# Calculation of the Vacuum Polarization

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# Vacuum polarization

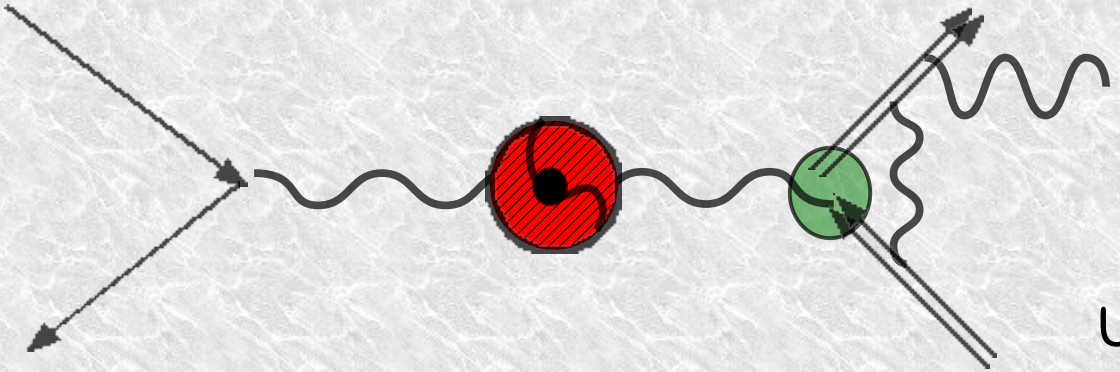
- Used in calculation of radiative correction
- to calculate "bare" cross-section
  used in some dispersion integral
- to calculate $\alpha(M_Z^2)$
  which is used in predictions of electroweak model
  the least known EW parameter from $G_\mu$, $M_z$, $\alpha(M_Z^2)$



# “Bare” & “dressed”

Vacuum polarization – sum of all loop correction for the photon propagator

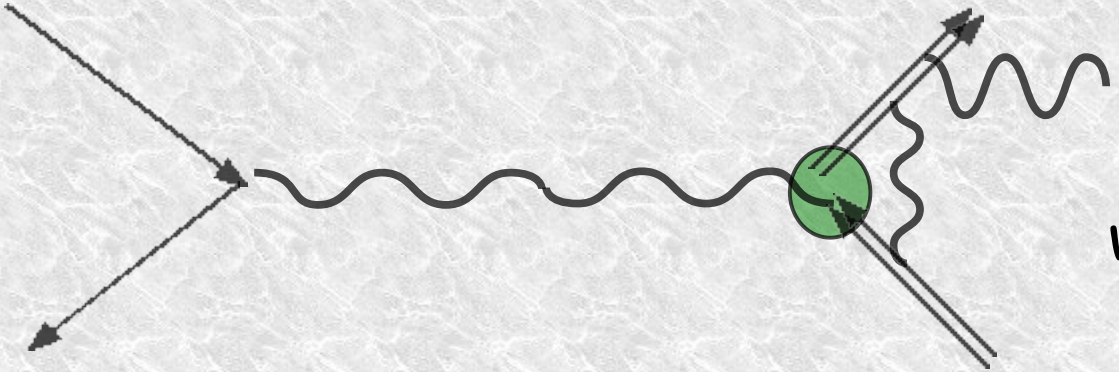
$$e^+ e^- \to \gamma * \to \text{everything}$$



“dressed” cross sections - measured cross sections

Used in hadron spectroscopy

(to get rho-meson mass,width, …)



“bare” cross sections - without vacuum polarization

used in R

for some dispersion integral



# Vacuum polarization

How has P(s) been calculated in past ?

- Well known analytical expressions for lepton Born cross sections were used. Today it is not enough to get P(s) with 0.05% accuracy.
- Experimental values for hadronic cross sections were used to evaluate hadronic contribution to P(s). It was done early a little not correctly!

How lepton and hadron parts of P(s)
should be calculated to improve it's accuracy?

- It is necessary to take into account all first order in a QED corrections. Two effects – namely: FSR and Coulomb interaction in final state should be incorporated into Born cross sections (it was done early for electrons only).
- For hadronic cross sections it is enough to do for $\pi^+\pi^-$ and $K^+K^-$ channels only. They gives dominant contribution at low energies.
- To obtain hadronic Born cross section VP effects must be removed from "dress" cross section.



# Vacuum polarization

VP can be calculated from dispersion relation, based on analyticity and unitarity

$$P(s),\Pi(s)=\frac{s}{4\pi^2\alpha}\left[PV\int_{4m_\pi^2}^{\infty}\frac{\sigma^{bare,dressed}_{ee\to\gamma\to everything}(s')ds'}{s-s'}-i\sigma^{bare,dressed}_{ee\to\gamma\to everything}(s)\right]$$

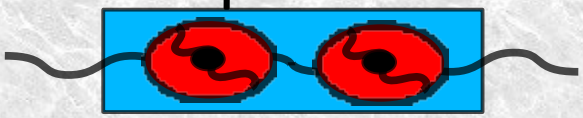
$$P(s)=P_{leptons}(s)+P_{hadrons}(s)$$

"bare" cross sections — result in — "dressed" cross sections

Polarization operator

$$\frac{1}{1-P(s)}=1+P+P^2+\ldots=1+\Pi(s)$$

Self-energy function of photon



It is necessary

- "undress" experimental cross sections
- nontrivial task to correct physical parameters of resonances

- "dress" leptonic part and pQCD
- "dress" experimental data which was normalized to 2 muons cross sections

Iterations is necessary in all case

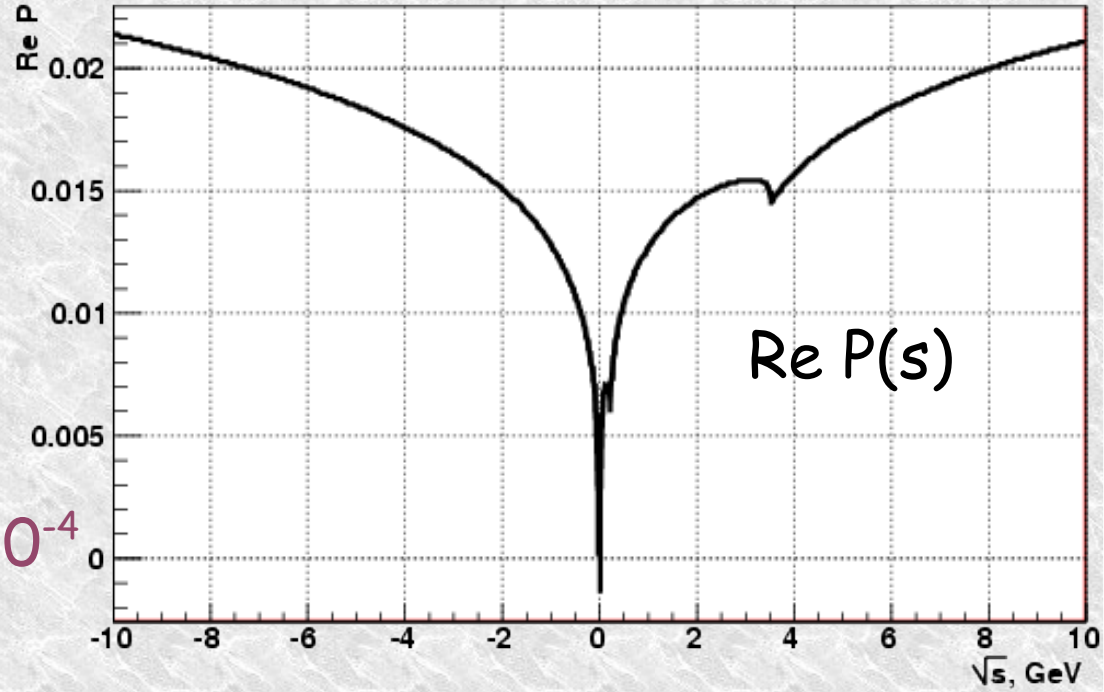


# Leptonic contribution

$$P_l(s)=\frac{\alpha}{\pi}P_1(s)+\left(\frac{\alpha}{\pi}\right)^2 P_2(s)+\ldots$$

Exact analytical formula are known for first order, for second order at $|s|\gg m_l^2$

Contribution of second order ~ $10^{-4}$




At final, the contribution was evaluated from dispersion integral with inclusion of FSR, Coulomb factor and vacuum polarization:

$$\sigma^{\text{dress}}_{ee\to\gamma^*\to ll(\gamma)}=\sigma^{\text{Born}}_{ee\to\gamma^*\to ll}\left(1+\frac{2\alpha}{\pi}\delta_l^{\text{FSR}}-\frac{z}{2}\right)f(z)|1+\Pi(s)|^2,\quad \delta_l^{\text{FSR}}\to 3/8$$

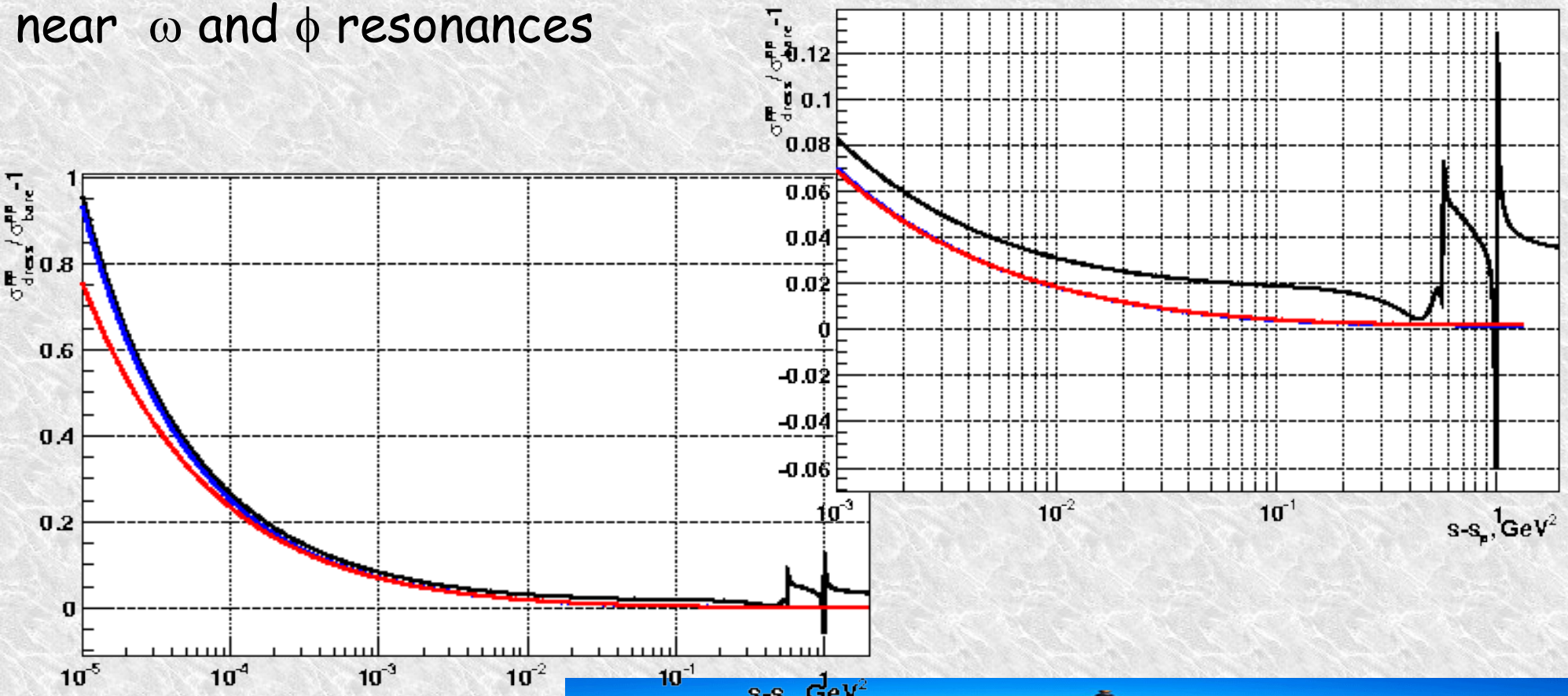
Vacuum polarization was included iteratively



# Leptonic contribution

- ✗ with final state radiation
- ✗ in addition exact Coulomb interaction
- ✗ with vacuum polarization ("dressed" cross sections)

Muons cross section are deviated by ~ 6% and ~25% near $\omega$ and $\phi$ resonances



# narrow resonances

$$J/\psi(1S),\psi(2S),\Upsilon(1S,2S,3S,4S,10860,11020)$$

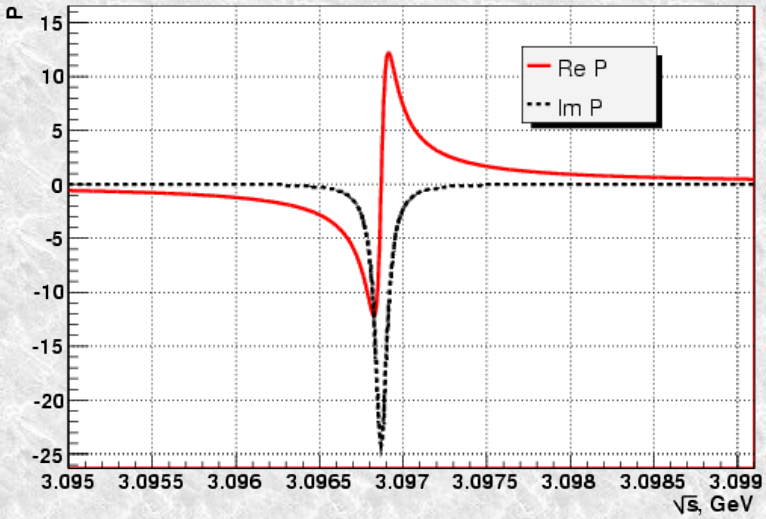
$$\sigma_x^{res}(s)=\frac{12\pi\,Br_x^{e^+e^-}}{m_x^2}\frac{m_x^2\Gamma_x^2}{|s-m_x^2-i\,m_x\Gamma_x|^2} \Rightarrow \Pi_x^{res}(s)=\frac{3Br_x^{e^+e^-}}{\alpha}\frac{s}{m_x^2}\frac{m_x\Gamma_x}{s-m_x^2+i\,m_x\Gamma_x}$$

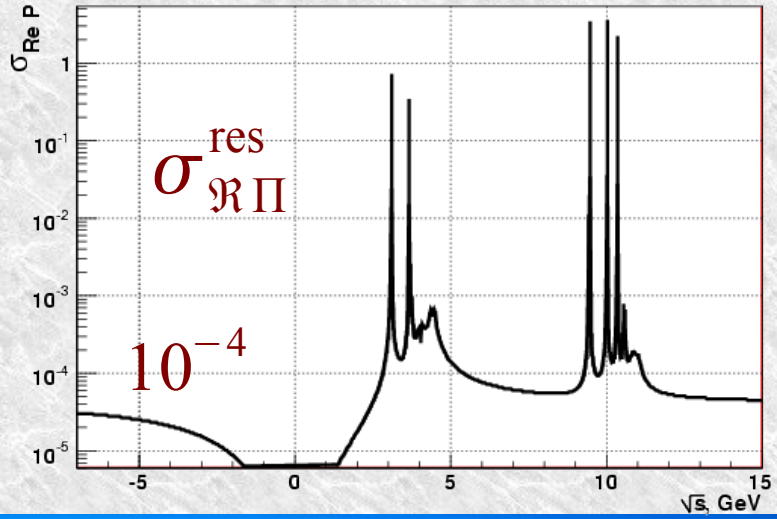
$$\delta m_x = m_x^{dress}-m_x^{bare}\approx\frac{3\Gamma_x^{ee}}{2\alpha}\Re\frac{1}{1+\Pi^{not\ res}(m_x^2)}$$

$$\delta\Gamma_x\approx\Gamma_x^{ee}R^{not\ res}|1+\Pi^{not\ res}(m_x^2)|^2$$

$$\Gamma_x^{ee\ bare}\approx\frac{\Gamma_x^{ee\ dress}}{|1+\Pi^{not\ res}(m_x^2)|^2}$$

The difference between "dress" and "bare" masses for $\phi$ and $J/\psi$ mesons are 254 keV (253 from fitting) and 1117 keV. It is much more than experimental precision.






# hadronic contribution

- As Input: It was used all available experimental data

$$e^+ e^- \to \text{hadrons}$$

(except of old data, which are inconsistent with new data)

ACO, BABAR, BBar, BCF, BES, CBALL, CMD, CMD2, DASP, DM1, DM2, FENICE, GG2, LENA, M3N, MARK2, MD1, MEA, MUPI, ND, OLYA, PLUTO, SND, SPEAR, TOF

- Some old experiments was corrected for vacuum polarization, when they used only leptonic contribution in VP (in luminosity determination from ee or in hadrons cross sections itself) (OLYA, CMD etc. )
- It was used exclusive channels for energies less than 2 GeV
- from isospin relations: $\sigma(K\bar{K}\pi\pi)=2\sigma(K^+K^-\pi\pi)$, $\sigma(K_L\pi K)=\sigma(K_s\pi K)$, $\sigma(K\bar{K}\pi^0)=2\sigma(K^+K^-\pi^0)$
- $\sigma(\pi^0\pi^0\gamma)$ from $\sigma(\omega\pi^0)$, $\sigma(3\pi\gamma)$ from $\sigma(\omega\pi^+\pi^-)$ and $\sigma(\eta\pi^+\pi^-)$
- For s>10 GeV it was used pQCD



# Averaging Data from Different Experiments

- Integration was done by trapezoidal method. Integration of one dataset have form:

$$\Pi_{k\,exp}(s) = A\sum_{\sigma_i - \text{experimental point}} \sigma_i\left(C_i(s) - i\pi D_i(s)\right)$$

- Than you can calculate error matrix for needed energy range and some experiment $\langle \delta\Pi(s)\delta\Pi(s)\rangle$

- Integration region was divided on intervals, which are correspond to energy range of some experiments or intersection between different experiments

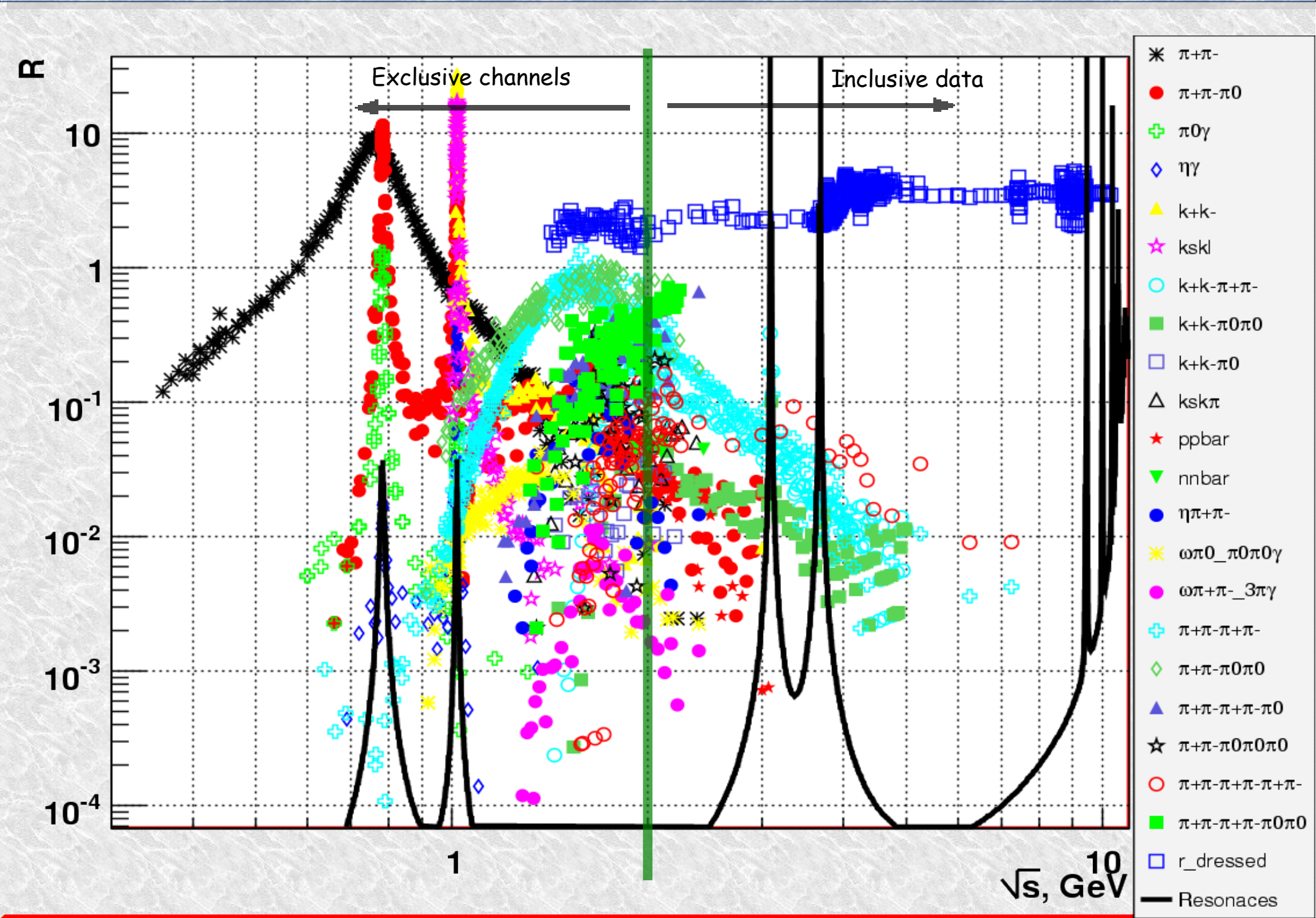
- Averaging of different experiments was done by minimization of (G.D'Agostini NIM A346(1994)306):

$$\chi^2 = \frac{\sum_{k=1}^{N_{exp}}\sum_i \left[P_{i,k} - (1+\lambda_k\alpha_{i,k})P_i^{fit}\right]^2}{\sigma_{i,k}^2(stat)} + \sum_{k=1}^{N_{exp}}\lambda_k^2, \quad \alpha_{i,k} = \sigma_{i,k}(syst)/P_i^k$$

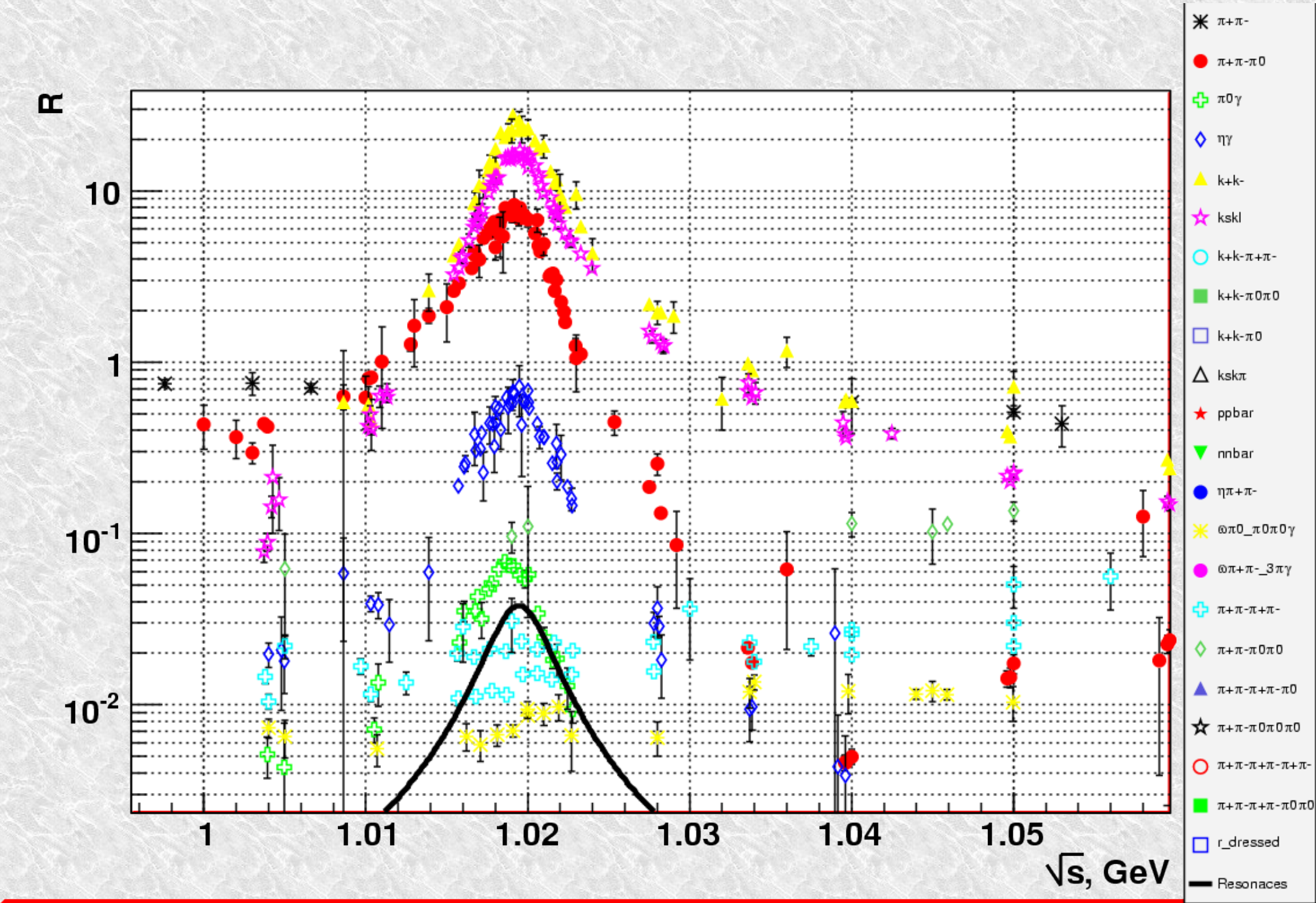
$i-\text{interval}, k-\text{experiment}$



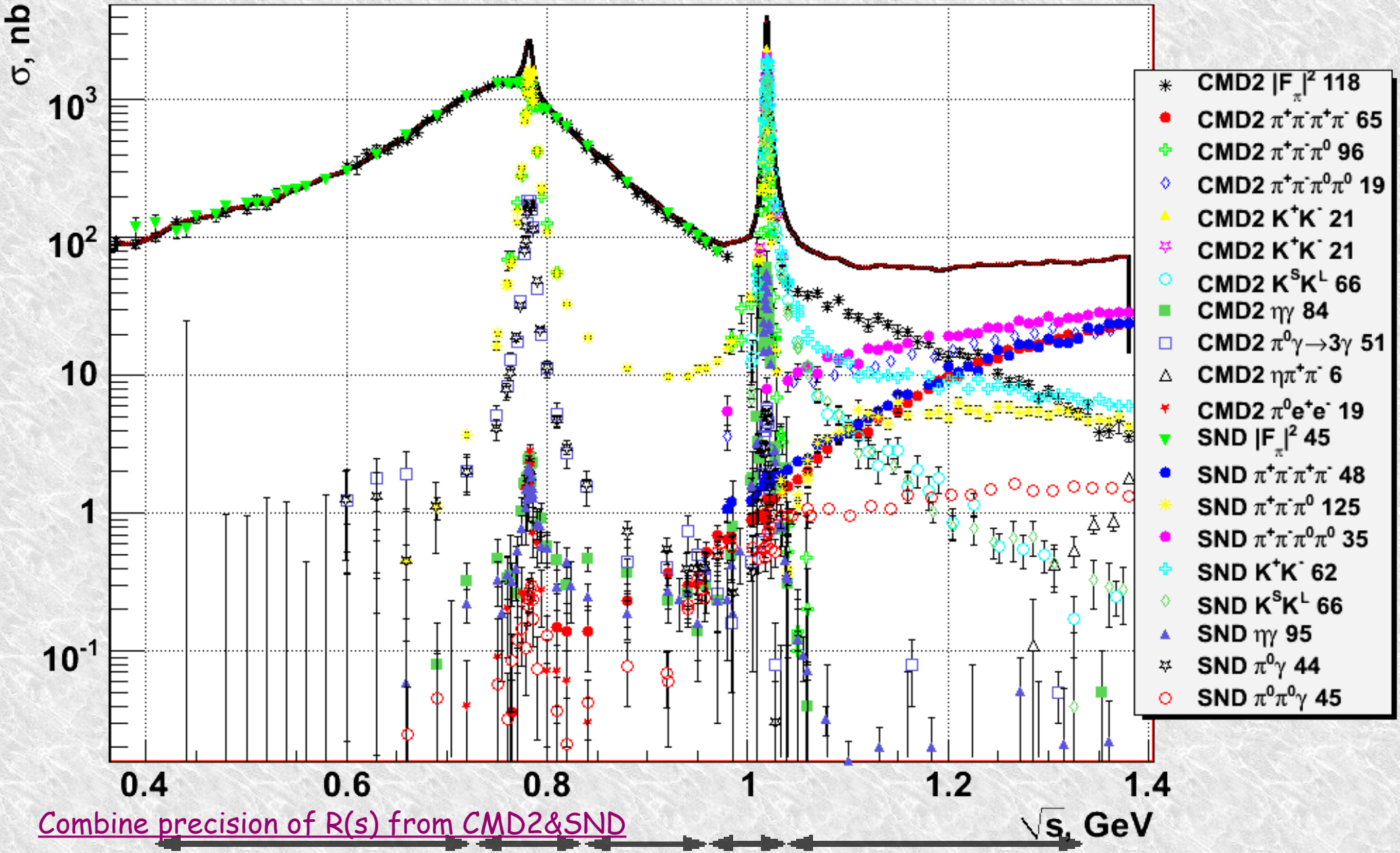
# All experimental data



# Φ meson


R
10
1
$10^{-1}$
$10^{-2}$
1
1.01
1.02
1.03
1.04
1.05
√s, GeV
π+π-
π+π-π0
π0γ
ηγ
k+k-
kskl
k+k-π+π-
k+k-π0π0
k+k-π0
kskπ
ppbar
nnbar
ηπ+π-
ωπ0_π0π0γ
ωπ+π-_3πγ
π+π-π+π-
π+π-π0π0
π+π-π+π-π0
π+π-π0π0π0
π+π-π+π-π+π-
π+π-π+π-π0π0
r_dressed
Resonaces

# Inclusive Hadronic Cross-Sections with CMD2&SND




Combine precision of R(s) from CMD2&SND

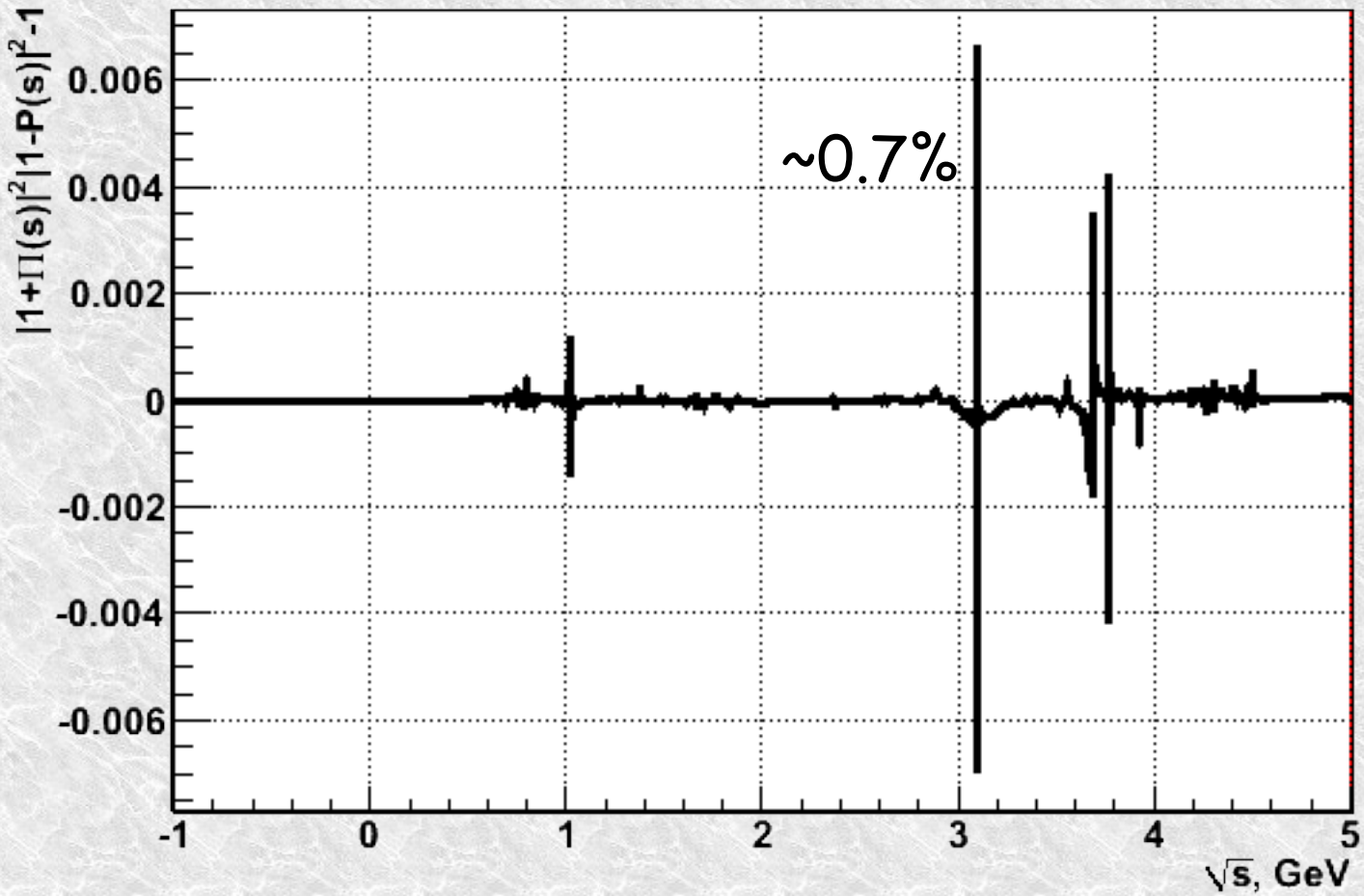
| | | | | | |
|---|---|---|---|---|---|
| Systematic error: | ~0.6-0.7% | 1.0% | 0.6% | 1.5% | 1.5 -- 3.5 % |
| Total error: | ~ 6 -- 1% | 1.5% | 1--2% | 2.0% | 2.5 -- 3.5 % |



# Bare & Dressed

Comparison of calculated vacuum polarization with "bare" and "dress" cross sections

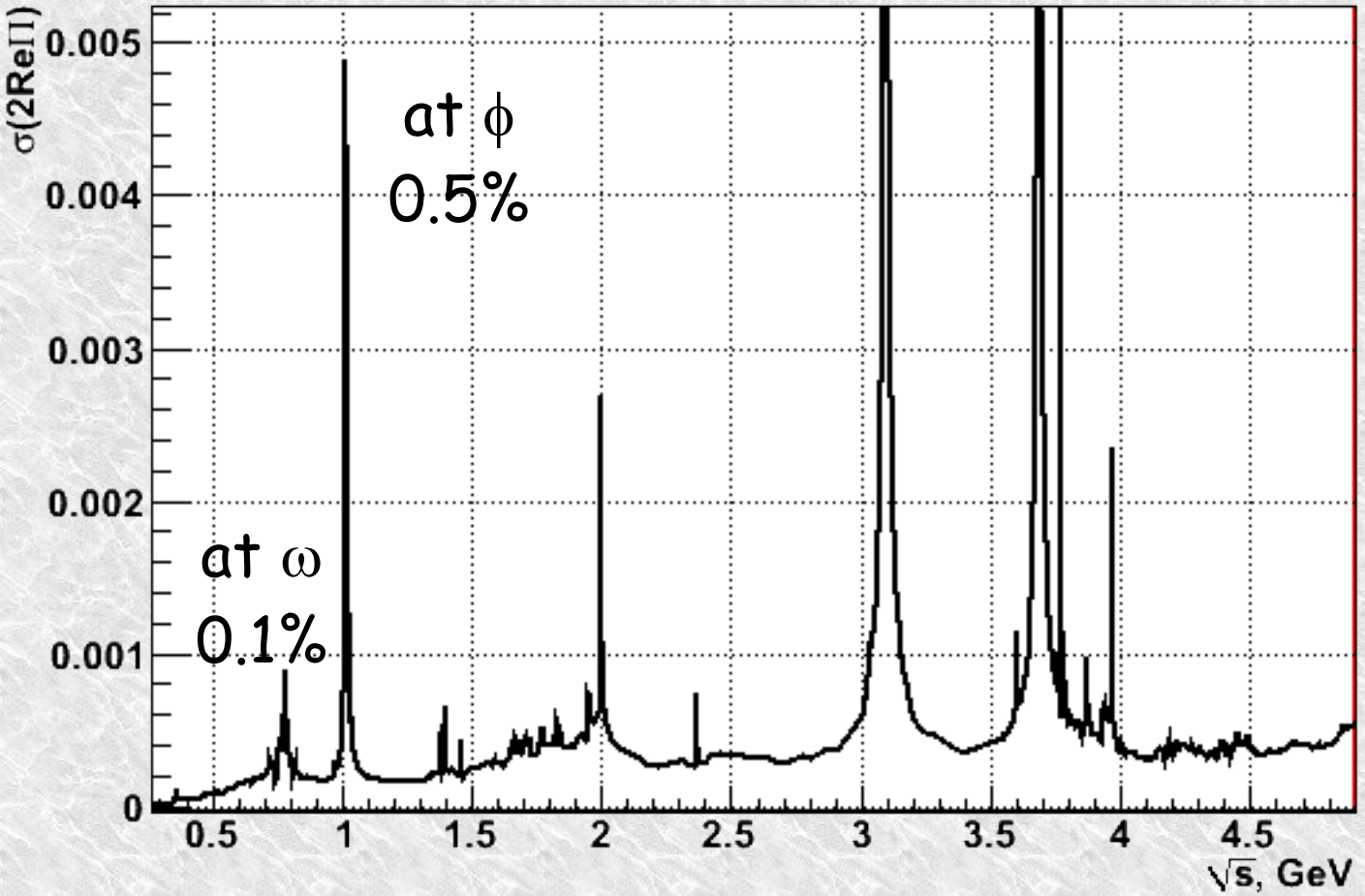
$$|1+\Pi(s)|^2|1-P(s)|^2-1$$






# Precision of Vacuum Polarization

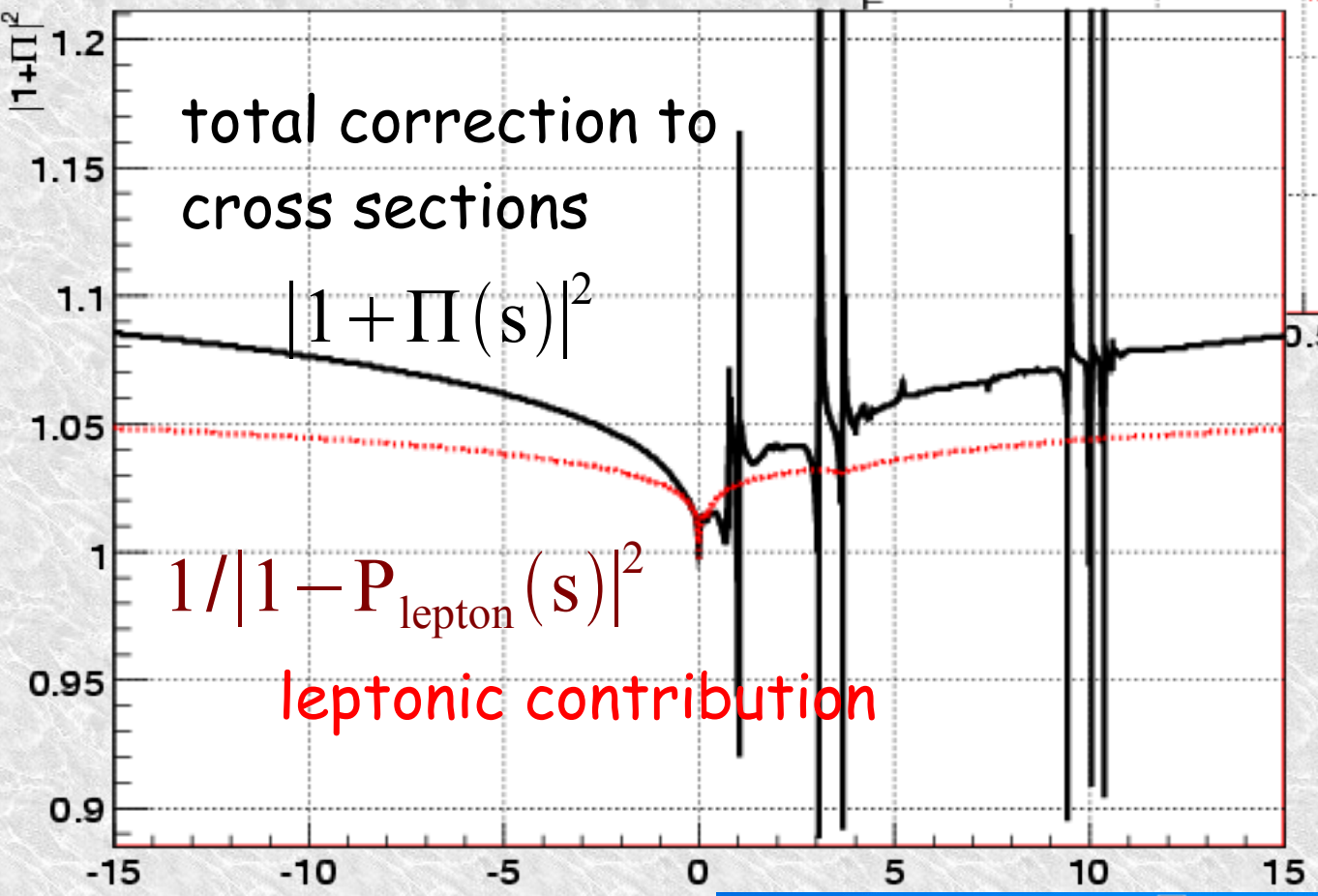
Except of resonances, statistical precision is ~ 0.05%

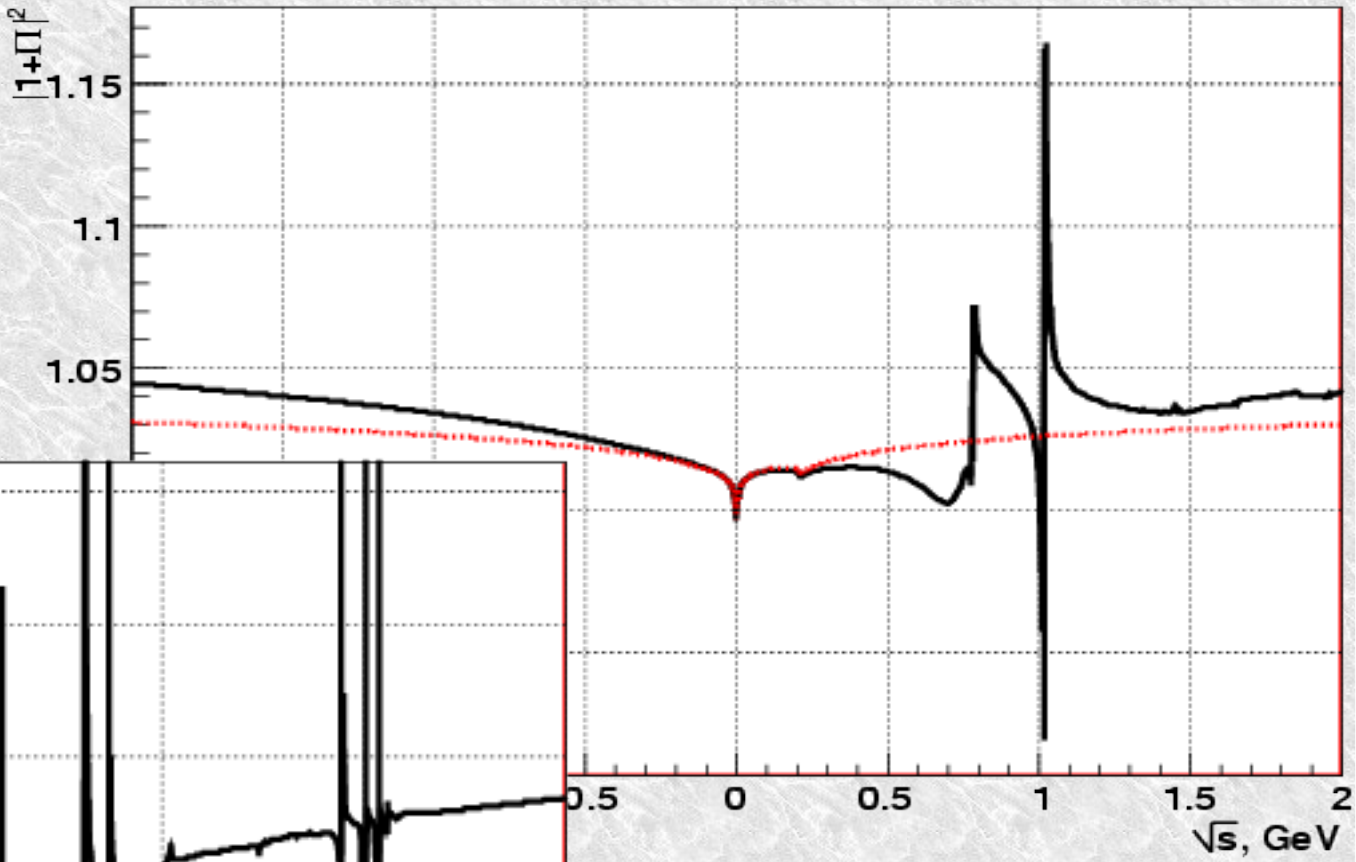



0.1%



# Vacuum Polarization






# Summary

- For processes e+e− → e+e− + γ , e+e− → μ+μ− + γ , e+e− → τ+τ−+ γ, e+ e− → π+π− + γ, K+K-+γ total cross section are calculated with first order α corrections.
- Coulomb interaction in FS taken into account correctly without expansion in series α/v (integration from threshold).
- Hadron contribution to operator $\Pi(s)$ is calculated using "dress " cross sections
- "Bare" cross sections with first order α corrections are required for different dispersion evaluations (VP is removed).
- Analytical expression for narrow resonance contribution to operator $\Pi(s)$ is derived in compact form.
- Vacuum polarization effects in propagator of virtual photon was evaluated with accuracy better than 0.05 % in VEPP-2000 energy region
- $\Delta\alpha(M_Z^2), 10^{-4} = 592.17 \pm 1.38$ from bare, $592.26 \pm 1.45$ from dress

K.Hagiwara et al., hep-ph/0611102v3

591.1 +- 1.7 with inclusive at 1.43 - 2 GeV

(from previous paper exclusive higher by 1.4)



# Bare & Dressed