Study of Michel parameters in leptonic τ decays at Belle

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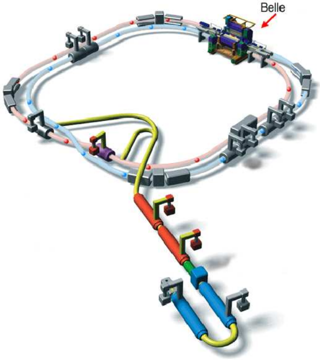
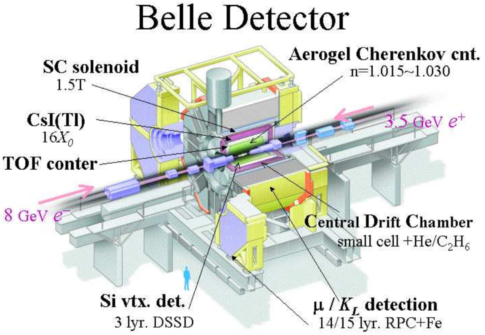
Belle experiment





Process	σ, nb
$e^+e^- \to e^+e^-(\gamma)$ $15^o \le \theta \le 165^o$	123.5
$e^+e^- \to \mu^+\mu^-(\gamma)$	1.005
$e^+e^- \to q\bar{q}$ $(q = u, d, s, c)$	3.39
$e^+e^- \to b\bar{b}$	1.05
$e^+e^- \to e^+e^- f\bar{f}$ $(f = u, d, s, c, e, \mu, \tau)$	72.6
$e^+e^- \to \tau^+\tau^-(\gamma)$	0.919

- $E_{e^-} = 8$ GeV, $E_{e^+} = 3.5$ GeV
- Peak luminosity: $L = 2.11 \times 10^{34}$ cm^{-2}s^{-1}
- Integrated luminosity: $\int Ldt \simeq 1$ ab^{-1}, $N_{\tau\tau} \simeq 10^9$
- B-factory is also τ-factory

Introduction

- The world largest statistics of τ leptons $\sim 2 \times 10^9$ collected at Belle opens the new era in precision tests of the SM in τ decays.
- In the SM charged weak interaction is described by the exchange of $W^{\pm}$ with a pure vector coupling to only left-handed fermions providing "V-A$\otimes$ V-A" Lorentz structure of the effective four-fermion weak interaction (maximal parity violation).
- Deviations from "V-A" can be originated from:
  - CPV in leptonic sector.
  - Scalar contributions from $H^{\pm}$
  - Mixing of right-handed and left-handed vector currents (W_L and W_R).
- $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ $(\ell = e, \mu)$ decays provide clean laboratory to probe electroweak couplings.

General weak four-lepton interaction ansatz

The most general, Lorentz invariant four-lepton interaction matrix element:

$$\mathcal{M} = \frac{4G}{\sqrt{2}} \sum_{\substack{N=S,V,T \\ i,j=L,R}} g^N_{ij} \left[\bar{u}_i(l^-) \Gamma^N v_n(\bar{\nu}_l) \right] \left[\bar{u}_m(\nu_\tau) \Gamma_N u_j(\tau^-) \right],$$

$$\Gamma^S = 1,\ \Gamma^V = \gamma^\mu,\ \Gamma^T = \frac{i}{2\sqrt{2}}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$$

Ten coupling constants g^N_{ij}:

g^S_{RR}, g^S_{RL}, g^S_{LR}, g^S_{LL}, g^V_{RR}, g^V_{RL}, g^V_{LR}, g^V_{LL}, g^T_{RL}, g^T_{LR}.

The indices i and j label the right- or lefthandedness (R, L) of the charged leptons. For a given i, j and N, the handedness of the neutrinos (n, m) are fixed. Couplings can be complex, with arbitrary total phase → 19 independent parameters.

In the SM the only non-zero coupling constant is $g^V_{LL} = 1$.

$\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$

Without measuring neutrinos and spin of the outgoing charged lepton, only four bilinear combinations of g^N_{ij} are experimentally accessible. They are called Michel parameters (MP): ρ, η, ξ and δ. They appear in the energy spectrum of the outgoing lepton:

$$\frac{d\Gamma(\tau^\mp)}{d\Omega dx} = \frac{4G_F^2 M_\tau E_{max}^4}{(2\pi)^4}\sqrt{x^2 - x_0^2}\Bigg(x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)$$

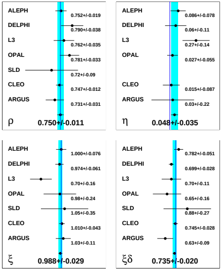
$$\mp\frac{1}{3}P_\tau \cos\theta_\ell \xi\sqrt{x^2 - x_0^2}\left[1 - x + \frac{2}{3}\delta\left(4x - 4 + \sqrt{1 - x_0^2}\right)\right]\Bigg)$$

- $x = \frac{E_\ell}{E_{max}}$, $E_{max} = \frac{M_\tau}{2}(1 + \frac{m_\ell^2}{M_\tau^2})$, $x_0 = \frac{m_\ell}{E_{max}}$;
- θ_ℓ - angle between τ spin and $\vec{p}_\ell$;
- P_τ - τ polarization

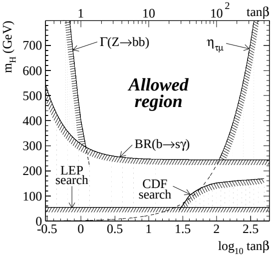
In the SM: $\rho = \frac{3}{4}$, $\eta = 0$, $\xi = 1$, $\delta = \frac{3}{4}$

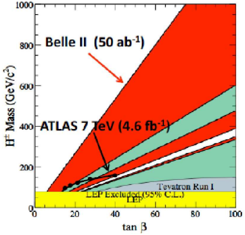
We are sensitive to the η parameter only with $\tau \to \mu\nu\nu$ decays.

Michel par.	Measured value	Experiment	SM value
ρ (e or μ)	$0.747 \pm 0.010 \pm 0.006$ 1.5%	CLEO-97	3/4
η (e or μ)	$0.012 \pm 0.026 \pm 0.004$ 2.6%	ALEPH-01	0
ξ (e or μ)	$1.007 \pm 0.040 \pm 0.015$ 4.3%	CLEO-97	1
$\xi\delta$ (e or μ)	$0.745 \pm 0.026 \pm 0.009$ 3.8%	CLEO-97	3/4
ξ_h (all hadr.)	$0.992 \pm 0.007 \pm 0.008$ 1.1%	ALEPH-01	1



In the MSSM with 2 Higgs doublets: $\eta_\mu = \frac{m_\mu M_\tau}{2}\left(\frac{\tan^2\beta}{M^2_{H^\pm}}\right)^2$





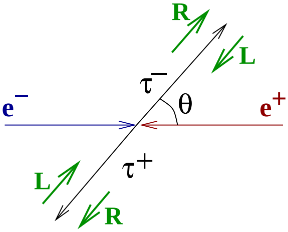
With ×300 statistics at Belle we can improve the uncertainties of MP at least by one order of magnitude !

Method, spin-spin correlation in $\tau^+\tau^-$

To measure ξ and δ MP we have to know τ spin direction. Effect of τ spin-spin correlation in $e^+e^- \to \tau^+(\vec{\zeta}^+)\tau^-(\vec{\zeta}^-)$ can be used:

$$\frac{d\sigma(\vec{\zeta}^-,\vec{\zeta}^+)}{d\Omega} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0+D_{ij}\zeta_i^-\zeta_j^+)$$

$$D_0 = 1+\cos^2\theta + \frac{1}{\gamma_\tau^2}\sin^2\theta$$



CMS frame

$$D_{ij} = \begin{pmatrix} (1+\frac{1}{\gamma_\tau^2})\sin^2\theta & 0 & \frac{1}{\gamma_\tau}\sin 2\theta \\ 0 & -\beta_\tau^2\sin^2\theta & 0 \\ \frac{1}{\gamma_\tau}\sin 2\theta & 0 & 1+\cos^2\theta-\frac{1}{\gamma_\tau^2}\sin^2\theta \end{pmatrix}$$

τ^- and τ^+ helicities are 95% anti-correlated, so if we know helicity of τ on the tag side we can identify helicity of τ on the signal side.

$\tau^- \to h^- \nu_\tau$, $h = \pi,\ \rho$

$$J^\mu = < h|\bar{d}\gamma^\mu(c_V + c_A\gamma^5)u|0 >$$

Michel formalism for the $\tau^- \to h^- \nu_\tau$ includes
$\xi_h = -\frac{2Re(c_V^* c_A)}{|c_V|^2 + |c_A|^2} = -h_{\nu_\tau}$ (=1 in SM):

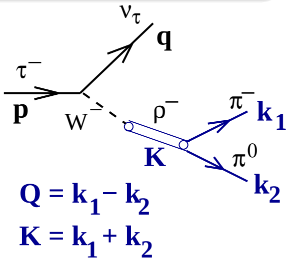
$$\frac{d\Gamma(\tau^\mp \to \pi^\mp \nu)}{d\Omega_\pi} = C\big(1 \pm \xi_\pi P_\tau \cos\theta_\pi\big)$$

$$\frac{d\Gamma(\tau^\mp \to \rho^\mp \nu)}{dm^2_{\pi\pi} d\Omega_\rho d\Omega^*_\pi} = f(\vec{k}_1, \vec{k}_2) \pm \xi_\rho \vec{P}_\tau \vec{g}(\vec{k}_1, \vec{k}_2) = f(\vec{k}_1, \vec{k}_2)(1 \pm \xi_\rho \vec{P}_\tau \vec{H}_\rho)$$

$\vec{H}_\rho = M_\tau \frac{2(q,Q)\vec{Q} + Q^2\vec{K}}{2(p,Q)(q,Q) - Q^2(p,q)}$

$\vec{H}_\rho$- polarimeter vector (indicates ρ spin direction)

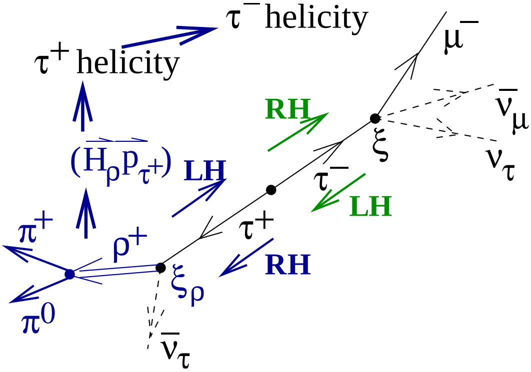
$\tau^\pm \to \rho^\pm(\to \pi^\pm\pi^0)\nu$ can be used as a spin analyzer



It was suggested to use events:

- $\tau^\mp \to \ell^\mp \nu\nu$ vs. $\tau^\pm \to \rho^\pm \nu$ ($\ell = \mathrm{e},\ \mu$) to measure MP: ρ, η, $\xi_\rho \xi$, $\xi_\rho \xi \delta$.
- $\tau^\mp \to \rho^\mp \nu$ vs. $\tau^\pm \to \rho^\pm \nu$ to measure ξ_ρ^2.
  Sign of ξ_ρ was reliably established by ARGUS:
  H. Albrecht et al., Phys. Lett. B 250 (1990) 164.

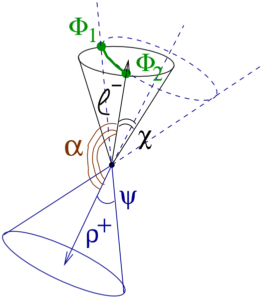
In the $\ell - \rho$ events $\tau \to \rho(\to \pi\pi^0)\nu$ serves as spin-analyzer.

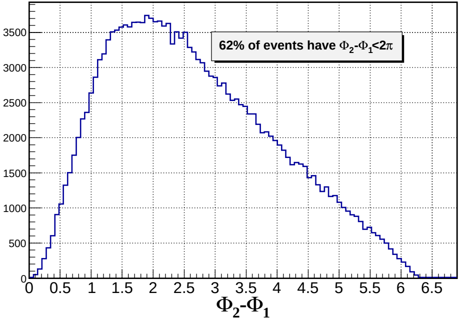


In the $\ell-\rho$ events the direction of τ axis is constrained by arc, which is determined by measurable angles ($m_{\nu_\tau}^{had}=0,\ 0<m_{\nu\nu}^{lep}\le(M_\tau-m_{lep})$):

$$\cos\psi=\frac{2E_\tau E_\rho-M_\tau^2-m_{\pi\pi^0}^2}{2p_\tau p_\rho},\quad \frac{2E_\tau E_\ell-M_\tau^2-m_\ell^2}{2p_\tau p_\ell}<\cos\chi\le\frac{E_\tau E_\ell-M_\tau m_\ell}{p_\tau p_\ell}$$

$$\cos\alpha=\sin\theta_\rho\sin\theta_\pi\cos(\varphi_\rho-\varphi_\pi)+\cos\theta_\rho\cos\theta_\pi$$



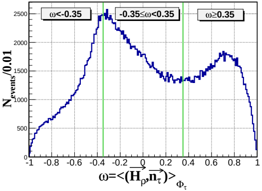


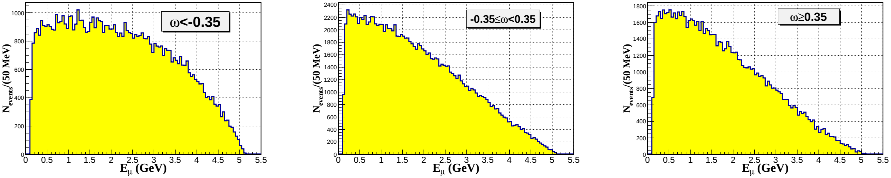
$$\Phi_1=\pi+\arcsin\left(\frac{\cos\psi\cos\alpha+\cos\chi}{\sin\psi\sin\alpha}\right),\quad \Phi_2=2\pi-\arcsin\left(\frac{\cos\psi\cos\alpha+\cos\chi}{\sin\psi\sin\alpha}\right)$$

M. Davier et. al Phys. Lett. B 306 (1993) 411.

Helicity sensitive variable ω is introduced as:

$$\omega = \frac{1}{\Phi_2 - \Phi_1}\int_{\Phi_1}^{\Phi_2} (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) d\Phi = < (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) >_{\Phi_\tau}$$





Spin-spin correlation manifests itself through momentum-momentum correlations of final lepton and pions.

Theoretical framework

- W. Fetscher, Phys. Rev. D 42 (1990) 1544.
  $\ell_1^{\mp} - \ell_2^{\pm}$, $\ell^{\mp} - \mathrm{h}^{\pm}$, $\ell = \mathrm{e},\ \mu$; $\mathrm{h} = \pi,\ \mathrm{K}$.
- K. Tamai, Nucl. Phys. B 668 (2003) 385. (KEK Preprint 2003-14, Belle note 471) $\ell^{\mp} - \rho^{\pm}(\to \pi^{\pm}\pi^0)$ + feasibility study.

$$\frac{d\sigma(\vec{\zeta},\vec{\zeta}')}{d\Omega} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i\zeta'_j)$$

$$\frac{d\Gamma(\tau^{\mp}(\vec{\zeta}^*) \to \ell^{\mp}\nu\nu)}{dx^* d\Omega^*_\ell} = \kappa_\ell(A(x^*) \mp \xi\vec{n}^*_\ell\vec{\zeta}^* B(x^*)),\ x^* = E^*_\ell/E^*_{\ell\max}$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*),\ B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\Gamma(\tau^{\pm}(\vec{\zeta'}^*) \to \rho^{\pm}\nu)}{dm^2_{\pi\pi} d\Omega^*_\rho d\tilde{\Omega}_\pi} = \kappa_\rho(A' \mp \xi_\rho \vec{B'}\vec{\zeta'}^*)W(m^2_{\pi\pi})$$

$$A' = 2(q,Q)Q^*_0 - Q^2 q^*_0,\ \vec{B'} = Q^2\vec{K}^* + 2(q,Q)\vec{Q}^*,\ W = |F_\pi(m^2_{\pi\pi})|^2 \frac{p_\rho(m^2_{\pi\pi})\tilde{p}_\pi(m^2_{\pi\pi})}{M_\tau m_{\pi\pi}}$$

$$\frac{d\sigma(\ell^{\mp},\rho^{\pm})}{dE^*_\ell d\Omega^*_\ell d\Omega^*_\rho dm^2_{\pi\pi} d\tilde{\Omega}_\pi d\Omega_\tau} = \kappa_\ell\kappa_\rho \frac{\alpha^2\beta_\tau}{64E^2_\tau}(D_0 A' A(E^*_\ell) + \xi_\rho\xi_\ell D_{ij} n^*_{\ell i} B'_j B(E^*_\ell))W(m^2_{\pi\pi})$$

$$\frac{d\sigma(\ell^{\mp},\rho^{\pm})}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm^2_{\pi\pi} d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^{\mp},\rho^{\pm})}{dE^*_\ell d\Omega^*_\ell d\Omega^*_\rho dm^2_{\pi\pi} d\tilde{\Omega}_\pi d\Omega_\tau} \left|\frac{\partial(E^*_\ell,\Omega^*_\ell,\Omega^*_\rho,\Omega_\tau)}{\partial(p_\ell,\Omega_\ell,p_\rho,\Omega_\rho,\Phi_\tau)}\right| d\Phi_\tau$$

Multidimensional unbinned maximum likelihood fit

4 Michel parameters ($\vec{\Theta} = (1,\ \rho,\ \eta,\ \xi_\rho\xi_\ell,\ \xi_\rho\xi_\ell\delta_\ell)$) are extracted in the unbinned maximum likelihood fit of $\ell - \rho$ events in the 9D phase space ($\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$) in CMS.
The PDF for individual k-th event is written in the form:

$$\mathcal{P}^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})},\ \mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^{\mp},\rho^{\pm})}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm^2_{\pi\pi} d\tilde{\Omega}_\pi},\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z}$$

Likelihood function for N events:

$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)},\ \mathcal{L} = -\ln L = N\ln\mathcal{N}(\vec{\Theta}) - \sum_{k=1}^{N}\ln\mathcal{F}^{(k)},\ \mathcal{F}^{(k)} = \mathcal{F}(\vec{z}^{(k)})$$

$$\mathcal{F}^{(k)} = A_0^{(k)}\Theta_0 + A_1^{(k)}\Theta_1 + A_2^{(k)}\Theta_2 + A_3^{(k)}\Theta_3 + A_4^{(k)}\Theta_4 = \sum_{i=0}^{4} A_i^{(k)}\Theta_i,\ \mathcal{N}(\vec{\Theta}) = \sum_{i=0}^{4} C_i\Theta_i$$

$$\mathcal{L} = N\ln(C_i\Theta_i) - \sum_{k=1}^{N}\ln(A_i^{(k)}\Theta_i)$$

As a result fitted statistics is represented by a set of $5 \times N$ values of $A_i^{(k)}$ ($k = 1 \div N,\ i = 0 \div 4$), which is calculated only once. C_i ($i = 0 \div 4$) are calculated using MC simulation.

Suppose we have N_{MC} MC events, which were simulated with particular set $\vec{\Theta}^{MC}$. By reweighting each event we can calculate normalization for arbitrary set $\vec{\Theta}$:

$$\mathcal{N}(\vec{\Theta}) \approx \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} w^{(k)}, \; w^{(k)} = \frac{A_i^{(k)}\Theta_i}{A_j^{(k)}\Theta_j^{MC}} = B_m^{(k)}\Theta_m, \; B_m^{(k)} = \frac{A_m^{(k)}}{A_j^{(k)}\Theta_j^{MC}}$$

$$\mathcal{N}(\vec{\Theta}) = C_i\Theta_i, \; C_i = \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} B_i^{(k)}$$

This algorithm can be easily extended to take into account selection efficiency:

$$\mathcal{F}(\vec{z}) \to \mathcal{F}'(\vec{z}) = \mathcal{F}(\vec{z})\epsilon(\vec{z}), \; \mathcal{N}'(\vec{\Theta}) = \int \mathcal{F}(\vec{z})\epsilon(\vec{z})d\vec{z}$$

$$\mathcal{L} = N_{sel} \ln \mathcal{N}'(\vec{\Theta}) - \sum_{k=1}^{N_{sel}} \ln(\mathcal{F}^{(k)}\epsilon(\vec{z})) = N_{sel} \ln(C_i'\Theta_i) - \sum_{k=1}^{N_{sel}} \ln(A_i^{(k)}\Theta_i) - \sum_{k=1}^{N_{sel}} \ln \epsilon(\vec{z})$$

$$C_i = \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}^{sel}} B_i^{(k)}$$

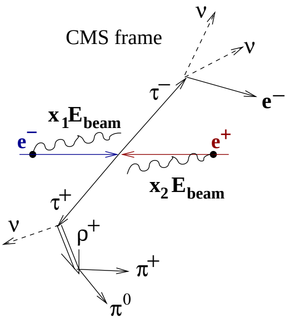
Intermediate results

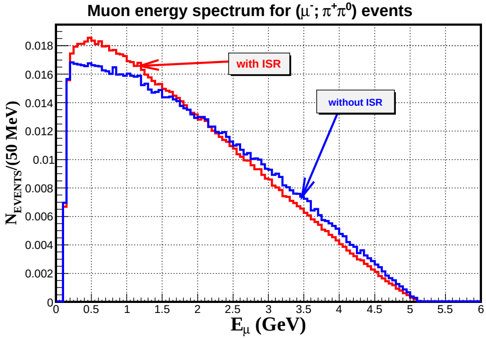
- Michel parameter formalism was embedded into TAUOLA generator.
  M. Schmidtler IEKP-KA/93-14 1993.
  About 100M MC events were generated for different tests of the fitting procedure.
- Belle pion form factor was also embedded into TAUOLA.
  M. Fujikawa et al. [Belle Collaboration], Phys. Rev. D 78 (2008) 072006.
- With high statistics of generated events we checked that spin-spin correlation of taus is implemented correctly in KKMC/TAUOLA.
- Integration over Φ_τ results in only ~ 1.4 MP sensitivity decrease.
- We confirmed that even with large Belle statistics we are not sensitive to η MP in $\tau^- \to \mathrm{e}^-\bar{\nu}_\mathrm{e}\nu_\tau$ (only $\tau^- \to \mu^-\bar{\nu}_\mathrm{e}\nu_\tau$ can be used to extract η).
- It was confirmed that with Belle statistics it is possible to measure Michel parameters in particular event configuration $(\ell^\mp;\ \rho^\pm)$ $\ell = \mathrm{e},\ \mu$ with the statistical uncertainty of $\mathrm{O}(10^{-3})$.

Physical corrections

- Initial state radiation (ISR) $e^+e^- \to \tau^+\tau^-\gamma_{\rm ISR}$
  - Analytical approach based on:
    E. A. Kuraev and V. S. Fadin, "On Radiative Corrections to e+ e- Single Photon Annihilation at High-Energy," Sov. J. Nucl. Phys. 41, 466 (1985)
  - KKMC based approach:
    We generate table of ISR photons and then use it to calculate visible differential cross section in CMS.
- Radiative leptonic decays $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau\gamma$
  - Analytical approach based on:
    A. B. Arbuzov, Phys. Lett. B 524 (2002) 99. $\mathcal{O}(\alpha)$.
    A. Arbuzov, A. Czarnecki and A. Gaponenko, Phys. Rev. D 65 (2002) 113006. $\mathcal{O}(\alpha^2 \ln^2(\frac{m_\mu}{m_e}))$.
    A. Arbuzov and K. Melnikov, Phys. Rev. D 66 (2002) 093003. $\mathcal{O}(\alpha^2 \ln(\frac{m_\mu}{m_e}))$.
  - TAUOLA based approach:
    M. Jezabek, Comput. Phys. Commum. 70 (1992) 69.
    A. Czarnecki, M. Jezabek and J. H. Kuhn, Nucl. Phys. B 351 (1991) 70.
- $\tau^- \to \pi^-\pi^0\nu_\tau\gamma$
  - Analytical approach based on:
    F. Flores-Baez et al, Phys. Rev. Lett. D 74 (2006) 071301(R).
    A. Flores-Tlalpa et al, Nucl. Phys. B (Proc. Suppl.) 169 (2007) 250.
  - PHOTOS based approach

Initial state radiation (ISR), Fadin & Kuraev formalism

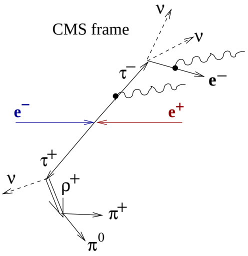


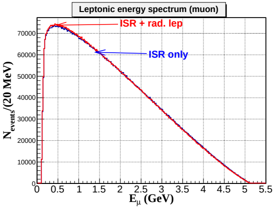


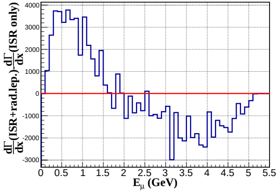
$$\frac{d\sigma_{vis}(s)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm^2_{\pi\pi} d\tilde{\Omega}_\pi} = \iint_0^1 dx_1 dx_2 D(x_1) D(x_2) \frac{d\sigma(s(1-x_1)(1-x_2))}{dp'_\ell d\Omega'_\ell dp'_\rho d\Omega'_\rho dm^2_{\pi\pi} d\tilde{\Omega}_\pi} \left| \frac{\partial(p'_\ell, \Omega'_\ell)}{\partial(p_\ell, \Omega_\ell)} \right| \left| \frac{\partial(p'_\rho, \Omega'_\rho)}{\partial(p_\rho, \Omega_\rho)} \right|$$

- $D(x) = x^{\beta/2-1} h(x)$ - probability function for initial $e^\mp$ to emit a γ-quantum jet carrying $x_{1,2}$ part of $e^\mp$ energy $E_{beam} = \sqrt{s}/2$. $\beta = \frac{2\alpha}{\pi}(\ln \frac{s}{m^2} - 1)$, $h(x)$ - smooth limited function.
- $\left| \frac{\partial(p'_i, \Omega'_i)}{\partial(p_i, \Omega_i)} \right|$ $(i = \ell,\ \rho)$ - Jacobian of transformation from the $\tau^+\tau^-$ rest frame to the Belle CMS.

Radiative leptonic decays







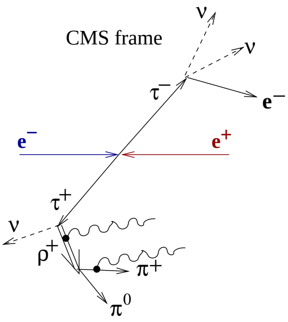
$$\frac{d\Gamma(\tau^{\mp}(\vec{\zeta}^*) \to \ell^{\mp}\nu\nu)}{dx^* d\Omega^*_\ell} = \kappa_\ell(A(x^*) \mp \xi \vec{n}^*_\ell \vec{\zeta}^* B(x^*)), \;\; x^* = E^*_\ell / E^*_{\ell \max}$$

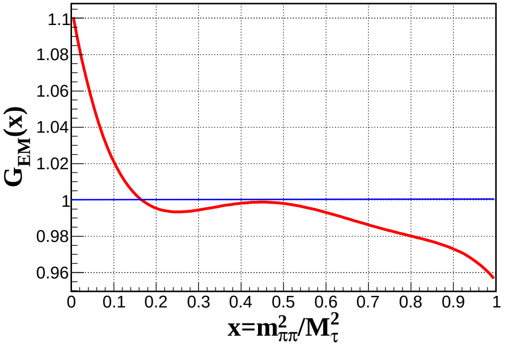
$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*) + A_{rad}(x^*), \;\; B(x^*) = B_1(x^*) + \delta B_2(x^*) + B_{rad}(x^*)$$

$$A_{rad}(x^*) = \alpha \cdot a_1(x^*) + \alpha^2 \ln^2\left(\frac{M_\tau}{m_\ell}\right) \cdot a_2(x^*) + \alpha^2 \ln\left(\frac{M_\tau}{m_\ell}\right) \cdot a_3(x^*)$$

$$B_{rad}(x^*) = \alpha \cdot b_1(x^*) + \alpha^2 \ln^2\left(\frac{M_\tau}{m_\ell}\right) \cdot b_2(x^*) + \alpha^2 \ln\left(\frac{M_\tau}{m_\ell}\right) \cdot b_3(x^*)$$

$a_{1,2,3}(x^*)$, $b_{1,2,3}(x^*)$ - known form factors





$$\frac{d\Gamma(\tau^\pm(\vec{\zeta'}^*) \to \rho^\pm \nu)}{dm^2_{\pi\pi} d\Omega^*_\rho d\tilde{\Omega}_\pi} = \kappa_\rho (A' \mp \xi_\rho \vec{B'} \vec{\zeta'}^*) W(m^2_{\pi\pi})$$

$$W(m^2_{\pi\pi}) \to W(m^2_{\pi\pi}) \cdot G_{EM}(m^2_{\pi\pi})$$

Detector effects

In our fit procedure the probability density function for individual event after all physical corrections applied is:

$$\mathcal{P}^{\rm vis}(\vec{z}|\vec{\Theta}) = \frac{\mathcal{F}^{\rm vis}(\vec{z}|\vec{\Theta})}{\mathcal{N}(\vec{\Theta})},\ \mathcal{F}^{\rm vis}(\vec{z}|\vec{\Theta}) = \frac{d\sigma_{\rm vis}(\ell^{\mp},\rho^{\pm})}{dp_{\ell}d\Omega_{\ell}dp_{\rho}d\Omega_{\rho}dm^2_{\pi\pi}d\tilde{\Omega}_{\pi}} = A_i^{\rm vis}(\vec{z})\Theta_i,$$

$$\mathcal{N}(\vec{\Theta}) = \int \mathcal{F}^{\rm vis}(\vec{z})d\vec{z} = C_i^{\rm vis}\Theta_i,\ C_i^{\rm vis} = \int A_i^{\rm vis}(\vec{z})d\vec{z}$$

$$\vec{z} = (p_{\ell},\ \cos\theta_{\ell},\ \phi_{\ell},\ p_{\rho},\ \cos\theta_{\rho},\ \phi_{\rho},\ m_{\pi\pi},\ \cos\tilde{\theta}_{\pi},\ \tilde{\phi}_{\pi})$$

For the reconstructed values $\vec{\tilde{z}}$:

$$\mathcal{P}_{\rm R}^{\rm vis}(\vec{\tilde{z}}|\vec{\Theta}) = \frac{1}{\tilde{\mathcal{N}}(\vec{\Theta})}\int \mathcal{F}^{\rm vis}(\vec{z}|\vec{\Theta})\cdot\mathcal{R}(\vec{\tilde{z}},\vec{z})d\vec{z} = \frac{1}{\tilde{\mathcal{N}}(\vec{\Theta})}\tilde{A}_i^{\rm vis}(\vec{\tilde{z}})\Theta_i,\ \tilde{\mathcal{N}}(\vec{\Theta}) = \tilde{C}_i^{\rm vis}\Theta_i$$

$\mathcal{R}(\vec{\tilde{z}},\vec{z})$ includes:

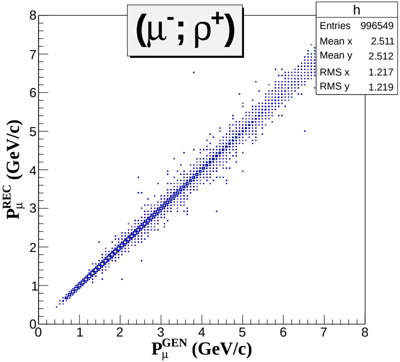
- Track momentum resolution ($\ell^{\mp}$, $\pi^{\pm}$)
- γ energy and angular resolution (π^0)
- Effect of external bremsstrahlung for $e-\rho$ events
- Beam energy spread

Track momentum resolution

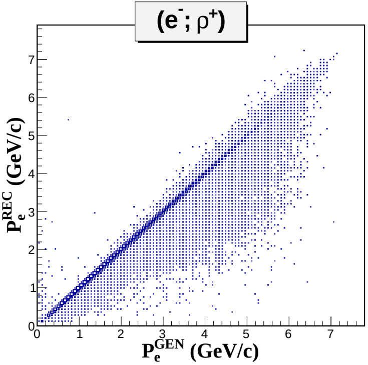
Uncertainties for the track helix parameters (d_ρ, ϕ_0, κ, d_z, $\tan\lambda$) are taken from the Mdst_trk_fit panther table (array error(15)). They are propagated to the uncertainties of (p_x, p_y, p_z) giving covariance matrix $\hat{D}$. The resolution function for 1 track:

$$\mathcal{R}(\Delta p = \tilde{\vec{p}} - \vec{p}) = \frac{1}{(2\pi)^{3/2}\sqrt{\det(\hat{D})}} e^{-\frac{1}{2}\Delta p^T \hat{D}^{-1} \Delta p}$$

Muon



Electron



Bremsstrahlung photon spectrum per unit of length ($E_e >> \frac{m}{\alpha Z^{1/3}}$): $\frac{dN_\gamma}{dx dE_\gamma} \simeq \frac{1}{X_0 E_\gamma}$. As a result the probability density function to emit bremsstrahlung photon:

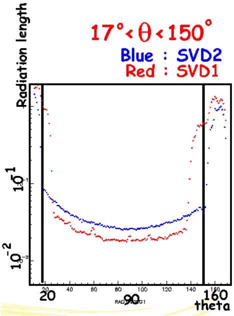
$$f(\varepsilon, \theta_{electron}) = (1 - p)\delta(\varepsilon) + pH(\varepsilon - \varepsilon_{min})\frac{1}{\varepsilon \ln(\frac{1}{\varepsilon_{min}})}$$

$$\varepsilon = \frac{E_\gamma}{E_e},\ \varepsilon_{min} = \frac{E_{\gamma min}}{E_e} = 10^{-4}$$

$$p = \frac{L}{1 - \varepsilon_{min}}\ln(\frac{1}{\varepsilon_{min}}),\ L = \frac{d_{SVD} + d_{vac.chamber}}{\sin\theta_{electron}}$$

$$d_{SVD} + d_{vac.chamber}(SVD1) = 1.9\% X_0$$

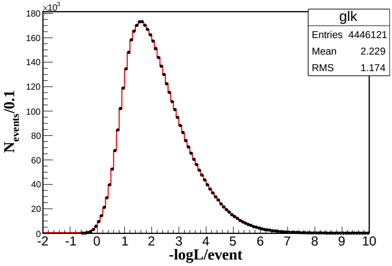
$$d_{SVD} + d_{vac.chamber}(SVD2) = 2.7\% X_0$$

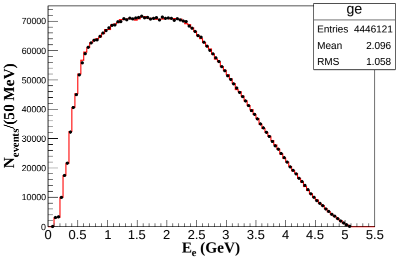


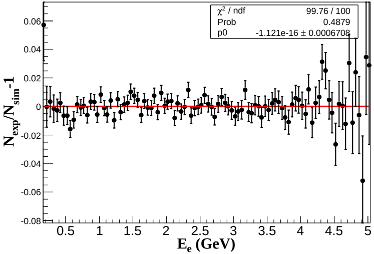
Validation of the fitter with 5M ($e^{\mp}$, $\rho^{\pm}$) MC sample

$(e^+; \pi^-\pi^0)$

ρ	$=$	0.7506	$\pm$	0.0010
η	$=$	0	-	fixed
ξ	$=$	1.0026	$\pm$	0.0043
$\xi\delta$	$=$	0.7544	$\pm$	0.0027

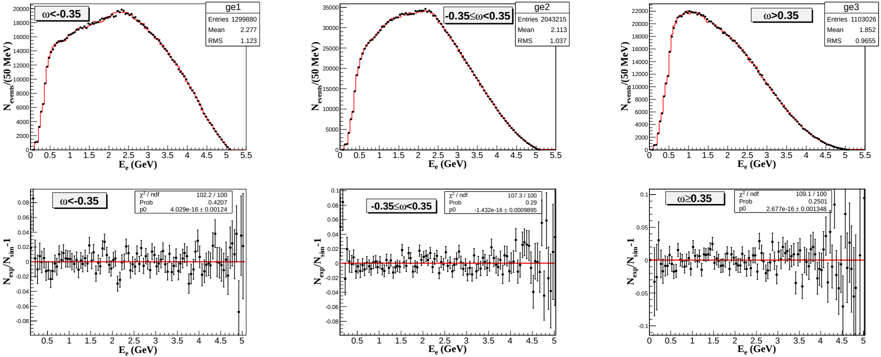




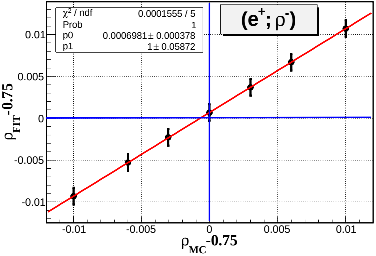


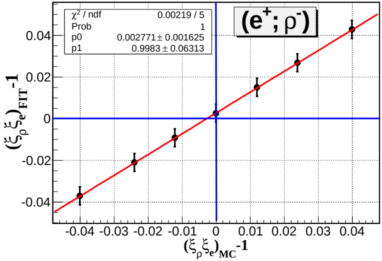
Validation of the fitter with 5M ($e^{\mp}$, $\rho^{\pm}$) MC sample

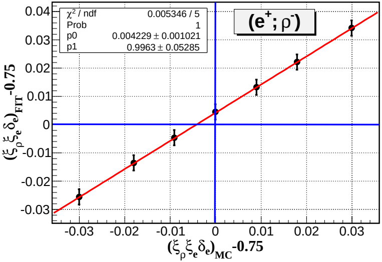
Electron energy spectra for different ω



Study of the fitter bias for (e^+, ρ^-)







Background

$(e^{\mp};\pi^{\pm}\pi^{0})$

Mode	Contribution (%)
$(e^{\mp};\pi^{\pm}\pi^{0})$	88
$(e^{\mp};\pi^{\pm}2\pi^{0})$	10
$(e^{\mp};K^{\pm}\pi^{0})$	
$(e^{\mp};\pi^{\pm}\pi^{0}K_{L})$	2
$(e^{\mp};\pi^{+}\pi^{-}\pi^{\pm}\pi^{0})$	
other	

$(\mu^{\mp};\pi^{\pm}\pi^{0})$

Mode	Contribution (%)
$(\mu^{\mp};\pi^{\pm}\pi^{0})$	88.0
$(\mu^{\mp};\pi^{\pm}2\pi^{0})$	8.1
$(\pi^{\mp};\pi^{\pm}\pi^{0})$	1.4
$(\mu^{\mp};K^{\pm}\pi^{0})$	
$(\mu^{\mp};\pi^{\pm}\pi^{0}K_{L})$	2.5
other	

Background from the non-$\tau\tau$ events is $\lesssim 0.1\%$.

To include background PDF for individual event is written as:

$$\mathcal{P} = (1-\lambda_1-\ldots-\lambda_k)\mathcal{P}^{\text{signal}} + \lambda_1\mathcal{P}_1^{\text{BG}} + \ldots + \lambda_k\mathcal{P}_k^{\text{BG}}$$

The main background modes ($(e^{\mp};\pi^{\pm}2\pi^{0})$, $(\mu^{\mp};\pi^{\pm}2\pi^{0})$, $(\pi^{\mp};\pi^{\pm}\pi^{0})$) are included in the fit in the same way as the signal one, writing PDF explicitly and applying all corrections.

The remaining sources are taken into account using MC-based approach:

D. Schmidt, R. Morrison and M. Witherell, Nucl. Instr. and Meth. A328 (1993) 547.

Likelihood per event

$$\mathcal{P}(x) = (1-\lambda_{3\pi}-\lambda_\pi)\frac{\varepsilon(x)S(x)}{\int\varepsilon(x)S(x)dx} + \lambda_{3\pi}\frac{\varepsilon(x)\int(1-\varepsilon_{\pi^0}(y))\varepsilon_{add}(y)B_{3\pi}(x,y)dy}{\int\varepsilon(x)(1-\varepsilon_{\pi^0}(y))\varepsilon_{add}(y)B_{3\pi}(x,y)dxdy} + \lambda_\pi\frac{\varepsilon_\pi(x)B_\pi(x)}{\int\varepsilon_\pi(x)B_\pi(x)dx}$$

- $x = (p_\ell,\ \Omega_\ell,\ p_\rho,\ \Omega_\rho,\ m^2_{\pi\pi},\ \tilde{\Omega}_\pi)$; $y = (p_{\pi^0},\ \Omega_{\pi^0})$;
- $S(x)$ - density of signal $(\ell^\mp,\ \pi^\pm\pi^0)$ events;
- $B_{3\pi}(x,y)$ - density of background $(\ell^\mp,\ \pi^\pm 2\pi^0)$ events;
- $B_\pi(x)$ - density of background $(\pi^\mp,\ \pi^\pm\pi^0)$ events;
- $\varepsilon(x)$ - detection efficiency for signal events;
- $\varepsilon_{\pi^0}(y)$ - π^0 detection efficiency;
- $\varepsilon_{add}(y)$ - additional efficiency for $(\ell^\mp,\ \pi^\pm 2\pi^0)$ events;
- $\varepsilon_\pi(x)$ - detection efficiency for $(\pi^\mp,\ \pi^\pm\pi^0)$ events;

$$\mathcal{P}(x) = \varepsilon(x)\left((1-\lambda_{3\pi}-\lambda_\pi)\frac{S(x)}{\int\varepsilon(x)S(x)dx} + \lambda_{3\pi}\frac{\tilde{B}_{3\pi}(x)}{\int\varepsilon(x)\tilde{B}_{3\pi}(x)dx} + \lambda_\pi\frac{\tilde{B}_\pi(x)}{\int\varepsilon(x)\tilde{B}_\pi(x)dx}\right)$$

$$\tilde{B}_{3\pi}(x) = \int(1-\varepsilon_{\pi^0}(y))\varepsilon_{add}(y)B_{3\pi}(x,y)dy,\ \tilde{B}_\pi(x) = \frac{\varepsilon_\pi(x)}{\varepsilon(x)}B_\pi(x)$$

D. M. Asner et al. [CLEO] Phys. Rev. D 61 (1999) 012002.

$$J^\mu = \beta_1 j_1^\mu(\rho\pi^0)_{S-wave} + \beta_2 j_2^\mu(\rho'\pi^0)_{S-wave} + \beta_3 j_3^\mu(\rho\pi^0)_{D-wave} + \beta_4 j_4^\mu(\rho'\pi^0)_{D-wave} +$$
$$+\beta_5 j_5^\mu(f_2(1270)\pi)_{P-wave} + \beta_6 j_6^\mu(f_0(500)\pi)_{P-wave} + \beta_7 j_7^\mu(f_0(1370)\pi)_{P-wave}$$

$$\frac{d\Gamma(\tau^\mp \to \pi^\mp 2\pi^0\nu)}{d\Omega^*_{3\pi} dm^2_{3\pi} d\tilde{\Omega}_\rho dm^2_{\pi\pi} d\tilde{\Omega}_\pi} = \kappa_{3\pi}(A' \pm \vec{B'}\vec{\zeta'}^*)W$$

$$A' = H_1 + \xi_{a_1} H_2, \quad \vec{B'} = \xi_{a_1}\vec{G_1} + \vec{G_2}, \quad \xi_{a_1} = 1$$

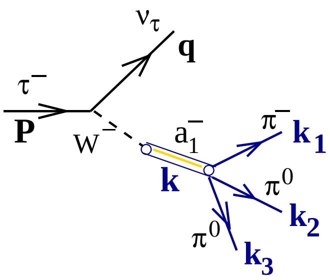
$$W = \frac{p_\pi(m^2_{\pi\pi})}{m_{\pi\pi}} \frac{p_\rho(m^2_{\pi\pi}, m^2_{3\pi})}{m_{3\pi}} \frac{p_{a_1}(m^2_{3\pi})}{M_\tau}$$

$$H_1 = (P, J^*)(q, J) + (P, J)(q, J^*) - (J, J^*)(P, q)$$

$$H_2 = ie^{\mu\nu\sigma\delta} J_\mu J^*_\nu P_\sigma q_\delta$$

$$G_1^\lambda = M_\tau((q, J)J^{*\lambda} + (q, J^*)J^\lambda - (J, J^*)q^\lambda)$$

$$G_2^\lambda = iM_\tau e^{\lambda\mu\nu\sigma} J_\mu J^*_\nu q_\sigma$$



Calculation of $\tilde{B}_{3\pi}(x)$

$$\frac{d\sigma(\vec{\zeta},\vec{\zeta}')}{d\Omega} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i\zeta_j')$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*)\to\ell^\mp\nu\nu)}{dx^*d\Omega_\ell^*} = \kappa_\ell(A(x^*)\mp\xi\vec{n}_\ell^*\vec{\zeta}^*B(x^*)),\ x^* = E_\ell^*/E_{\ell\max}^*$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*),\ B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\Gamma(\tau^\mp\to\pi^\mp 2\pi^0\nu)}{d\Omega_{3\pi}^* dm_{3\pi}^2 d\tilde{\Omega}_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \kappa_{3\pi}(H_1 + \xi_{a_1}H_2 \pm (\xi_{a_1}\vec{G_1} + \vec{G_2}, \vec{\zeta'}^*))W(m_{\pi\pi}^2, m_{3\pi}^2)$$

$$\frac{d\sigma(\ell^\mp,\pi^\pm 2\pi^0)}{dE_\ell^* d\Omega_\ell^* d\Omega_{3\pi}^* dm_{3\pi}^2 d\tilde{\Omega}_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = \kappa_\ell\kappa_{3\pi}\frac{\alpha^2\beta_\tau}{64E_\tau^2}(D_0 A(E_\ell^*)(H_1 + \xi_{a_1}H_2) +$$

$$+B(E_\ell^*)D_{ij}n_{\ell i}^*(\xi_{a_1}G_{1j} + G_{2j}))W(m_{\pi\pi}^2, m_{3\pi}^2)$$

$$\frac{d\sigma(\ell^\mp,\pi^\pm 2\pi^0)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi dp_{\pi^0} d\Omega_{\pi^0}} = \int\limits_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp,\pi^\pm 2\pi^0)}{dE_\ell^* d\Omega_\ell^* d\Omega_{3\pi}^* dm_{3\pi}^2 d\tilde{\Omega}_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau}\times$$

$$\times\left|\frac{\partial(E_\ell^*,\Omega_\ell^*)}{\partial(p_\ell,\Omega_\ell)}\right|\cdot\left|\frac{\partial(\Omega_{3\pi}^*,\Omega_\tau)}{\partial(p_{3\pi},\Omega_{3\pi},\Phi_\tau)}\right|\cdot\left|\frac{\partial(m_{3\pi}^2,\tilde{\Omega}_\rho,p_{3\pi},\Omega_{3\pi})}{\partial(p_\rho,\Omega_\rho,p_{\pi^0},\Omega_{\pi^0})}\right| d\Phi_\tau$$

$$\frac{d\sigma_{obs}(\ell^\mp,\pi^\pm 2\pi^0)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \int\limits_{\text{missed }\pi^0} \frac{d\sigma(\ell^\mp,\pi^\pm 2\pi^0)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi dp_{\pi^0} d\Omega_{\pi^0}} dp_{\pi^0} d\Omega_{\pi^0}$$

$(\pi^\mp,\ \pi^\pm\pi^0)$ events, calculation of $\tilde{B}_\pi(x)$

In the $(\pi^\mp,\ \pi^\pm\pi^0)$ events the direction of τ axis is determined with two-fold ambiguity.

$$\frac{d\sigma(\vec{\zeta},\vec{\zeta}')}{d\Omega} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i\zeta_j')$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*)\to\pi^\mp\nu)}{d\Omega_\pi^*} = \kappa_\pi(1\pm\xi_\pi\vec{n}_\pi^*\vec{\zeta}^*)$$

$$\frac{d\Gamma(\tau^\pm(\vec{\zeta'}^*)\to\rho^\pm\nu)}{dm_{\pi\pi}^2 d\Omega_\rho^* d\tilde{\Omega}_\pi} = \kappa_\rho(A'\mp\xi_\rho\vec{B'}\vec{\zeta'}^*)W(m_{\pi\pi}^2)$$

$$A' = 2(q,Q)Q_0^* - Q^2 q_0^*,\ \vec{B'} = Q^2\vec{K}^* + 2(q,Q)\vec{Q}^*,\ W = |F_\pi(m_{\pi\pi}^2)|^2\frac{p_\rho(m_{\pi\pi}^2)\tilde{p}_\pi(m_{\pi\pi}^2)}{M_\tau m_{\pi\pi}}$$

$$\frac{d\sigma(\pi^\mp,\rho^\pm)}{d\Omega_\pi^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = \kappa_\pi\kappa_\rho\frac{\alpha^2\beta_\tau}{64E_\tau^2}(D_0 A' - \xi_\rho\xi_\pi D_{ij}n_{\pi i}^* B_j'),\ \ \xi_\rho\xi_\pi = 1$$

$$\frac{d\sigma(\pi^\mp,\rho^\pm)}{dp_\pi d\Omega_\pi dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \sum_{\Phi_1,\Phi_2}\frac{d\sigma(\pi^\mp,\rho^\pm)}{d\Omega_\pi^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau}\left|\frac{\partial(\Omega_\pi^*,\Omega_\rho^*,\Omega_\tau)}{\partial(p_\pi,\Omega_\pi,p_\rho,\Omega_\rho)}\right|$$

Efficiency ratio $\varepsilon_\pi(x)/\varepsilon(x) = \varepsilon_{\pi\to\mu}(p_\ell^{LAB},\theta_\ell^{LAB})/\varepsilon_{\mu\to\mu}(p_\ell^{LAB},\theta_\ell^{LAB})$ was tabulated from MC as a function of $(p_\ell^{LAB},\theta_\ell^{LAB})$ and used to get $\tilde{B}_\pi(x)$ for each event.

Validation of the fitter with dominant backgrounds

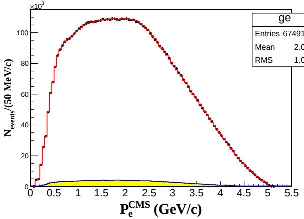
For each configuration 5M MC sample is fitted. The other, statistically independent, 5M MC sample was used to calculate normalization.

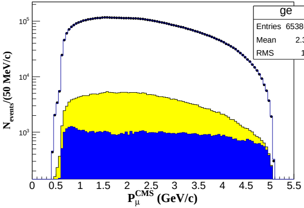
$(e^+; \pi^-\pi^0)$

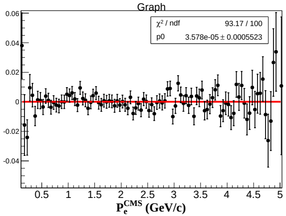
ρ	=	0.7517	±	0.0010
η	=	0	±	0
ξ	=	1.0092	±	0.0043
$\xi\delta$	=	0.7538	±	0.0027

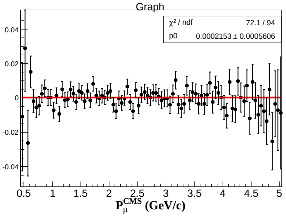
$(\mu^+; \pi^-\pi^0)$

ρ	=	0.7494	±	0.0027
η	=	0.0052	±	0.0101
ξ	=	0.9995	±	0.0050
$\xi\delta$	=	0.7519	±	0.0033



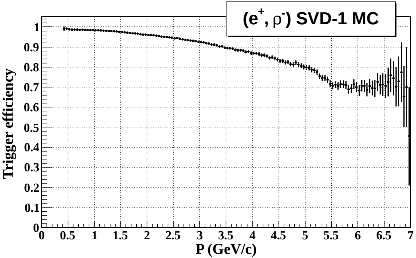




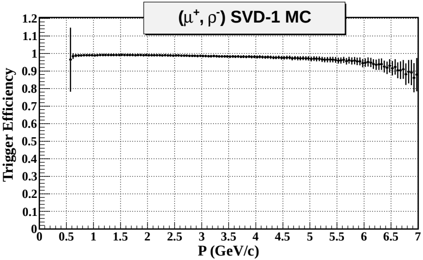


Trigger efficiency

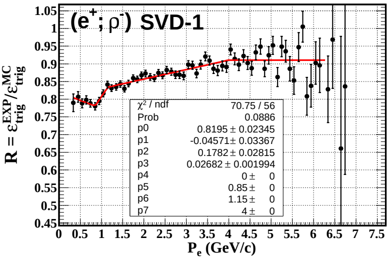
$$\bar{\varepsilon}_{\rm trigger}(e^{\pm},\ \rho^{\mp}) = 93\%$$

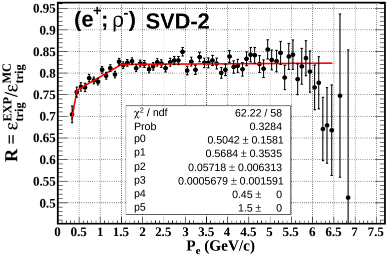


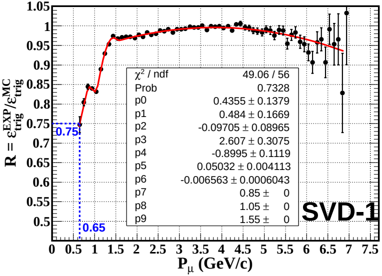
$$\bar{\varepsilon}_{\rm trigger}(\mu^{\pm},\ \rho^{\mp}) = 99\%$$

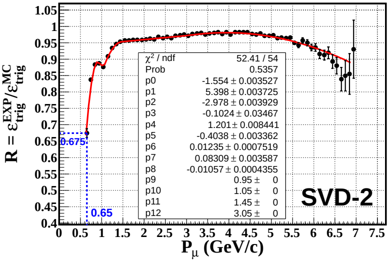


EXP/MC corr. to the trigger eff.

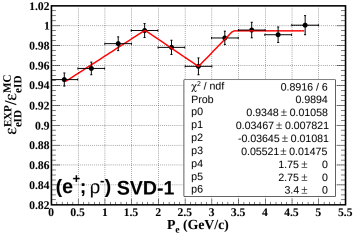


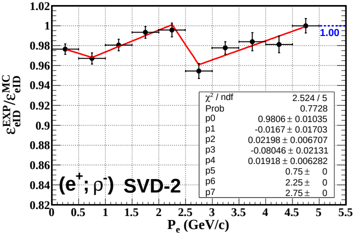


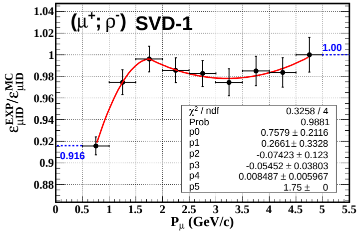


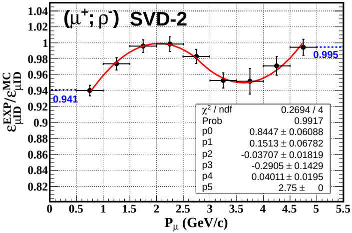


ℓID efficiency corrections





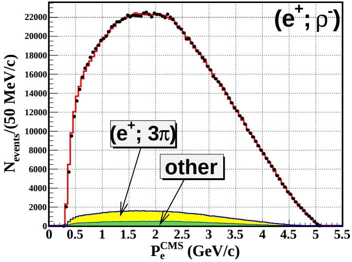


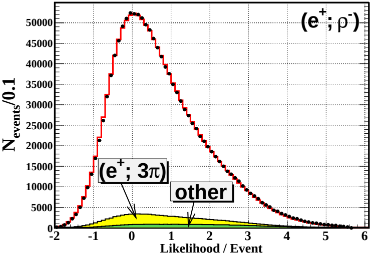


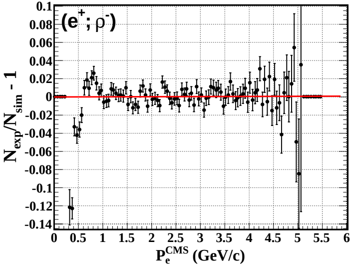
To get ℓID related systematic uncertainty of Michel parameters well below ∼1% level more precise study of eID and μID efficiencies will be needed (using large statistics of $e^+e^- \to e^+e^-\gamma$ and $e^+e^- \to \mu^+\mu^-\gamma$ events).

Fit of experimental (e^+; ρ^-) data

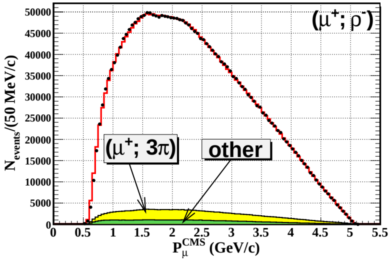
About 7M events of all 4 configurations were selected for the fit.

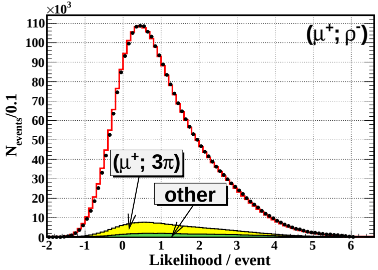


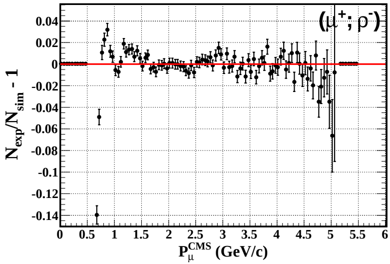




Fit of experimental (μ^+; ρ^-) data





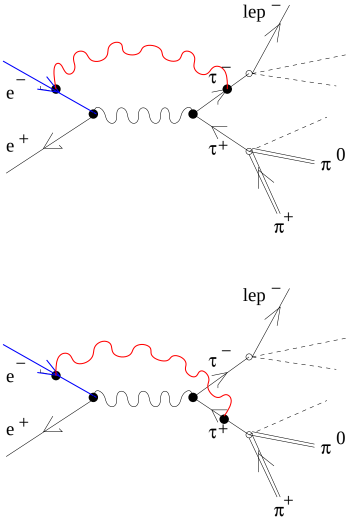


Summary

- The procedure to measure 4 Michel parameters (ρ, η, ξ, $\xi\delta$) in leptonic τ decays has been developed and tested.
- The trigger and lepton identification EXP/MC efficiency corrections were determined and taken into account in the procedure to fit experimental data.
- The first fit of experimental data ($\sim$7M events) has been done.
- Systematic uncertainty of about few percent is seen for both (e; ρ) and (μ; ρ) events.
- Study of systematic effects is going on.

Backup slides

Validity of the factorization



Contribution of the right diagrams violating factorization ansatz is additionally suppressed by a factor of $\frac{\Gamma_\tau}{M_\tau} \sim 10^{-12}$.

Preselection of $\tau\tau$ events

- $2 \leq N_{trk} \leq 8$
- $|Q_{total}| \leq 2$
- $P_{\perp\ max}^{LAB} > 0.5$ GeV/c
- Event vertex $|R| < 1.0$ cm, $|Z| < 3.0$ cm
- For $N_{trk} = 2$:
  - $\sum_{i=1}^{N_{clusters}} E_i^{LAB}(ECL) < 11$ GeV
  - $5^o < \theta_{missing}^{LAB} < 175^o$
- $E_{rec} = \sum_{i=1}^{N_{trk}} |\vec{P_i}|^{CMS} + \sum_{j=1}^{N_\gamma} |\vec{K_j}|^{CMS} > 3$ GeV/c OR $P_{\perp\ max}^{LAB} > 1.0$ GeV/c
- If $2 \leq N_{trk} \leq 4$:
  - $E_{tot} = E_{rec} + |\sum_{i=1}^{N_{trk}} \vec{P_i}^{CMS} + \sum_{j=1}^{N_\gamma} \vec{K_j}^{CMS}| <$ 9 GeV/c OR Maximum opening angle$< 175^o$ OR 2 GeV$<$ $\sum_{i=1}^{N_{clusters}} E_i^{LAB}(ECL) < 10$ GeV
  - $N_{barrel} \geq 2$ OR $\sum_{All\ clusters} E^{CMS} - \sum_{photons} E_\gamma^{CMS} < 5.3$ GeV

- Standard preselection.
- Additional criteria ($\varepsilon \simeq 46\%$):
  - $2 \le \mathrm{N_{tracks}} \le 4$ ($\mathrm{P_\perp^{CMS}} > 0.1$ GeV/c, $|\triangle r| < 0,5$ cm, $|\triangle z| < 2.5$ cm)
  - $|\mathrm{Q_{total}}| \le 1$
  - $\mathrm{N_\gamma} \le 5$ ($\mathrm{E_\gamma^{CMS}} > 0.08$ GeV)
  - $\sum_{i=1}^{\mathrm{N_{clusters}}} \mathrm{E_i^{LAB}}(\mathrm{ECL}) < 9$ GeV
  - $1\ \mathrm{GeV/c^2} \le \mathrm{M_{missing}} \le 7\ \mathrm{GeV/c^2}$
  - $30^o \le \theta^{\mathrm{CMS}}_{\mathrm{missing}} \le 150^o$
- We select events with two oppositely charged tracks, one of them is identified as lepton (eID, μID > 0.9) and the other one as pion (PID(π/K) > 0.4).
- π^0 candidate is reconstructed from the pair of gammas ($\mathrm{E_\gamma^{LAB}} > 80$ MeV) satisfying $115\ \mathrm{MeV/c^2} < \mathrm{M_{\gamma\gamma}} < 150\ \mathrm{MeV/c^2}$.
- $\cos(\vec{\mathrm{P}}_{\mathrm{lep}}, \vec{\mathrm{P}}_\pi) > 0$, $\cos(\vec{\mathrm{P}}_{\mathrm{lep}}, \vec{\mathrm{P}}_{\pi^0}) > 0$.
- $0.3\ \mathrm{GeV/c^2} < \mathrm{M_{\pi\pi^0}} < 1.8\ \mathrm{GeV/c^2}$, $\mathrm{P_{\pi^0}^{CMS}} > 0.3$ GeV/c
- $\mathrm{E_\gamma^{LABextra}} < 0.1$ GeV (ρ side) and $\mathrm{E_\gamma^{LABextra}} < 0.1$ GeV (lepton side)

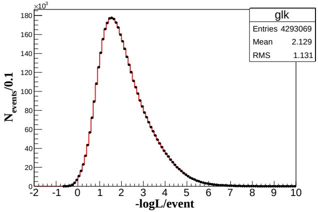
Detection efficiency $\varepsilon_{\mathrm{det}} \simeq 12\%$

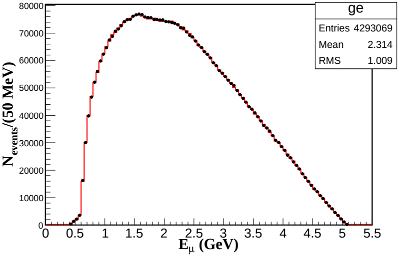
Validation of the fitter with MC

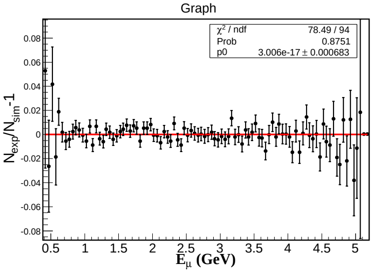
- From the total $\tau\tau$ generic MC data sample we selected about 4×15M events of all configurations $(\ell^{\mp},\ \rho^{\pm})$ $(\ell = \mathrm{e},\ \mu)$. We expect to have ∼100M selected MC events for each configuration (in total 4×100M events), which is large enough to keep related systematic uncertainty several times smaller than the expected experimental statistical error of MP.
- For each configuration 10M MC data sample has been splitted into two statistically independent 5M parts. The first subsample is used to calculate normalization. And the second one is fitted to extract MP.
- To check biases of the fitter we reweighted initial MC sample (generated with SM values of MP) to get series of samples with shifted MP. For each MP we produced samples with $\pm 3\sigma$, $\pm 6\sigma$ and $\pm 10\sigma$ shifted MP values. During the fit all 4 MP were free.

Fit of the 5M $(\mu^{\mp}, \rho^{\pm})$ MC sample

$(\mu^+; \pi^-\pi^0)$				
ρ	=	0.7468	$\pm$	0.0026
η	=	-0.0083	$\pm$	0.0101
ξ	=	0.9933	$\pm$	0.0050
$\xi\delta$	=	0.7501	$\pm$	0.0032

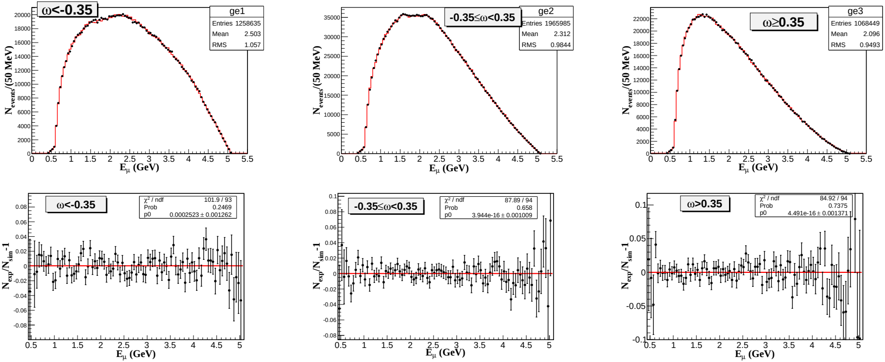






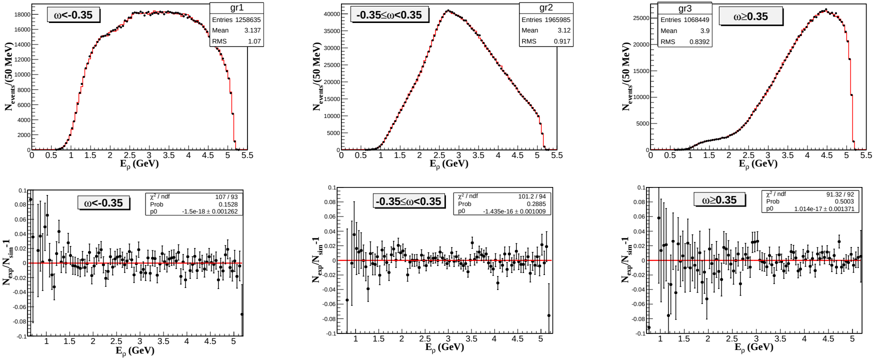
Fit of the 5M ($\mu^{\mp}$, $\rho^{\pm}$) MC sample

Muon energy spectra for different ω

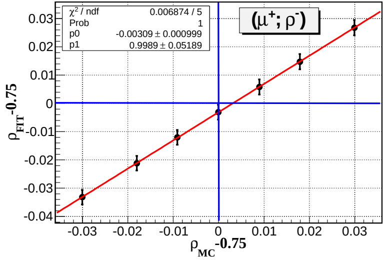


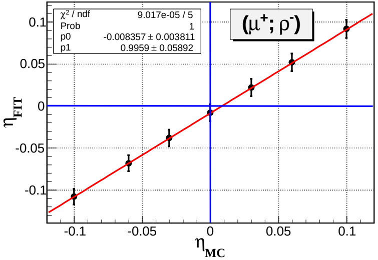
Fit of the 5M ($\mu^{\mp}$, $\rho^{\pm}$) MC sample

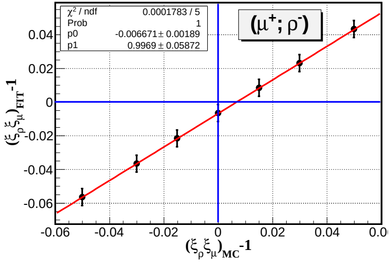
ρ energy spectra for different ω

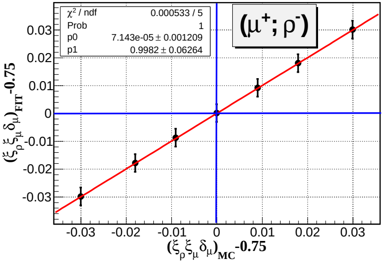


Study of fitter bias for (μ^+, ρ^-)



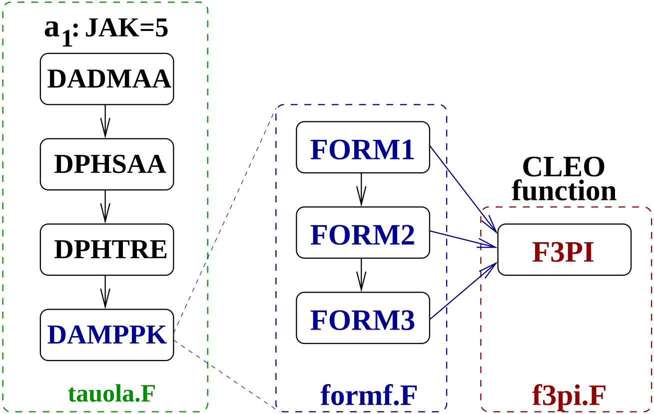






$\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$ in TAUOLA

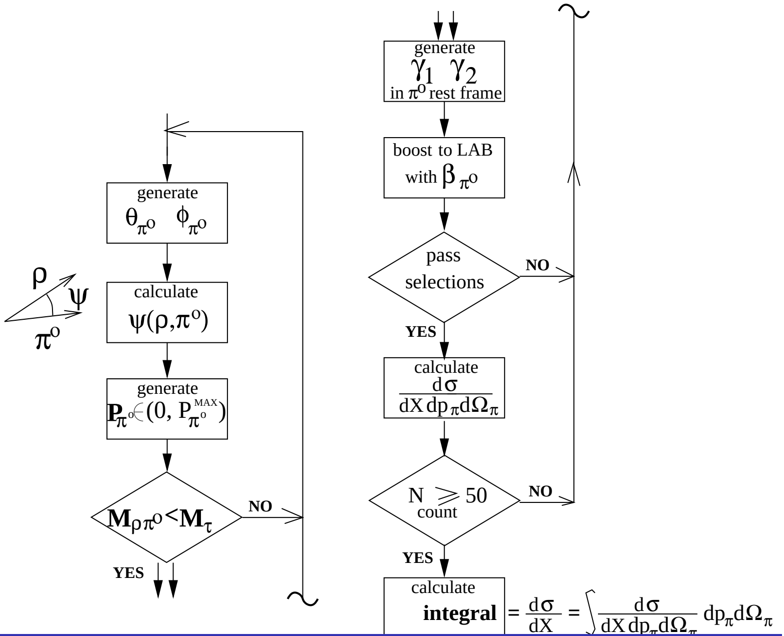
D. M. Asner et al. [CLEO] Phys. Rev. D 61 (1999) 012002.



$$J^\mu = \beta_1 j_1^\mu(\rho\pi^0)_{S-wave} + \beta_2 j_2^\mu(\rho'\pi^0)_{S-wave} + \beta_3 j_3^\mu(\rho\pi^0)_{D-wave} + \beta_4 j_4^\mu(\rho'\pi^0)_{D-wave} +$$

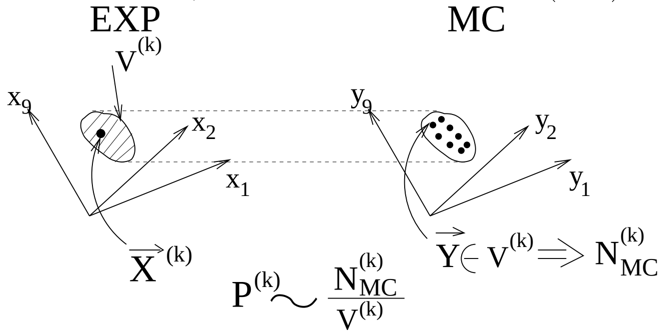
$$+\beta_5 j_5^\mu(f_2(1270)\pi)_{P-wave} + \beta_6 j_6^\mu(f_0(500)\pi)_{P-wave} + \beta_7 j_7^\mu(f_0(1370)\pi)_{P-wave}$$

Calculation of $\tilde{B}_{3\pi}(x)$



"Schmidt et al." method for the remaining background

D. Schmidt et al., Nucl. Instr. and Meth. A328 (1993) 547.



$$\mathcal{P}^{(k)} = \frac{N_{MC}^{sel\ (k)}/V^{(k)}}{N_{MC}^{sel\ TOT}},\ k = 1 \div N_{exp}$$

Volume around experimental point can be chosen large enough to calculate likelihood function precisely, but should be kept at the level, where the nonlinearity of the likelihood function within the volume is still small.
Technically bulky because of the $\sim$ 10M $\times$ 10M double cycle.

Description of the remaining background

We use 5D phase subspace: p_ℓ, $\cos\theta_\ell$, p_ρ, $\cos\theta_\rho$, $m^2_{\pi\pi}$ (distributions over φ_ℓ, φ_ρ, $\tilde{\Omega}_\pi$ are uniform) to evaluate likelihood function:

Background likelihood component

$$\mathcal{P}_{\rm other}(x_i) = \frac{N^{\rm sel}_{\rm other}(x_i)/V_i}{N^{\rm sel\ TOT}_{\rm other}}$$

$$\mathcal{L}_{\rm other}(x_i) = \frac{\bar{\varepsilon}}{\varepsilon(x_i)}\mathcal{P}_{\rm other}(x_i), \quad \frac{\bar{\varepsilon}}{\varepsilon(x_i)} = \frac{N^{\rm sel\ TOT}_{\rm signal}}{N^{\rm sel}_{\rm signal}(x_i)}\frac{N^{\rm gen}_{\rm signal}(x_i)}{N^{\rm gen\ TOT}_{\rm signal}}$$

$$\mathcal{L}_{\rm other}(x_i) = \frac{1-\lambda_{3\pi}-\lambda_\pi-\lambda_{\rm other}}{\lambda_{\rm other}}\frac{N^{\rm sel}_{\rm other}(x_i)}{N^{\rm sel}_{\rm signal}(x_i)} \times p_{\rm signal}(x_i)$$

$$\text{Binning}: \ p_{\rm signal}(x_i) = \frac{1}{V_i}\frac{N^{\rm gen}_{\rm signal}(x_i)}{N^{\rm gen\ TOT}_{\rm signal}}, \quad \text{"Schmidt et al."}: \ p_{\rm signal}(x_i) = S(x_i)$$

Total likelihood per event

$$\mathcal{P}(x) = \frac{\varepsilon(x)}{\bar{\varepsilon}}\left((1-\sum_i \lambda_i)\frac{S(x)}{\int \frac{\varepsilon(x)}{\bar{\varepsilon}}S(x)dx} + \lambda_{3\pi}\frac{\tilde{B}_{3\pi}(x)}{\int \frac{\varepsilon(x)}{\bar{\varepsilon}}\tilde{B}_{3\pi}(x)dx} + \lambda_\pi\frac{\tilde{B}_\pi(x)}{\int \frac{\varepsilon(x)}{\bar{\varepsilon}}\tilde{B}_\pi(x)dx} + \lambda_{\rm other}\mathcal{L}_{\rm other}(x)\right)$$