Michel parameters at Belle II and SCTF with polarized e^- beam

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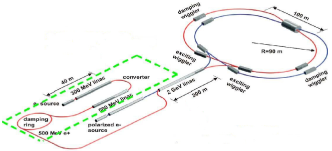
Seminar dedicated to the memory of Prof. E.A.Kuraev, 17-18 October 2023, JINR, Dubna

Outline:

1. Introduction
2. Method
3. Toy MC studies of the effect of polarized e^- beam
4. Summary



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Introduction: Michel parameters in τ decays

In the SM, charged weak interaction is described by the exchange of $W^{\pm}$ with a pure vector coupling to only left-handed fermions ("V-A" Lorentz structure). Deviations from "V-A" indicate New Physics. $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ ($\ell = e, \mu$) decays provide clean laboratory to probe electroweak couplings.
The most general, Lorentz invariant four-lepton interaction matrix element:

$$\mathcal{M} = \frac{4G}{\sqrt{2}} \sum_{\substack{N=S,V,T \\ i,j=L,R}} g^N_{ij} \left[\bar{u}_i(l^-)\Gamma^N v_n(\bar{\nu}_l)\right] \left[\bar{u}_m(\nu_\tau)\Gamma_N u_j(\tau^-)\right],$$

$$\Gamma^S = 1,\ \Gamma^V = \gamma^\mu,\ \Gamma^T = \frac{i}{2\sqrt{2}}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

Ten couplings g^N_{ij}, in the SM the only non-zero constant is $g^V_{LL} = 1$

Four bilinear combinations of g^N_{ij}, which are called as Michel parameters (MP): $\boldsymbol{\rho}$, $\boldsymbol{\eta}$, $\boldsymbol{\xi}$ and $\boldsymbol{\delta}$ appear in the energy spectrum of the outgoing lepton:

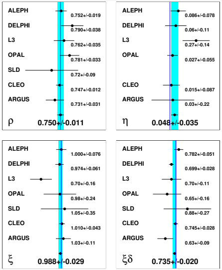
$$\frac{d\Gamma(\tau^\mp)}{d\Omega dx} = \frac{4G_F^2 M_\tau E^4_{\max}}{(2\pi)^4}\sqrt{x^2 - x_0^2}\Bigg(x(1-x) + \frac{2}{9}\boldsymbol{\rho}(4x^2 - 3x - x_0^2) + \boldsymbol{\eta} x_0(1-x)$$

$$\mp\frac{1}{3}P_\tau cos\theta_\ell \boldsymbol{\xi}\sqrt{x^2 - x_0^2}\left[1 - x + \frac{2}{3}\boldsymbol{\delta}\left(4x - 4 + \sqrt{1 - x_0^2}\right)\right]\Bigg),\ x = \frac{E_\ell}{E_{\max}},\ x_0 = \frac{m_\ell}{E_{\max}}$$

In the SM: $\boldsymbol{\rho} = \frac{3}{4}$, $\boldsymbol{\eta} = 0$, $\boldsymbol{\xi} = 1$, $\boldsymbol{\delta} = \frac{3}{4}$

Introduction: Current status

Michel par.	Measured value	Experiment	SM value
ρ (e or μ)	$0.747 \pm 0.010 \pm 0.006$ **1.2%**	CLEO-97	3/4
η (e or μ)	$0.012 \pm 0.026 \pm 0.004$ **2.6%**	ALEPH-01	0
ξ (e or μ)	$1.007 \pm 0.040 \pm 0.015$ **4.3%**	CLEO-97	1
$\xi\delta$ (e or μ)	$0.745 \pm 0.026 \pm 0.009$ **2.8%**	CLEO-97	3/4
ξ_h (all hadr.)	$0.992 \pm 0.007 \pm 0.008$ **1.1%**	ALEPH-01	1



Current systematic uncertainties at Belle (study is going on)

Source	$\Delta(\rho)$, %	$\Delta(\eta)$, %	$\Delta(\xi_\rho\xi)$, %	$\Delta(\xi_\rho\xi\delta)$, %
Physical corrections				
ISR+$\mathcal{O}(\alpha^3)$	0.10	0.30	0.20	0.15
$\tau \to \ell\nu\nu\gamma$	0.03	0.10	0.09	0.08
$\tau \to \rho\nu\gamma$	0.06	0.16	0.11	0.02
Background	0.20	0.60	0.20	0.20
Apparatus corrections				
Resolution $\oplus$ brems.	0.10	0.33	0.11	0.19
$\sigma(E_{\rm beam})$	0.07	0.25	0.03	0.15
Normalization				
$\Delta\mathcal{N}$	0.11	0.50	0.17	0.13
without EXP/MC corr.	**0.3**	**1.0**	**0.4**	**0.4**

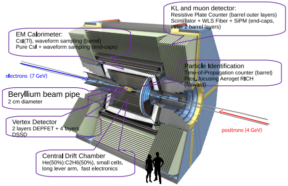
At Belle we are working on the various EXP/MC efficiency corrections which produce the systematic uncertainties in MP of about few percent.

Belle II with unpolarized beams

Planned integrated luminosity is 50 ab^{-1}

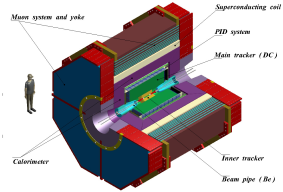
$$\sigma(b\bar{b}) = 1.05 \text{ nb} \quad N_{b\bar{b}} = 53 \times 10^9$$
$$\sigma(c\bar{c}) = 1.30 \text{ nb} \quad N_{c\bar{c}} = 65 \times 10^9$$
$$\boldsymbol{\sigma(\tau\tau) = 0.92 \text{ nb} \quad N_{\tau\tau} = 46 \times 10^9}$$

Belle II detector



Super Charm-Tau factory (SCTF) with polarized e^- beam

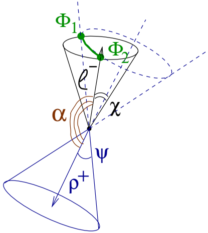
In five c.m.s. energy points
($2E = 3.554,\ 3.686,\ 3.770,\ 4.170,\ 4.650\,\text{GeV}$)
it is planned to accumulate 7 ab^{-1}, which
corresponds to $\boldsymbol{N_{\tau\tau} = 21 \times 10^9}$

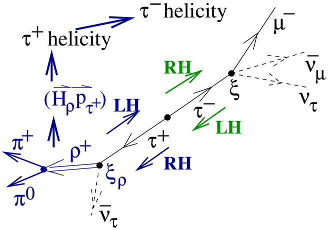


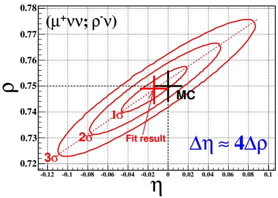
The polarized e^- beam results in the nonzero average polarization of single tau, which provide advantages in the measurement of ξ and δ Michel parameters.

Method: e^+e^- factory with unpolarized beams

Effect of τ spin-spin correlation is used to measure ξ and δ MP.
Events of the $(\tau^\mp \to \ell^\mp\nu\nu;\ \tau^\pm \to \rho^\pm\nu)$ topology are used to measure: ρ, η, $\xi_\rho\xi$ and $\xi_\rho\xi\delta$, while $(\tau^\mp \to \rho^\mp\nu;\ \tau^\pm \to \rho^\pm\nu)$ events are used to extract ξ_ρ^2.







$$\frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dE_\ell^*\,d\Omega_\ell^*\,d\Omega_\rho^*\,dm^2_{\pi\pi}\,d\tilde{\Omega}_\pi\,d\Omega_\tau} = A_0 + \rho A_1 + \eta A_2 + \xi_\rho\xi A_3 + \xi_\rho\xi\delta A_4 = \sum_{i=0}^{4} A_i\Theta_i$$

$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dp_\ell\,d\Omega_\ell\,dp_\rho\,d\Omega_\rho\,dm^2_{\pi\pi}\,d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dE_\ell^*\,d\Omega_\ell^*\,d\Omega_\rho^*\,dm^2_{\pi\pi}\,d\tilde{\Omega}_\pi\,d\Omega_\tau}\left|\frac{\partial(E_\ell^*,\Omega_\ell^*,\Omega_\rho^*,\Omega_\tau)}{\partial(p_\ell,\Omega_\ell,p_\rho,\Omega_\rho,\Phi_\tau)}\right|d\Phi_\tau$$

$$L = \prod_{k=1}^{N}\mathcal{P}^{(k)},\ \mathcal{P}^{(k)} = \mathcal{F}(\vec{z}^{(k)})/\mathcal{N}(\vec{\Theta}),\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z},\ \vec{\Theta} = (1,\rho,\eta,\xi_\rho\xi_\ell,\xi_\rho\xi_\ell\delta_\ell)$$

$$\boldsymbol{\mathcal{P}}_{\mathbf{total}} = (1-\sum_{i=1}^{4}\lambda_i)\boldsymbol{\mathcal{P}}^{\ell-\rho}_{signal} + \lambda_1\mathcal{P}^{\ell-3\pi}_{bg} + \lambda_2\mathcal{P}^{\pi-\rho}_{bg} + \lambda_3\mathcal{P}^{\rho-\rho}_{bg} + \lambda_4\boldsymbol{\mathcal{P}}^{other}_{bg}\ \text{(MC)}$$

MP are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu;\ \rho\nu)$ events in the 9D phase space $\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m^2_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$ in CMS.

Method: theoretical framework

W. Fetscher, Phys. Rev. D **42** (1990) 1544. K. Tamai, Nucl. Phys. B **668** (2003) 385.

$$\frac{d\sigma(\vec{\zeta},\vec{\zeta}')}{d\Omega} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i\zeta_j')$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*) \to \ell^\mp\nu\nu)}{dx^* d\Omega_\ell^*} = \kappa_\ell(A(x^*) \mp \xi\vec{n}_\ell^*\vec{\zeta}^* B(x^*)),\ x^* = E_\ell^*/E_{\ell max}^*$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*),\ B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\Gamma(\tau^\pm(\vec{\zeta'}^*) \to \rho^\pm\nu)}{dm_{\pi\pi}^2 d\Omega_\rho^* d\tilde{\Omega}_\pi} = \kappa_\rho(A' \mp \xi_\rho\vec{B'}\vec{\zeta'}^*)W(m_{\pi\pi}^2) = \kappa_\rho A'(1 \mp \xi_\rho\vec{H}_\rho\vec{\zeta'}^*)W(m_{\pi\pi}^2)$$

$$\vec{H}_\rho = \frac{\vec{B'}}{A'} - \text{polarimeter vector},\ \xi_\rho = -\frac{2Re(c_V^* c_A)}{|c_V|^2 + |c_A|^2} = -h_{\nu_\tau}\ (h_{\nu_\tau} = -1 \text{ in the SM})$$

$$A' = 2(q,Q)Q_0^* - Q^2 q_0^*,\ \vec{B'} = Q^2\vec{K}^* + 2(q,Q)\vec{Q}^*,\ W = |F_\pi(m_{\pi\pi}^2)|^2\frac{p_\rho(m_{\pi\pi}^2)\tilde{p}_\pi(m_{\pi\pi}^2)}{M_\tau m_{\pi\pi}}$$

$$\frac{d\sigma(\ell^\mp,\rho^\pm)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = \kappa_\ell\kappa_\rho\frac{\alpha^2\beta_\tau}{64E_\tau^2}\left(D_0 A' A(E_\ell^*) + \xi_\rho\xi_\ell D_{ij} n_{\ell i}^* B_j' B(E_\ell^*)\right) W(m_{\pi\pi}^2)$$

$$\frac{d\sigma(\ell^\mp,\rho^\pm)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2}\frac{d\sigma(\ell^\mp,\rho^\pm)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau}\left|\frac{\partial(E_\ell^*,\Omega_\ell^*,\Omega_\rho^*,\Omega_\tau)}{\partial(p_\ell,\Omega_\ell,p_\rho,\Omega_\rho,\Phi_\tau)}\right| d\Phi_\tau$$

Effect of the e^- beam polarization

At the Super Charm-Tau factory with polarized electron beam the average polarization of single τ is nonzero, hence the differental decay probability will contain both, τ spin-dependent and spin-independent parts.

$$\frac{d\sigma(\vec{\zeta}^-, \vec{\zeta}^+)}{d\Omega_\tau} = \frac{\alpha^2}{64 E_\tau^2}\beta_\tau (D_0 + D_{ij}\zeta_i^-\zeta_j^+ + \mathcal{P}_e(F_i^-\zeta_i^- + F_j^+\zeta_j^+))$$

$$D_0 = 1 + \cos^2\theta + \frac{1}{\gamma_\tau^2}\sin^2\theta, \; \mathcal{P}_e = \frac{N_e(+) - N_e(-)}{N_e(+) + N_e(-)}$$

$$D_{ij} = \begin{pmatrix} (1 + \frac{1}{\gamma_\tau^2})\sin^2\theta & 0 & \frac{1}{\gamma_\tau}\sin 2\theta \\ 0 & -\beta_\tau^2 \sin^2\theta & 0 \\ \frac{1}{\gamma_\tau}\sin 2\theta & 0 & 1 + \cos^2\theta - \frac{1}{\gamma_\tau^2}\sin^2\theta \end{pmatrix}$$

Single τ studies at the Super Charm-Tau factory:

$$\frac{d\sigma(\vec{\zeta}^-)}{d\Omega_\tau} = \frac{\alpha^2}{32 E_\tau^2}\beta_\tau (D_0 + \mathcal{P}_e F_i^- \zeta_i^-)$$

As a result, there are two methods to measure MP:

- **(I) Unbinned fit of the $(\ell,\ \rho)$ events in 9D phase space (spin-spin correlations + polarized e^- beam)**
- **(II) Unbinned fit of the $(\ell,\ \mathrm{all})$ events in 3D lepton phase space (only polarized e^- beam)**

$(\ell,\ \rho)$ in 9D

$$\frac{d\sigma(\vec{\zeta},\vec{\zeta}')}{d\Omega} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i\zeta_j' + \mathcal{P}_e(F_i^-\zeta_i^- + F_j^+\zeta_j^+))$$

$$\frac{d\Gamma(\tau^-(\vec{\zeta}^*) \to \ell^-\nu\nu)}{dx^* d\Omega_\ell^*} = \kappa_\ell(A(x^*) - \xi\vec{n}_\ell^*\vec{\zeta}^* B(x^*)),\ x^* = E_\ell^*/E_{\ell max}^*$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*),\ B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\Gamma(\tau^+(\vec{\zeta'}^*) \to \rho^+\nu)}{dm_{\pi\pi}^2 d\Omega_\rho^* d\tilde{\Omega}_\pi} = \kappa_\rho(A' - \xi_\rho\vec{B'}\vec{\zeta'}^*)W(m_{\pi\pi}^2) = \kappa_\rho A'(1 - \xi_\rho\vec{H}_\rho\vec{\zeta'}^*)W(m_{\pi\pi}^2)$$

$$\vec{H}_\rho = \frac{\vec{B'}}{A'} - \text{polarimeter vector},\ \xi_\rho = -\frac{2Re(c_V^*c_A)}{|c_V|^2 + |c_A|^2} = -h_{\nu_\tau}\ (h_{\nu_\tau} = -1 \text{ in the SM})$$

$$A' = 2(q,Q)Q_0^* - Q^2q_0^*,\ \vec{B'} = Q^2\vec{K}^* + 2(q,Q)\vec{Q}^*,\ W = |F_\pi(m_{\pi\pi}^2)|^2\frac{p_\rho(m_{\pi\pi}^2)\tilde{p}_\pi(m_{\pi\pi}^2)}{M_\tau m_{\pi\pi}}$$

$$\frac{d\sigma(\ell^\mp,\rho^\pm)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = \kappa_\ell\kappa_\rho\frac{\alpha^2\beta_\tau}{64E_\tau^2}(D_0A'A(E_\ell^*) + \xi_\rho\xi_\ell D_{ij}n_{\ell i}^*B_j'B(E_\ell^*) -$$

$$-\mathcal{P}_e(\xi A'B(x^*)F_i^-n_{\ell i}^* + \xi_\rho A(x^*)F_j^+B_j'))W(m_{\pi\pi}^2),$$

$$\frac{d\sigma(\ell^\mp,\rho^\pm)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2}\frac{d\sigma(\ell^\mp,\rho^\pm)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau}\left|\frac{\partial(E_\ell^*,\Omega_\ell^*,\Omega_\rho^*,\Omega_\tau)}{\partial(p_\ell,\Omega_\ell,p_\rho,\Omega_\rho,\Phi_\tau)}\right|d\Phi_\tau$$

$(\ell,\ \text{all})$ in 3D

$$\frac{d\sigma(\vec{\zeta})}{d\Omega_\tau} = \frac{\alpha^2}{32E_\tau^2}\beta_\tau(D_0 + \mathcal{P}_e F_i \zeta_i)$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*) \to \ell^\mp \nu\nu)}{dx^* d\Omega_\ell^*} = \kappa_\ell(A(x^*) \mp \xi_\ell \vec{n}_\ell^* \vec{\zeta}^* B(x^*)),\ x^* = E_\ell^*/E_{\ell max}^*$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*),\ B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\sigma(\ell^\mp)}{dE_\ell^* d\Omega_\ell^* d\Omega_\tau} = \kappa_\ell \frac{\alpha^2 \beta_\tau}{32E_\tau^2}(D_0 A(E_\ell^*) \mp \mathcal{P}_e \xi_\ell F_i n_{\ell i}^* B(E_\ell^*))$$

$$\frac{d\sigma(\ell^\mp)}{dp_\ell d\Omega_\ell} = \int\limits_{\Omega_\tau-\text{sector}} \frac{d\sigma(\ell^\mp)}{dE_\ell^* d\Omega_\ell^* d\Omega_\tau} \left|\frac{\partial(E_\ell^*, \Omega_\ell^*)}{\partial(p_\ell, \Omega_\ell)}\right| d\Omega_\tau$$

Ω_τ-sector is determined by the kinematical constraint $m_{\nu\nu} > 0$

- All Michel parameters (ρ, η, $\mathcal{P}_e\xi$, $\mathcal{P}_e\xi\delta$) are measured in the unbinned maximum likelihood fit of ($\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$; $\tau^+ \to$ all) events in the **3D** phase space.
- The reduced 3D phase space allows one to tabulate various EXP/MC corrections to the detection efficiency more precisely.
- **The crucial point in this method is to have high-efficiency 1-track trigger.**

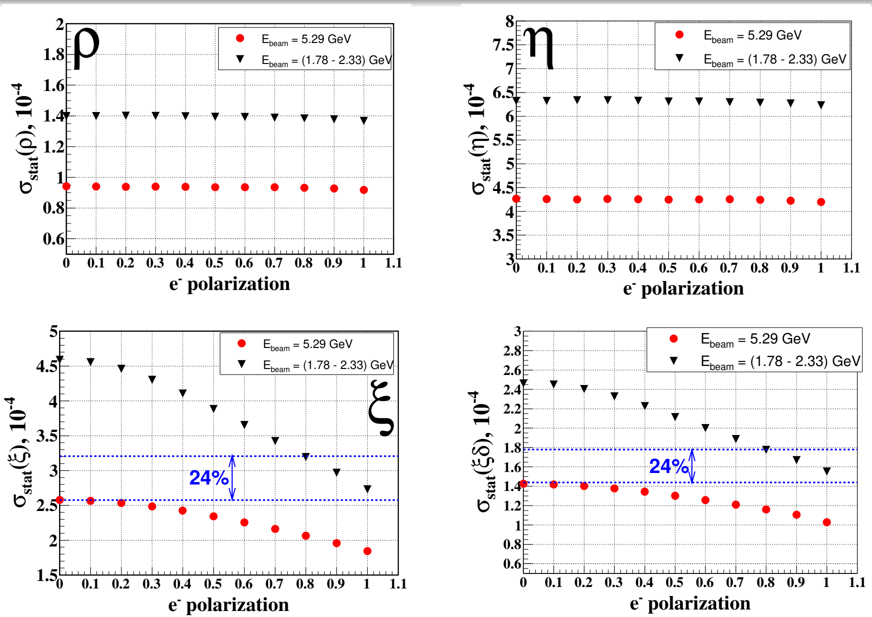
Physical corrections

- **Radiative corrections to $e^+e^- \to \tau^+\tau^-$**
  - All $\mathcal{O}(\alpha^3)$ QED and electroweak higher order corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$ are included:
    A. B. Arbuzov, ..., E. A. Kuraev, ... *et al*, JHEP **9710** (1997) 001.
    S. Jadach and Z. Was, Acta Phys. Polon. B **15** (1984) 1151 [Erratum-ibid. B **16** (1985) 483].
  - KKMC based approach:
    We generate table of ISR photons and then use it to calculate visible differential cross section in CMS.
- **Radiative leptonic decays $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau\gamma$**
  - Analytical approach based on:
    A. B. Arbuzov, Phys. Lett. B **524** (2002) 99. $\mathcal{O}(\alpha)$.
    A. Arbuzov, A. Czarnecki and A. Gaponenko, Phys. Rev. D **65** (2002) 113006. $\mathcal{O}(\alpha^2 \ln^2(\frac{m_\mu}{m_e}))$.
    A. Arbuzov and K. Melnikov, Phys. Rev. D **66** (2002) 093003. $\mathcal{O}(\alpha^2 \ln(\frac{m_\mu}{m_e}))$.
  - TAUOLA based approach:
    M. Jezabek, Comput. Phys. Commum. **70** (1992) 69.
    A. Czarnecki, M. Jezabek and J. H. Kuhn, Nucl. Phys. B **351** (1991) 70.
- **Radiative corrections to $\tau^- \to \pi^-\pi^0\nu_\tau$**
  - Analytical approach based on:
    F. Flores-Baez *et al*, Phys. Rev. Lett. D **74** (2006) 071301(R).
    A. Flores-Tlalpa *et al*, Nucl. Phys. B (Proc. Suppl.) **169** (2007) 250.
  - PHOTOS based approach

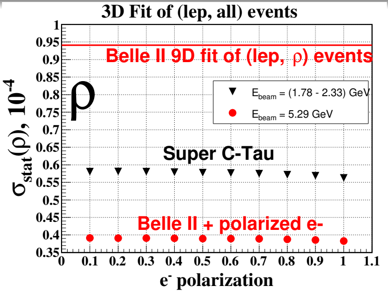
Toy MC studies of the effect of polarized e^- beam

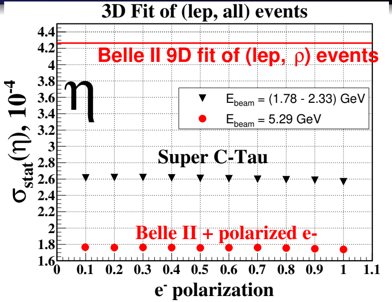
- The generator of the $(\ell,\ \rho)$ events has been developed, effects of spin-spin correlation of taus and e^- beam polarization are taken into account.
- Effects of ISR and FSR are not simulated. The development of the full fitter at Belle showed that radiative corrections can be taken into account properly in the fitter and they don't decrease the statistical sensitivity to MP.
- The direction of tau was taken from the generator. Studies of the sensitivity to MP at Belle showed that the integration over the allowed tau directions results in the sensitivity degradation factor of 1.4 only, this factor was additionally taken into account in our results.
- CLEO model (ρ, ρ') for $F_\pi(m^2_{\pi\pi})$ (used in the current version of TAUOLA) was utilized in our generator.
- 66 10M $(\mu,\ \rho)$ samples, at 6 center-of-mass (c.m.s.) energies (according to Table 1.1 in Super Charm-Tau factory CDR part I) : $2E = 3.554$ GeV ($\tau^+\tau^-$ production threshold), $2E = 3.686$ GeV ($\psi(2S)$), $2E = 3.770$ GeV ($\psi(3770)$), $2E = 4.170$ GeV ($\psi(4160)$), $2E = 4.650$ GeV (maximum of the $\sigma(e^+e^- \to \Lambda_c^+\Lambda_c^-)$), $2E = 10.58$ GeV (Belle II), for 11 values of e^- beam polarization: 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, were generated for the calculation of the normalizations. 66 statistically independent 1M samples at the same energies and polarizations were generated for the fit.
- To evaluate MP sensitivities (rescaling the sensitivities obtained in the fits of 1M samples) we took the detection efficiency of $(\mu,\ \rho)$ events to be 20% (to be compared with 12% efficiency obtained at Belle, where the π^0 rec. efficiency is only 40%). The detection efficiency of (μ, all) events was taken to be 30%.
- To measure ρ, ξ and $\xi\delta$ MP, samples with $\ell = e,\ \mu$ were taken into account, while η MP is measured in samples with $\ell = \mu$ only.

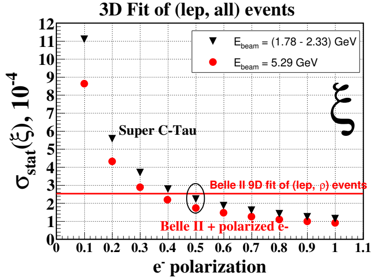
Fit of (ℓ, ρ) in 9D at Belle II/Super C-Tau

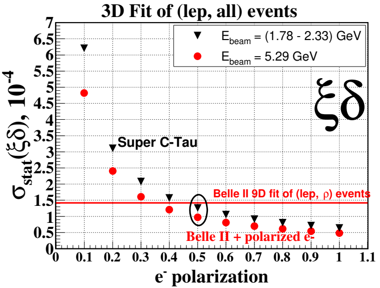


Fit of $(\ell,\ \text{all})$ in 3D at Belle II/Super C-Tau









The sensitivity to the ξ and $\xi\delta$ parameters at the Super Charm-Tau factory becomes better than that at Belle II (with unpolarized e^- beam) for the e^- beam polarizations larger than 0.5.

Summary

- Feasibility study of Michel parameters at the Super Charm-Tau factory and Belle II with polarized e^- beam has been carried out. This simple generator level study allows us to estimate the statistical sensitivities to MP as a function of e^- beam polarization.
- Two methods were studied, (I) 9D fit of the $(\ell,\ \rho)$ events, (II) 3D fit of the $(\ell,\ \mathrm{all})$ events.
- **In the method (I)**, the sensitivities to ρ and η parameters for the expected Belle II (with unpolarized e^- beam) and Super Charm-Tau factory statistics differ by only a factor of 1.5, Belle II has the best sensitivities. The sensitivities to the ξ and $\xi\delta$ MP differ by only 25% (with unpolarized e^- beam for Belle II and e^- beam polarization of 0.8 for Super Charm-Tau factory), with Belle II best sensitivities.
- **In the method (II)**, the sensitivities to ρ and η parameters for the expected Belle II (with unpolarized e^- beam) and Super Charm-Tau factory statistics differ by only a factor of 1.5, Super Charm-Tau factory has the best sensitivities. The sensitivities to the ξ and $\xi\delta$ MP become equal with unpolarized e^- beam for Belle II and e^- beam polarization of 0.5 for Super Charm-Tau factory. For the higher e^- beam polarization the sensitivities to ξ and $\xi\delta$ MP improve as $1/\mathcal{P}_e$, and Super Charm-Tau factory wins Belle II. For the high e^- beam polarizations there is some notable room to decrease luminosity while keeping priority in the sensitivities to ξ and $\xi\delta$ MP at Super Charm-Tau factory. The reduced 3D phase space in method (II) allows one to tabulate various EXP/MC corrections to the detection efficiency more precisely.
- **It is seen that the expected MP statistical uncertainties are of the order of 10^{-4}, to reach similar level systematic uncertainty, the NNLO corrections to the $e^+e^- \to \tau^+\tau^-$ cross section are mandatory.**

Backup slides

Michel parameters

$$\rho = \frac{3}{4} - \frac{3}{4}\left(|g^V_{LR}|^2 + |g^V_{RL}|^2 + 2|g^T_{LR}|^2 + 2|g^T_{RL}|^2 + \Re(g^S_{LR}g^{T*}_{LR} + g^S_{RL}g^{T*}_{RL})\right)$$

$$\eta = \frac{1}{2}\Re\left(6g^V_{RL}g^{T*}_{LR} + 6g^V_{LR}g^{T*}_{RL} + g^S_{RR}g^{V*}_{LL} + g^S_{RL}g^{V*}_{LR} + g^S_{LR}g^{V*}_{RL} + g^S_{LL}g^{V*}_{RR}\right)$$

$$\xi = 4\Re(g^S_{LR}g^{T*}_{LR}) - 4\Re(g^S_{RL}g^{T*}_{RL}) + |g^V_{LL}|^2 + 3|g^V_{LR}|^2 - 3|g^V_{RL}|^2 - |g^V_{RR}|^2 + $$
$$+ 5|g^T_{LR}|^2 - 5|g^T_{RL}|^2 + \frac{1}{4}|g^S_{LL}|^2 - \frac{1}{4}|g^S_{LR}|^2 + \frac{1}{4}|g^S_{RL}|^2 - \frac{1}{4}|g^S_{RR}|^2$$

$$\xi\delta = \frac{3}{16}|g^S_{LL}|^2 - \frac{3}{16}|g^S_{LR}|^2 + \frac{3}{16}|g^S_{RL}|^2 - \frac{3}{16}|g^S_{RR}|^2 - \frac{3}{4}|g^T_{LR}|^2 + \frac{3}{4}|g^T_{RL}|^2 + $$
$$+ \frac{3}{4}|g^V_{LL}|^2 - \frac{3}{4}|g^V_{RR}|^2 + \frac{3}{4}\Re(g^S_{LR}g^{T*}_{LR}) - \frac{3}{4}\Re(g^S_{RL}g^{T*}_{RL})$$

Search for New Physics in leptonic τ decays

In BSM models the couplings to τ are expected to be larger than those to μ. Contribution from New Physics in τ decays can be enhanced by a factor of $(\frac{m_\tau}{m_\mu})^2$.

- **Type II 2HDM:** $\eta_\mu(\tau) = \frac{m_\mu M_\tau}{2}\left(\frac{\tan^2\beta}{M^2_{H^\pm}}\right)^2$; $\frac{\eta_\mu(\tau)}{\eta_e(\mu)} = \frac{M_\tau}{m_e} \approx 3500$
- **Tensor interaction:**
  $\mathcal{L} = \frac{g}{2\sqrt{2}} W^\mu \left\{\bar{\nu}\gamma_\mu(1-\gamma^5)\tau + \frac{\kappa^W_\tau}{2m_\tau}\partial^\nu\left(\bar{\nu}\sigma_{\mu\ nu}(1-\gamma^5)\tau\right)\right\}$,
  $-0.096 < \kappa^W_\tau < 0.037$: DELPHI Abreu EPJ C16 (2000) 229.
- **Unparticles:** Moyotl PRD 84 (2011) 073010, Choudhury PLB 658 (2008) 148.
- **Lorentz and CPTV:** Hollenberg PLB **701** (2011) 89
- **Heavy Majorana neutrino:** M. Doi *et al.*, Prog. Theor. Phys. **118** (2007) 1069.
- $\mu - \tau$ **LFV Yukawa couplings in** ξ_μ**:** K. Tobe, JHEP **1610** (2016) 114

Multidimensional unbinned maximum likelihood fit

4 Michel parameters ($\vec{\Theta} = (1,\ \rho,\ \eta,\ \xi_\rho\xi_\ell,\ \xi_\rho\xi_\ell\delta_\ell)$) are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu;\ \rho\nu)$ events in the 9D phase space in CMS, $\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$. The PDF for individual k-th event is written in the form:

$$\mathcal{P}^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})},\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z}$$

Likelihood function for N events:

$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)},\ \mathcal{L} = -\ln L = N \ln \mathcal{N}(\vec{\Theta}) - \sum_{k=1}^{N} \ln \mathcal{F}^{(k)},\ \mathcal{F}^{(k)} = \mathcal{F}(\vec{z}^{(k)})$$

$$\mathcal{F}^{(k)} = A_0^{(k)}\Theta_0 + A_1^{(k)}\Theta_1 + A_2^{(k)}\Theta_2 + A_3^{(k)}\Theta_3 + A_4^{(k)}\Theta_4 = \sum_{i=0}^{4} A_i^{(k)}\Theta_i$$

$$\mathcal{N} = C_0\Theta_0 + C_1\Theta_1 + C_2\Theta_2 + C_3\Theta_3 + C_4\Theta_4,\ C_j = \frac{1}{N}\sum_{k=1}^{N} C_j^{(k)},\ C_j^{(k)} = \frac{A_j^{(k)}}{\sum_{i=0}^{4} A_i^{(k)}\Theta_i^{MC}}$$

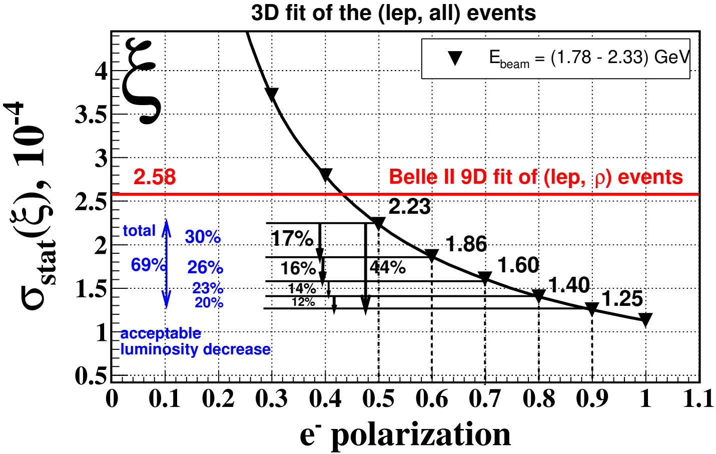
$$\vec{\Theta}^{MC} = (1,\ 0.75,\ 0,\ 1,\ 0.75),\ \mathcal{L} = N\ln\left(\sum_{j=0}^{4} C_j\Theta_j\right) - \sum_{k=1}^{N}\ln\left(\sum_{i=0}^{4} A_i^{(k)}\Theta_i\right)$$

As a result fitted statistics is represented by a set of $5 \times N$ values of $A_i^{(k)}$ $(k = 1 \div N,\ i = 0 \div 4)$, which is calculated only once.

C_i $(i = 0 \div 4)$ are calculated using MC simulation.

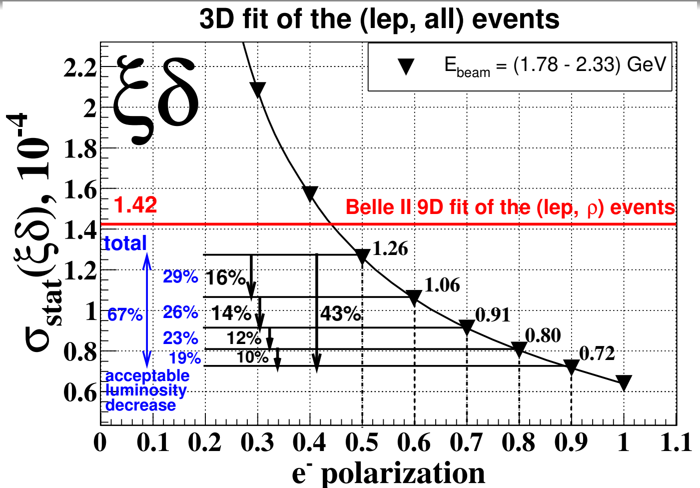
In ideal case (no rad. corr., $\varepsilon = 100\%$): $C_0 = 1$, $C_2 = 4m_\ell/m_\tau$, $C_{1,3,4} = 0$

ξ from the fit of $(\ell, \text{ all})$ in 3D at Super C-Tau



For the increases of the e^- beam polarizations, 0.5 $\to$ 0.6, 0.6 $\to$ 0.7, 0.7 $\to$ 0.8, 0.8 $\to$ 0.9, the corresponding improvements in the sensitivities to the ξ parameter, 17%, 16%, 14%, 12%, respectively. If we move from the polarization of 0.5 to the higher polarizations: 0.5 $\to$ 0.6, 0.5 $\to$ 0.7, 0.5 $\to$ 0.8, 0.5 $\to$ 0.9, the acceptable luminosity decrease factors (to keep the sensitivity at the level of that we have for polarization 0.5) are: $(1.86/2.23)^2 = 0.70$, $(1.60/2.23)^2 = 0.51$, $(1.40/2.23)^2 = 0.39$, $(1.25/2.23)^2 = 0.31$, respectively.

$\xi\delta$ from the fit of $(\ell,\ \mathrm{all})$ in 3D at Super C-Tau



For the increases of the e^- beam polarizations, 0.5 → 0.6, 0.6 → 0.7, 0.7 → 0.8, 0.8 → 0.9, the corresponding improvements in the sensitivities to the $\xi\delta$ parameter, 16%, 14%, 12%, 10%, respectively. If we move from the polarization of 0.5 to the higher polarizations: 0.5 → 0.6, 0.5 → 0.7, 0.5 → 0.8, 0.5 → 0.9, the acceptable luminosity decrease factors (to keep the sensitivity at the level of that we have for polarization 0.5) are: $(1.06/1.26)^2 = 0.71$, $(0.91/1.26)^2 = 0.52$, $(0.80/1.26)^2 = 0.40$, $(0.72/1.26)^2 = 0.33$, respectively.