.038	DELPHI -	i+/@.11
r (1 2%				13	0.712+/-0.025	L3 021	¥/4.14
(e or μ)			0		SLD -	0.781+/-0.033	GPAL	7+1-0.055
'1	$0.012 \pm 0.020 \pm 0.004$	ALEFH-01	0		CLEO	0.72-0.00	CLEO	5+1-0.087
(e or µ)	2.0%				ARGUS	s 0.721+/-0.021	ARGUS	+/0.22
ε	$1.007 \pm 0.040 \pm 0.015$	CLEO-97	1		ρ	0.750 0.011	η 0.048	5
5 (0.07)	4 3%							
	4.070			•	ALEPH	1.000+/-0.076	ALEPH + 671	2+/-8.851
ξ0	$0.745 \pm 0.026 \pm 0.009$	CLEO-97	3/4		13	0.976+/-0.061	13	6+1-8.828
(e or µ)	2.8%				OPAL	0.99+10.24		He/4.11
<u> </u>					SLD	1.05+/-0.35	SLD	+/-0.37
ζh	$0.992 \pm 0.007 \pm 0.008$	ALEPH-01	1		CLEO	1.010+/-0.043	CLEO 674	5+10.020
(all hadr.)	1.1%				ARGOS	1.02+/-0.11	ARG05 841	+/-0.09
	-	-			ξ	0.988+-0.029	ξδ 0.735-0.02	0
	Current systematic uncertainties at Belle (study is going on)							
					•			
	Source	Δ	$(\rho), \%$	$\Delta(\eta), \%$	$\dot{\Delta}(\xi_{\rho}\xi),$	$\% \Delta(\xi_{\rho}\xi)$	δ), %	
	Source	Δ Phy	(ρ), % /sical c	$\Delta(\eta), \%$	$\Delta(\xi_{\rho}\xi),$	$ \Delta(\xi_{\rho}\xi) $	δ), %	
_	Source ISR+ $\mathcal{O}(\alpha^3)$	Δ Phy	(ρ), % /sical c 0.10	$\Delta(\eta), \%$ prrections 0.30	$\Delta(\xi_{\rho}\xi),$ 0.20	$\%^{-} \Delta(\bar{\xi}_{\rho}\xi$	δ), %	
_	Source ISR+ $\mathcal{O}(\alpha^3)$ $\tau \to \ell \nu \nu \gamma$	Δ) Phy ((ρ), % /sical c 0.10 0.03	$\Delta(\eta), \%$ prrections 0.30 0.10	$\Delta(\xi_{\rho}\xi),$ 0.20 0.09	<u>% Δ(ξ_ρξ</u> 0.1 0.0	δ), %	
	Source ISR+ $\mathcal{O}(\alpha^3)$ $\tau \rightarrow \ell \nu \nu \gamma$ $\tau \rightarrow \rho \nu \gamma$	Δ(Phy (((<i>ρ</i>), % /sical co 0.10 0.03 0.06	$\Delta(\eta), \%$ prrections 0.30 0.10 0.16	$\Delta(\xi_{\rho}\xi),$ 0.20 0.09 0.11	${0.1} {0.1}$	δ), % 5 8 2	
_	Source ISR+ $\mathcal{O}(\alpha^{3})$ $\tau \rightarrow \ell \nu \nu \gamma$ $\tau \rightarrow \rho \nu \gamma$ Background	Δ) Phy ((((<i>ρ</i>), % /sical co 0.10 0.03 0.06	$\Delta(\eta), \%$ prrections 0.30 0.10 0.16 0.60	$\Delta(\xi_{\rho}\xi),$ 0.20 0.09 0.11 0.20	<u>% Δ(ξ_ρξ</u> 0.1 0.0 0.0	δ), % 15 08 02	
_	Source ISR+ $\mathcal{O}(\alpha^{3})$ $\tau \rightarrow \ell \nu \nu \gamma$ $\tau \rightarrow \rho \nu \gamma$ Background	Δ) Phy (((((<i>ρ</i>), % /sical c 0.10 0.03 0.06 0.20	$\Delta(\eta), \%$ prrections 0.30 0.10 0.16 0.60	$\Delta(\xi_{\rho}\xi),$ 0.20 0.09 0.11 0.20	${0.1}$ 0.1 0.0 0.0 0.2	δ), % 15 08 02 20	
_	Source ISR+ $\mathcal{O}(\alpha^{3})$ $\tau \rightarrow \ell \nu \nu \gamma$ $\tau \rightarrow \rho \nu \gamma$ Background	Δ Phy ((((App;	(<i>ρ</i>), % /sical α 0.10 0.03 0.06 0.20 aratus α	$\Delta(\eta), \%$ princetions 0.30 0.10 0.16 0.60 corrections		${0.1} \frac$	δ), % 5 8 2 20	
	Source ISR+ $\mathcal{O}(\alpha^{3})$ $\tau \rightarrow \ell \nu \nu \gamma$ $\tau \rightarrow \rho \nu \gamma$ Background Resolution \oplus brem	Δ Phy ((((Appa s. ((<i>ρ</i>), % /sical α 0.10 0.03 0.06 0.20 aratus α 0.10	$\Delta(\eta), \%$ prrections 0.30 0.10 0.16 0.60 corrections 0.33	$\Delta(\xi_{\rho}\xi),$ 0.20 0.09 0.11 0.20 5 0.11	${0.1}$ 0.1 0.0 0.0 0.2 0.1	δ), % 5 08 02 20 9	
	$\begin{array}{c} \text{Source} \\ \\ \text{ISR+}\mathcal{O}(\alpha^{3}) \\ \tau \rightarrow \ell \nu \nu \gamma \\ \tau \rightarrow \rho \nu \gamma \\ \text{Background} \\ \\ \text{Resolution} \oplus \text{brem} \\ \sigma(E_{\text{beam}}) \end{array}$	Δ Phy ((((((Apps s. ()	$(\rho), \%$ /sical co 0.10 0.03 0.06 0.20 aratus co 0.10 0.07	$\Delta(\eta), \%$ prrections 0.30 0.10 0.16 0.60 corrections 0.33 0.25	$ \frac{\Delta(\xi_{\rho}\xi),}{0.20} \\ 0.09 \\ 0.11 \\ 0.20 \\ 0.11 \\ 0.03 $	% Δ(ξ _ρ ξ 0.1 0.0 0.2 0.1 0.1 0.1 0.2 0.1 0.1 0.1 0.1 0.1 0.2 0.1	8), % 5 08 02 20 9 5	
	$\begin{array}{c} \text{Source} \\ \\ \text{ISR+}\mathcal{O}(\alpha^3) \\ \tau \to \ell \nu \nu \gamma \\ \pi \to \rho \nu \gamma \\ \text{Background} \\ \\ \text{Resolution} \oplus \text{brem} \\ \sigma(E_{\text{beam}}) \end{array}$	Δ Phy (((((Apps s. (((ρ), % /sical co 0.10 0.03 0.06 0.20 aratus c 0.10 0.07 Normal	$\Delta(\eta), \%$ prrections 0.30 0.10 0.16 0.60 corrections 0.33 0.25 ization	$ \frac{\Delta(\xi_{\rho}\xi),}{0.20} \\ 0.09 \\ 0.11 \\ 0.20 \\ 0.11 \\ 0.03 $	$ \frac{\% \Delta(\bar{\xi}_{\rho}\xi)}{0.1} = 0.1 $ 0.1 0.2 0.2 0.1 0.1 0.1	8), % 5 08 02 20 9 5	

At Belle we are working on the various EXP/MC efficiency corrections which produce the systematic uncertainties in MP of about few percent.

1.0

0.4

0.4

0.3

Introduction: e^+e^- Super Factories

Belle II with unpolarized beams

Planned integrated luminosity is 50 ab⁻¹

$$\begin{aligned} \sigma(b\bar{b}) &= 1.05 \text{ nb} \quad N_{b\bar{b}} = 53 \times 10^9 \\ \sigma(c\bar{c}) &= 1.30 \text{ nb} \quad N_{c\bar{c}} = 65 \times 10^9 \\ \sigma(\tau\tau) &= 0.92 \text{ nb} \quad N_{\tau\tau} = 46 \times 10^9 \end{aligned}$$



Super Charm-Tau factory with polarized e⁻ beam

In five c.m.s. energy points (2E = 3.554, 3.686, 3.770, 4.170, 4.650 GeV) it is planned to accumulate 7 ab⁻¹, which corresponds to $N_{\tau\tau} = 21 \times 10^9$



The polarized e^- beam results in the nonzero average polarization of single tau, which provide advantages in the measurement of ξ and δ Michel parameters.

Method: e^+e^- factory with unpolarized beams

Effect of τ spin-spin correlation is used to measure ξ and δ MP. Events of the $(\tau^{\mp} \rightarrow \ell^{\mp}\nu\nu; \tau^{\pm} \rightarrow \rho^{\pm}\nu)$ topology are used to measure: ρ , η , $\xi_{\rho}\xi$ and $\xi_{\rho}\xi\delta$, while $(\tau^{\mp} \rightarrow \rho^{\mp}\nu; \tau^{\pm} \rightarrow \rho^{\pm}\nu)$ events are used to extract ξ_{ρ}^{2} .



$$\begin{aligned} \frac{d\sigma(\ell^{\mp}\nu\nu,\rho^{\pm}\nu)}{dE_{\ell}^{*}d\Omega_{\rho}^{*}d\Omega_{\rho}^{*}dm_{\pi\pi}^{2}d\tilde{\Omega}_{\pi}d\Omega_{\tau}} &= A_{0} + \rho A_{1} + \eta A_{2} + \xi_{\rho}\xi A_{3} + \xi_{\rho}\xi\delta A_{4} = \sum_{i=0}^{4} A_{i}\Theta_{i} \\ \mathcal{F}(\vec{z}) &= \frac{d\sigma(\ell^{\mp}\nu\nu,\rho^{\pm}\nu)}{dp_{\ell}d\Omega_{\ell}dp_{\rho}d\Omega_{\rho}dm_{\pi\pi}^{2}d\tilde{\Omega}_{\pi}} = \int_{\Phi_{1}}^{\Phi_{2}} \frac{d\sigma(\ell^{\mp}\nu\nu,\rho^{\pm}\nu)}{dE_{\ell}^{*}d\Omega_{\rho}^{*}dm_{\pi\pi}^{2}d\tilde{\Omega}_{\pi}d\Omega_{\tau}} \Big| \frac{\partial(E_{\ell}^{*},\Omega_{\ell}^{*},\Omega_{\rho}^{*},\Omega_{\tau})}{\partial(\rho_{\ell},\Omega_{\ell},\rho,\rho,\rho,\rho,\phi_{\tau})} \Big| d\Phi_{\tau} \\ \mathcal{L} &= \prod_{k=1}^{N} \mathcal{P}^{(k)}, \ \mathcal{P}^{(k)} = \mathcal{F}(\vec{z}^{(k)})/\mathcal{N}(\vec{\Theta}), \ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z}, \ \vec{\Theta} &= (1,\rho,\eta,\xi_{\rho}\xi_{\ell},\xi_{\rho}\xi_{\ell}\delta_{\ell}) \\ \mathcal{P}_{\text{total}} &= (1 - \sum_{i=1}^{4}\lambda_{i})\mathcal{P}_{\text{signal}}^{\ell-\rho} + \lambda_{1}\mathcal{P}_{\text{bg}}^{\ell-3\pi} + \lambda_{2}\mathcal{P}_{\text{bg}}^{\pi-\rho} + \lambda_{3}\mathcal{P}_{\text{bg}}^{\rho-\rho} + \lambda_{4}\mathcal{P}_{\text{bg}}^{\text{other}} (\text{MC}) \end{aligned}$$

MP are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu; \rho\nu)$ events in the 9D phase space $\vec{z} = (p_{\ell}, \cos\theta_{\ell}, \phi_{\ell}, p_{\rho}, \cos\theta_{\rho}, \phi_{\rho}, m_{\pi\pi}^2, \cos\tilde{\theta}_{\pi}, \tilde{\phi}_{\pi})$ in CMS.

Method: theoretical framework

W. Fetscher, Phys. Rev. D 42 (1990) 1544. K. Tamai, Nucl. Phys. B 668 (2003) 385.

$$\begin{aligned} \frac{d\sigma(\vec{\zeta},\vec{\zeta}')}{d\Omega} &= \frac{\alpha^2}{64E_{\tau}^2} \beta_{\tau} (D_0 + D_{ij}\zeta_i\zeta_j') \\ \frac{d\Gamma(\tau^{\mp}(\vec{\zeta}^*) \to \ell^{\mp}\nu\nu)}{dx^* d\Omega_{\ell}^*} &= \kappa_{\ell} (A(x^*) \mp \xi \vec{n}_{\ell}^* \vec{\zeta}^* B(x^*)), \ x^* = E_{\ell}^* / E_{\ell max}^* \\ A(x^*) &= A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*), \ B(x^*) &= B_1(x^*) + \delta B_2(x^*) \\ \frac{d\Gamma(\tau^{\pm}(\vec{\zeta}^*) \to \rho^{\pm}\nu)}{dm_{\pi\pi}^2 d\Omega_{\rho}^* d\Omega_{\pi}} &= \kappa_{\rho} (A' \mp \xi_{\rho} \vec{B'} \vec{\zeta'}^*) W(m_{\pi\pi}^2) = \kappa_{\rho} A' (1 \mp \xi_{\rho} \vec{H}_{\rho} \vec{\zeta'}^*) W(m_{\pi\pi}^2) \\ \vec{H}_{\rho} &= \frac{\vec{B'}}{A'} - \text{polarimeter vector}, \ \xi_{\rho} &= -\frac{2R\Theta(c_V^* c_A)}{|c_V|^2 + |c_A|^2} = -h_{\nu_{\tau}} (h_{\nu_{\tau}} = -1 \text{ in the SM}) \\ A' &= 2(q, Q)Q_0^* - Q^2 q_0^*, \ \vec{B'} &= Q^2 \vec{K}^* + 2(q, Q)\vec{Q}^*, \ W &= |F_{\pi}(m_{\pi\pi}^2)|^2 \frac{p_{\rho}(m_{\pi\pi}^2)\tilde{p}_{\pi}(m_{\pi\pi}^2)}{M_{\tau}m_{\pi\pi}} \\ \frac{d\sigma(\ell^{\mp}, \rho^{\pm})}{dE_{\ell}^* d\Omega_{\ell}^* d\Omega_{\rho}^* dm_{\pi\pi}^2 d\tilde{\Omega}_{\pi} d\Omega_{\tau}} &= \kappa_{\ell} \kappa_{\rho} \frac{\alpha^2 \beta_{\tau}}{64E_{\tau}^2} (D_0 A' A(E_{\ell}^*) + \xi_{\rho} \xi_{\ell} D_{ij} n_{\ell}^* |\vec{B}| E_{\ell}^*)) W(m_{\pi\pi}^2) \\ \frac{d\sigma(\ell^{\mp}, \rho^{\pm})}{d\rho_{\ell} d\Omega_{\ell} d\rho_{\rho} dm_{\pi\pi}^2 d\tilde{\Omega}_{\pi}} &= \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^{\mp}, \rho^{\pm})}{dE_{\ell}^* d\Omega_{\ell}^* d\Omega_{\mu}^* d\tilde{\Omega}_{\pi}^* d\tilde{\Omega}_{\pi}} d\tilde{\Omega}_{\pi} d\Omega_{\tau}} \left| \frac{\partial(E_{\ell}^*, \Omega_{\ell}^*, \Omega_{\rho}^*, \Omega_{\tau})}{\partial(\rho_{\ell}, \Omega_{\ell}, \rho_{\rho}, \Omega_{\rho}, \Phi_{\tau})} \right| d\Phi_{\tau} \end{aligned}$$

Effect of the e⁻ beam polarization

At the Super Charm-Tau factory with polarized electron beam the average polarization of single τ is nonzero, hence the differential decay probability will contain both, τ spin-dependent and spin-independent parts.

$$\begin{aligned} \frac{d\sigma(\vec{\zeta}^{-},\vec{\zeta}^{+})}{d\Omega_{\tau}} &= \frac{\alpha^{2}}{64E_{\tau}^{2}}\beta_{\tau}(D_{0}+D_{ij}\zeta_{i}^{-}\zeta_{j}^{+}+\mathcal{P}_{e}(F_{i}^{-}\zeta_{i}^{-}+F_{j}^{+}\zeta_{j}^{+}))\\ D_{0} &= 1+\cos^{2}\theta+\frac{1}{\gamma_{\tau}^{2}}\sin^{2}\theta, \ \mathcal{P}_{e} &= \frac{N_{e}(+)-N_{e}(-)}{N_{e}(+)+N_{e}(-)}\\ D_{ij} &= \begin{pmatrix} (1+\frac{1}{\gamma_{\tau}^{2}})\sin^{2}\theta & 0 & \frac{1}{\gamma_{\tau}}\sin2\theta\\ 0 & -\beta_{\tau}^{2}\sin^{2}\theta & 0\\ \frac{1}{\gamma_{\tau}}\sin2\theta & 0 & 1+\cos^{2}\theta-\frac{1}{\gamma_{\tau}^{2}}\sin^{2}\theta \end{pmatrix} \end{aligned}$$

Single τ studies at the Super Charm-Tau factory:

$$\frac{d\sigma(\vec{\zeta}^{-})}{d\Omega_{\tau}} = \frac{\alpha^2}{32E_{\tau}^2}\beta_{\tau}(D_0 + \mathcal{P}_{\mathsf{e}}\mathsf{F}_i^{-}\zeta_i^{-})$$

As a result, there are two methods to measure MP:

- (I) Unbinned fit of the (ℓ, ρ) events in 9D phase space (spin-spin correlations + polarized e^- beam)
- (II) Unbinned fit of the $(\ell, \text{ all})$ events in 3D lepton phase space (only polarized e^- beam)

(ℓ, ρ) in 9D

$$\begin{split} \frac{d\sigma(\vec{\zeta},\vec{\zeta}')}{d\Omega} &= \frac{\alpha^2}{64E_{\tau}^2} \beta_{\tau} (D_0 + D_{ij}\zeta_i \zeta_j' + \mathcal{P}_{\theta}(F_i^- \zeta_i^- + F_j^+ \zeta_j^+)) \\ \frac{d\Gamma(\tau^-(\vec{\zeta}^*) \to \ell^- \nu \nu)}{dx^* d\Omega_{\ell}^*} &= \kappa_{\ell} (A(x^*) - \xi \vec{n}_{\ell}^* \vec{\zeta}^* B(x^*)), \ x^* = E_{\ell}^* / E_{\ell max}^* \\ A(x^*) &= A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*), \ B(x^*) = B_1(x^*) + \delta B_2(x^*) \\ \frac{d\Gamma(\tau^+(\vec{\zeta}^{**}) \to \rho^+ \nu)}{dm_{\pi\pi}^2 d\Omega_{\rho}^* d\Omega_{\pi}} &= \kappa_{\rho} (A' - \xi_{\rho} \vec{B'} \vec{\zeta}^{**}) W(m_{\pi\pi}^2) = \kappa_{\rho} A' (1 - \xi_{\rho} \vec{H}_{\rho} \vec{\zeta}^{**}) W(m_{\pi\pi}^2) \\ \vec{H}_{\rho} &= \frac{\vec{B'}}{A'} - \text{polarimeter vector}, \ \xi_{\rho} &= -\frac{2R\theta(c_v^* c_A)}{|c_v|^2 + |c_A|^2} = -h_{\nu_{\tau}} \ (h_{\nu_{\tau}} = -1 \text{ in the SM}) \\ A' &= 2(q, Q)Q_0^* - Q^2 q_0^*, \ \vec{B'} &= Q^2 \vec{K}^* + 2(q, Q)\vec{Q}^*, \ W &= |F_{\pi}(m_{\pi\pi}^2)|^2 \frac{p_{\rho}(m_{\pi\pi}^2) \tilde{p}_{\pi}(m_{\pi\pi}^2)}{M_{\tau} m_{\pi\pi}} \\ \frac{d\sigma(\ell^{\mp}, \rho^{\pm})}{dE_{\ell}^* d\Omega_{\ell}^* d\Omega_{\rho}^* dm_{\pi\pi}^2 d\tilde{\Omega}_{\pi} d\Omega_{\tau}} &= \kappa_{\ell} \kappa_{\rho} \frac{\alpha^2 \beta_{\tau}}{64E_{\tau}^2} (D_0 A' A(E_{\ell}^*) + \xi_{\rho} \xi_{\ell} D_{ij} n_{\ell i}^* B'_j B(E_{\ell}^*) - -P_{\theta}(\xi A' B(x^*) F_i^- n_{\ell i}^* + \xi_{\rho} A(x^*) F_j^+ B'_j)) W(m_{\pi\pi}^2), \\ \frac{d\sigma(\ell^{\mp}, \rho^{\pm})}{d\rho_{\ell} d\Omega_{\ell} d\Omega_{\rho} dm_{\pi\pi}^2 d\tilde{\Omega}_{\pi}} d\tilde{\Omega}_{\pi}} &= \int_{\Phi_{\Phi}}^{\Phi_2} \frac{d\sigma(\ell^{\mp}, \rho^{\pm})}{dE_{\ell}^* d\Omega_{\ell}^* d\Omega_{\pi}^* d\tilde{\Omega}_{\pi}} d\tilde{\Omega}_{\pi}} &= \int_{\Phi_{\Phi}}^{\Phi_2} \frac{d\sigma(\ell^{\mp}, \rho^{\pm})}{dE_{\ell}^* d\Omega_{\mu}^* d\Omega_{\pi\pi}^* d\tilde{\Omega}_{\pi}} d\tilde{\Omega}_{\pi}} \left| \frac{\partial(E_{\ell}^*, \Omega_{\ell}^*, \Omega_{\rho}^*, \Omega_{\tau})}{\partial(\rho(\ell, \Omega_{\ell}, \rho_{\rho}, \Omega_{\rho}, \rho_{\tau}, \Phi_{\tau})} \right| d\Phi_{\tau} \end{split}$$

$(\ell, \text{ all})$ in 3D

$$\begin{aligned} \frac{d\sigma(\vec{\zeta})}{d\Omega_{\tau}} &= \frac{\alpha^2}{32E_{\tau}^2} \beta_{\tau} (D_0 + \mathcal{P}_{\theta} F_i \zeta_i) \\ \frac{d\Gamma(\tau^{\mp}(\vec{\zeta}^*) \to \ell^{\mp} \nu \nu)}{dx^* d\Omega_{\ell}^*} &= \kappa_{\ell} (A(x^*) \mp \xi_{\ell} \vec{n}_{\ell}^* \vec{\zeta}^* B(x^*)), \ x^* = E_{\ell}^* / E_{\ell max}^* \\ A(x^*) &= A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*), \ B(x^*) &= B_1(x^*) + \delta B_2(x^*) \\ \frac{d\sigma(\ell^{\mp})}{dE_{\ell}^* d\Omega_{\ell}^* d\Omega_{\tau}} &= \kappa_{\ell} \frac{\alpha^2 \beta_{\tau}}{32E_{\tau}^2} (D_0 A(E_{\ell}^*) \mp \mathcal{P}_{\theta} \xi_{\ell} F_i n_{\ell i}^* B(E_{\ell}^*)) \\ \frac{d\sigma(\ell^{\mp})}{d\rho_{\ell} d\Omega_{\ell}} &= \int_{\Omega_{\tau} - \text{sector}} \frac{d\sigma(\ell^{\mp})}{dE_{\ell}^* d\Omega_{\ell}^* d\Omega_{\tau}} \left| \frac{\partial(E_{\ell}^*, \Omega_{\ell}^*)}{\partial(\rho_{\ell}, \Omega_{\ell})} \right| d\Omega_{\tau} \end{aligned}$$

 $\Omega_{ au}$ -sector is determined by the kinematical constraint $m_{
u
u} > 0$

- All Michel parameters (ρ, η, P_eξ, P_eξδ) are measured in the unbinned maximum likelihood fit of (τ⁻ → ℓ⁻ν_ℓν_τ; τ⁺ → all) events in the **3D** phase space.
- The reduced 3D phase space allows one to tabulate various EXP/MC corrections to the detection efficiency more precisely.
- The crucial point in this method is to have high-efficiency 1-track trigger.

Toy MC studies of the effect of polarized e^- beam

- The generator of the (ℓ, ρ) events has been developed, effects of spin-spin correlation of taus and e⁻ beam polarization are taken into account.
- Effects of ISR and FSR are not simulated. The development of the full fitter at Belle showed that radiative corrections can be taken into account properly in the fitter and they don't decrease the statistical sensitivity to MP.
- The direction of tau was taken from the generator. Studies of the sensitivity to MP at Belle showed that the integration over the allowed tau directions results in the sensitivity degradation factor of 1.4 only, this factor was additionally applied to our results.
- CLEO model (ρ, ρ') for F_π(m²_{ππ}) (used in the current version of TAUOLA) was utilized in our generator.
- 66 10M (μ, ρ) samples, at 6 center-of-mass (c.m.s.) energies (according to Table 1.1 in Super Charm-Tau factory CDR part I) : 2E = 3.554 GeV (τ⁺τ⁻ production threshold), 2E = 3.686 GeV (ψ(2S)), 2E = 3.770 GeV (ψ(3770)), 2E = 4.170 GeV (ψ(4160)), 2E = 4.650 GeV (maximum of the σ(e⁺e⁻ → Λ_c⁺Λ_c⁻)), 2E = 10.58 GeV (Belle II), for 11 values of e⁻ beam polarization: 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, were generated for the calculation of the normalizations. 66 statistically independent 1M samples at the same energies and polarizations were generated for the fit.
- To evaluate MP sensitivities (rescaling the sensitivities obtained in the fits of 1M samples) we took the detection efficiency of (μ, ρ) events to be 20% (to be compared with 12% efficiency obtained at Belle, where the π^0 rec. efficiency is only 40%). The detection efficiency of $(\mu, \text{ all})$ events was taken to be 30%.
- To measure ρ, ξ and ξδ MP, samples with ℓ = e, μ were taken into account, while η MP is measured in samples with ℓ = μ only.

10/17

Fit of (ℓ, ρ) in 9D at Belle II



Visible spread of ρ and η parameters around the trend is associated with the finite accuracy of the normalization.

Fit of 1M (ℓ, ρ) samples in 9D



 e^- beam polarizations than that at $E_{\text{beam}} = 5.29 \text{ GeV}$.

At the e^- beam polarization of 100% the sensitivities become equal.

Fit of (ℓ, ρ) in 9D at Belle II/Super C-Tau



Fit of (ℓ, all) in 3D at Belle II/Super C-Tau



The sensitivity to the ξ and $\xi\delta$ parameters at the Super Charm-Tau factory becomes better

than that at Belle II (with unpolarized e^- beam) for the e^- beam polarizations larger than 0.5.

ξ from the fit of $(\ell, \text{ all})$ in 3D at Super C-Tau



For the increases of the e⁻ beam polarizations, $0.5 \rightarrow 0.6$, $0.6 \rightarrow 0.7$, $0.7 \rightarrow 0.8$, $0.8 \rightarrow 0.9$, the corresponding improvements in the sensitivities to the ξ parameter, 17%, 16%, 14%, 12%, respectively. If we move from the polarization of 0.5 to the higher polarizations: $0.5 \rightarrow 0.6$, $0.5 \rightarrow 0.8$, $0.5 \rightarrow 0.9$, the acceptable luminosity decrease factors (to keep the sensitivity at the level of that we have for polarization 0.5) are: $(1.86/2.23)^2 = 0.70$, $(1.60/2.23)^2 = 0.51$, $(1.40/2.23)^2 = 0.39$, $(1.25/2.23)^2 = 0.31$, respectively.

$\xi\delta$ from the fit of $(\ell, \text{ all})$ in 3D at Super C-Tau



E polarization For the increases of the e⁻ beam polarizations, $0.5 \rightarrow 0.6$, $0.6 \rightarrow 0.7$, $0.7 \rightarrow 0.8$, $0.8 \rightarrow 0.9$, the corresponding improvements in the sensitivities to the $\xi\delta$ parameter, 16%, 14%, 12%, 10%, respectively. If we move from the polarization of 0.5 to the higher polarizations: $0.5 \rightarrow 0.6$, $0.5 \rightarrow 0.7$, $0.5 \rightarrow 0.8$, $0.5 \rightarrow 0.9$, the acceptable luminosity decrease factors (to keep the sensitivity at the level of that we have for polarization 0.5) are: $(1.06/1.26)^2 = 0.71$, $(0.91/1.26)^2 = 0.52$, $(0.80/1.26)^2 = 0.40$, $(0.72/1.26)^2 = 0.33$, respectively.

Summary

Feasibility study of Michel parameters at the Super Charm-Tau factory and Belle II with polarized e⁻ beam has been caried out. This simple generator level study allows us to estimate the statistical sensitivities to MP as a function of e⁻ beam polarization.

• Two methods were studied, (I) 9D fit of the (ℓ, ρ) events, (II) 3D fit of the (ℓ, all) events.

- In the method (I), the sensitivities to ρ and η parameters for the expected Belle II (with unpolarized e⁻ beam) and Super Charm-Tau factory statistics differ by only a factor of 1.5, Belle II has the best sensitivities. The sensitivities to the ξ and ξδ MP differ by only 25% (with unpolarized e⁻ beam for Belle II and e⁻ beam polarization of 0.8 for Super Charm-Tau factory), with Belle II best sensitivities.
- In the method (II), the sensitivities to ρ and η parameters for the expected Belle II (with unpolarized e^- beam) and Super Charm-Tau factory statistics differ by only a factor of 1.5, Super Charm-Tau factory has the best sensitivities. The sensitivities to the ξ and $\xi\delta$ MP become equal with unpolarized e^- beam for Belle II and e^- beam polarization of 0.5 for Super Charm-Tau factory. For the higher e^- beam polarization the sensitivities to ξ and $\xi\delta$ MP improve as $1/P_e$, and Super Charm-Tau factory wins Belle II. For the high e^- beam polarizations there is some notable room to decrease luminosity while keeping priority in the sensitivities to ξ and $\xi\delta$ MP at Super Charm-Tau factory. The reduced 3D phase space in method (II) allows one to tabulate various EXP/MC corrections to the detection efficiency more precisely.
- Even with polarized e⁻ beam also at Belle II the difference of the MP sensitivities are about 1.5 for *ρ* and *η*, and about 1.3 for *ξ* and *ξδ*, which make Belle II and Super Charm-Tau factory, both with polarized e⁻ beam, equally strong competitors. The main struggle will be to decrease the systematic uncertainty AMAP.
- It is seen that the expected MP statistical uncertainties are of the order of 10⁻⁴, to reach similar level systematic uncertainty, the NNLO corrections to the e⁺e⁻ → τ⁺τ⁻ cross section are mandatory. BINP theorists participate in this activity.
- I am grateful to Shota Nagumo(U-Tokyo) and KMI(U-Nagoya) for the help.