



Изучение процесса электрон-позитронной аннигиляции в четыре пиона

Евгений Козырев
BINP, NSU
e.a.kozyrev@inp.nsk.su
20.06.2019

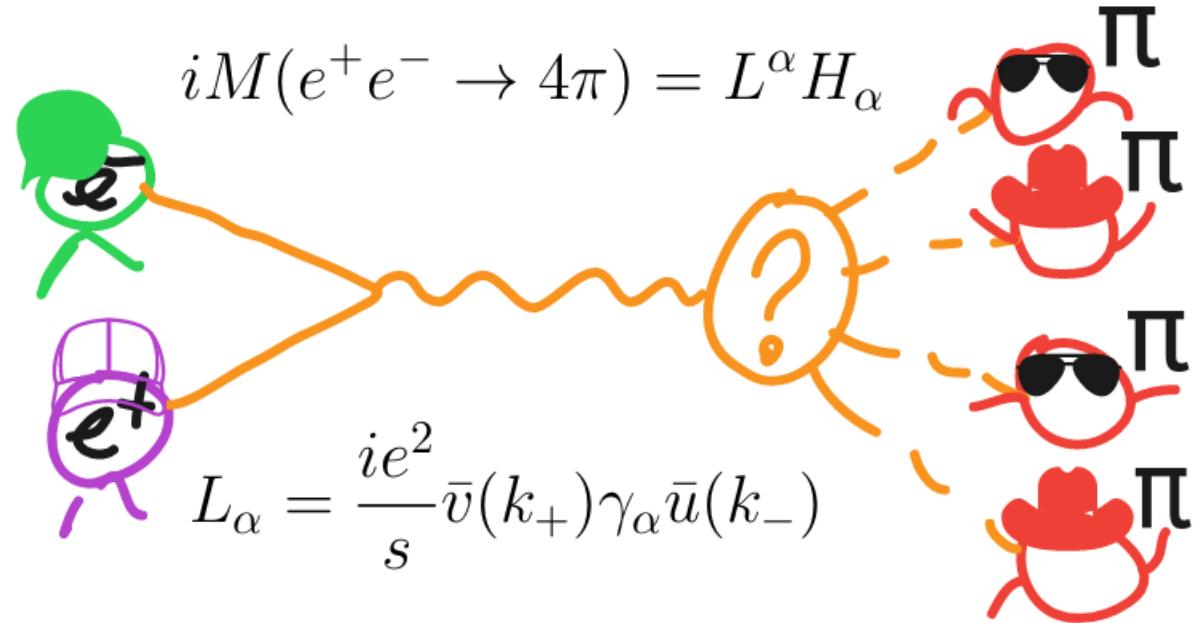
Научный руководитель
д.ф.-м.н. гл.н.с. ИЯФ СО РАН
Солодов Евгений Петрович

Оглавление

- Введение, цель работы
- История данного исследования
- Амплитудный анализ (*unbinned likelihood amplitude analysis*)
- Сравнение с экспериментом
- Сечение процесса
- Заключение

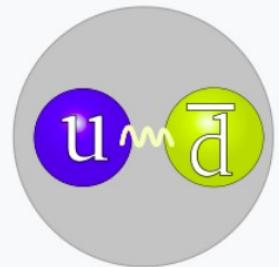
Цель работы

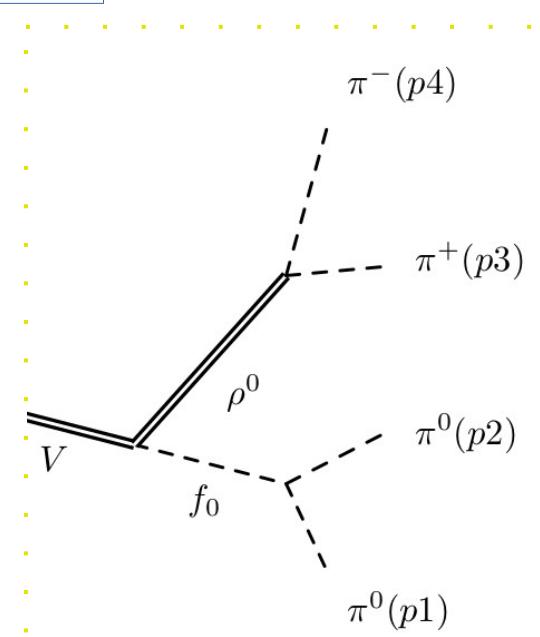
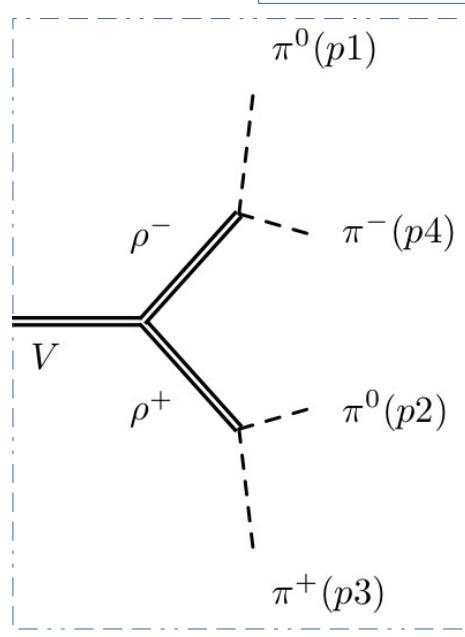
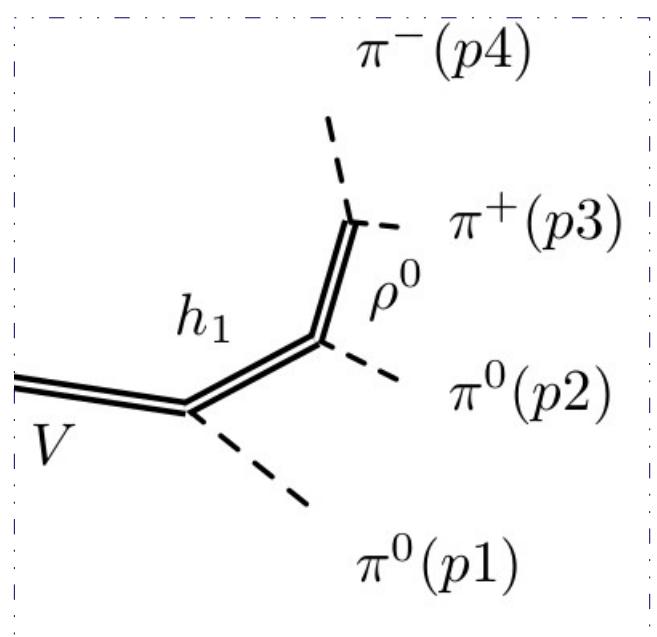
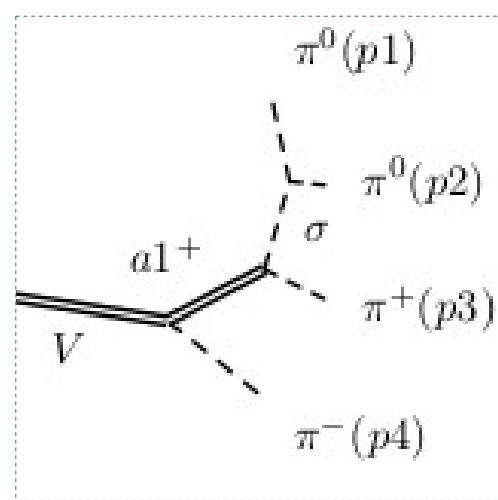
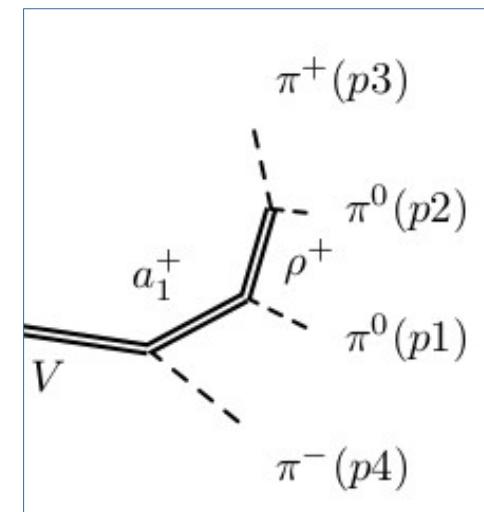
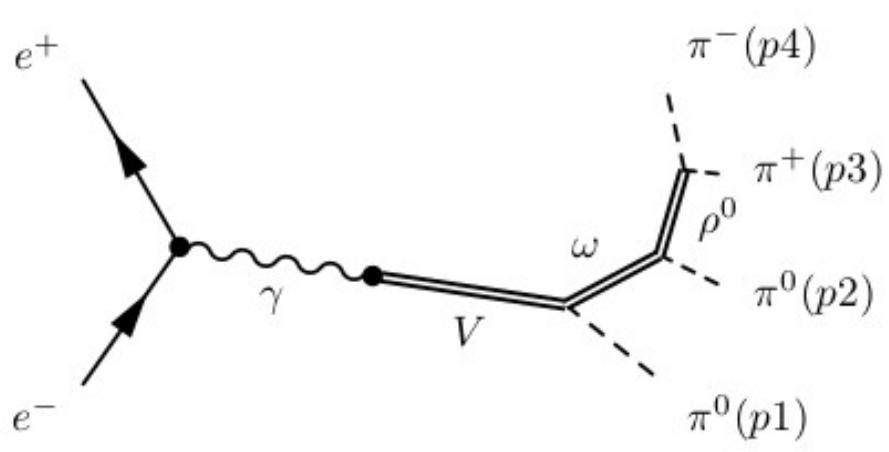
- Построить модель промежуточной динамики
- Померить сечение



$$\sigma = \frac{N}{L} - ?$$

Пион (π^\pm (π^0))

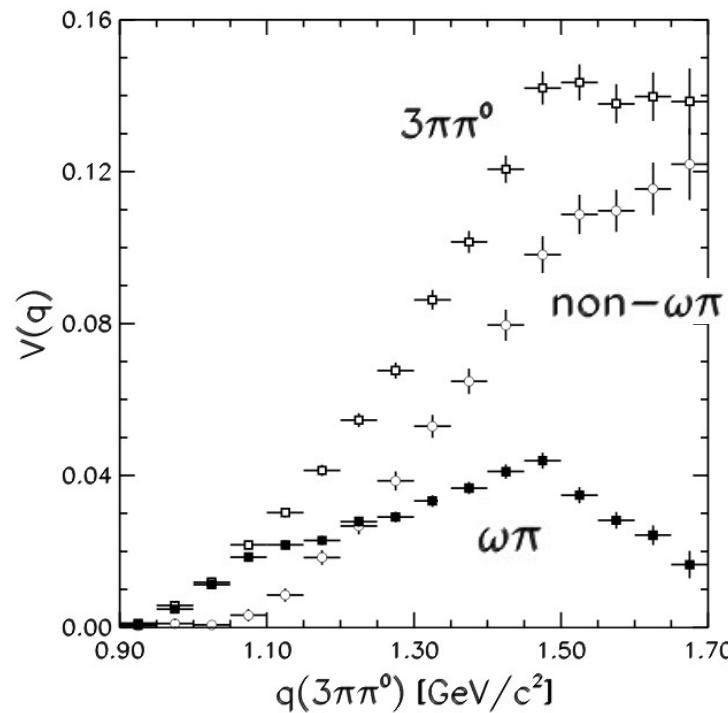




Амплитудный анализ с детектором КМД-2 (5.8 pb^{-1})

- The data in the $ee \rightarrow \pi^+\pi^-2\pi^0$ (22128 events) with $\sqrt{s} = [1.05-1.38] \text{ GeV}$ is used
 - The **dominance of the $\omega\pi$ and $a_1\pi$** is proved
 - The data in $ee \rightarrow \pi^+\pi^-2\pi^0$ and $ee \rightarrow 2\pi^+2\pi^-$ (28552) is used for the estimation: $B(a_1 \rightarrow \omega\pi)/B(a_1 \rightarrow \rho\pi) \sim 0.3$
 - The measured cross section are systematically shifted from other measurement.
- The results of a search for the admixture of other possible states
- | Model | L_{\min} | $r_X [\%]$ | U.L. [%] |
|---------------------------------------|------------|---------------------|----------|
| $\omega\pi^0 + a_1\pi$ | 1264 | — | — |
| $\omega\pi^0 + a_1\pi + \rho\sigma$ | 1256 | $2.1^{+1.2}_{-0.9}$ | 4.3 |
| $\omega\pi^0 + a_1\pi + h_1\pi$ | 1263 | $0.1^{+0.2}_{-0.1}$ | 0.4 |
| $\omega\pi^0 + a_1\pi + a_2\pi$ | 1263 | $0.2^{+0.4}_{-0.2}$ | 0.8 |
| $\omega\pi^0 + a_1\pi + \pi'\pi$ | 1250 | $9.5^{+3.2}_{-2.8}$ | 15. |
| $\omega\pi^0 + a_1\pi + \rho^+\rho^-$ | 1246 | $4.7^{+2.0}_{-1.6}$ | 7.7 |
- R.R. Akhmetshin et al., Physics Letters B **466**, 392–402 (1999)*

Амплитудный анализ процесса $\tau \rightarrow 3\pi\pi^0\nu_\tau$ с детектором CLEO (1999)



$$\frac{\Gamma(\tau^- \rightarrow \nu_\tau 2\pi^- \pi^+ \pi^0)}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = \frac{3 \cos^2 \theta_c}{2\pi \alpha^2 m_\tau^8} \int_0^{m_\tau^2} dQ^2 Q^2 (m_\tau^2 - Q^2)^2 (m_\tau^2 + 2Q^2) \cdot \left[\frac{1}{2} \sigma_{e^+ e^- \rightarrow 2\pi^- 2\pi^+}(Q^2) + \sigma_{e^+ e^- \rightarrow \pi^+ \pi^- 2\pi^0}(Q^2) \right]$$

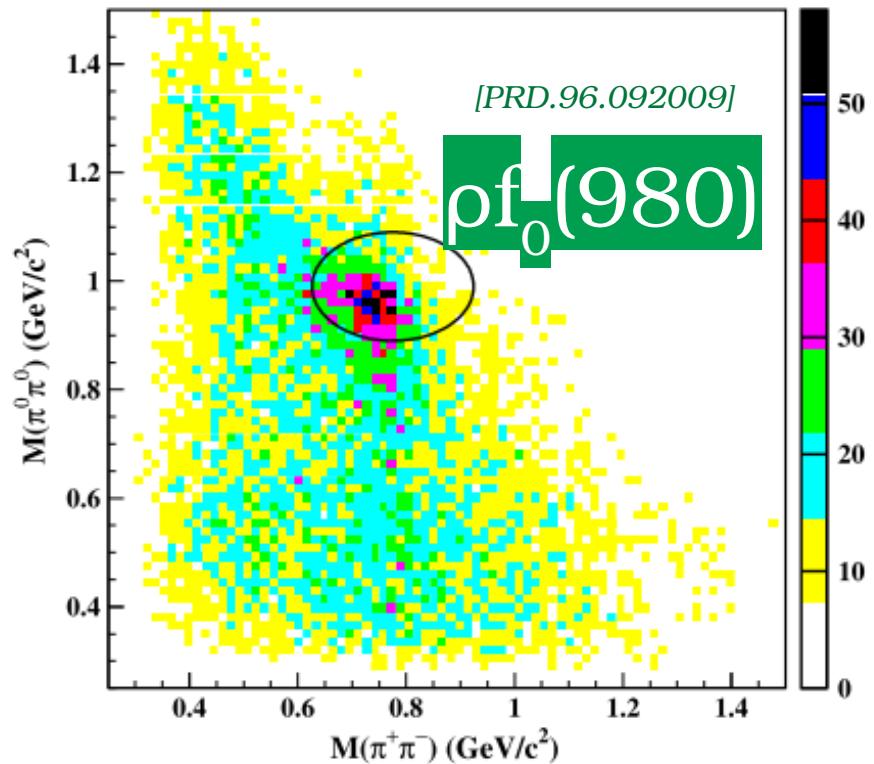
Model	Integrated amplitudes	Goodness-of-fit
Model 2	$R_{\omega\pi} = 0.38 \pm 0.02 \pm 0.02$	< 5%
	$R_{a_1\pi} = 0.43 \pm 0.02 \pm 0.02$	
Model 3	$R_{\omega\pi} = 0.38 \pm 0.02 \pm 0.01$	20%
	$R_{a_1\pi} = 0.49 \pm 0.02 \pm 0.02$	
	$R_{\sigma\rho} = 0.01 \pm 0.02 \pm 0.01$	
	$R_{f_0\rho} = 0.01 \pm 0.01 \pm 0.01$	

Fit results for various models

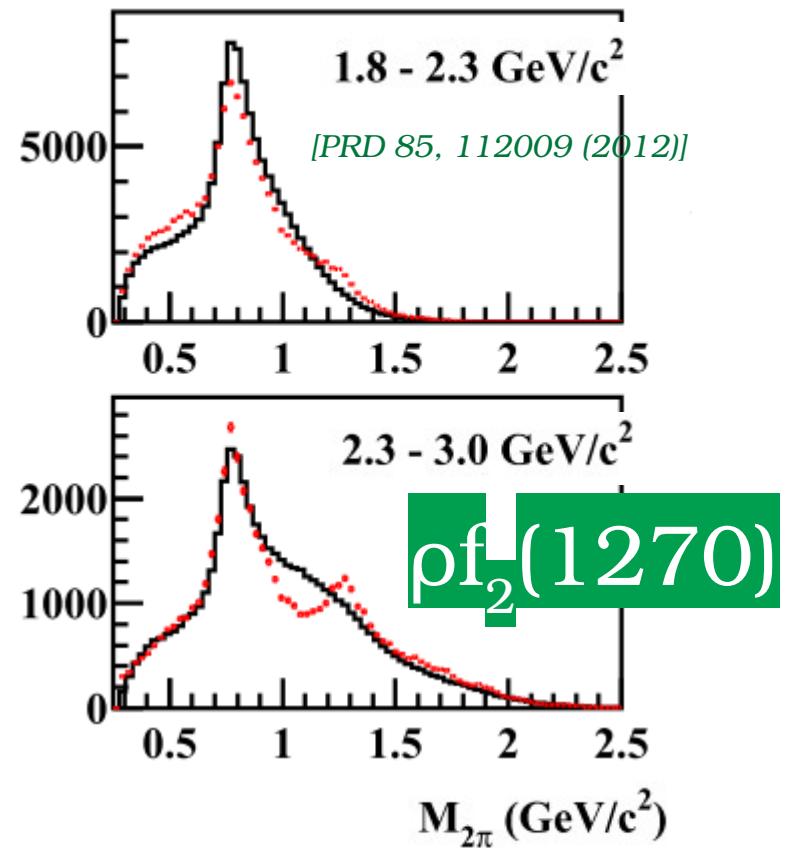
- Model with $\omega\pi$ and $a_1\pi$ and $\rho\sigma$, ρf_0 provides the best description of the data.

Physical Review D - Particles, Fields, Gravitation and Cosmology, **61**, 1-16 (2000).

Наблюдение состояний ρf_0 и ρf_2 с детектором BaBar при $E_{\text{с.м.}} > 1.8 \text{ ГэВ}$

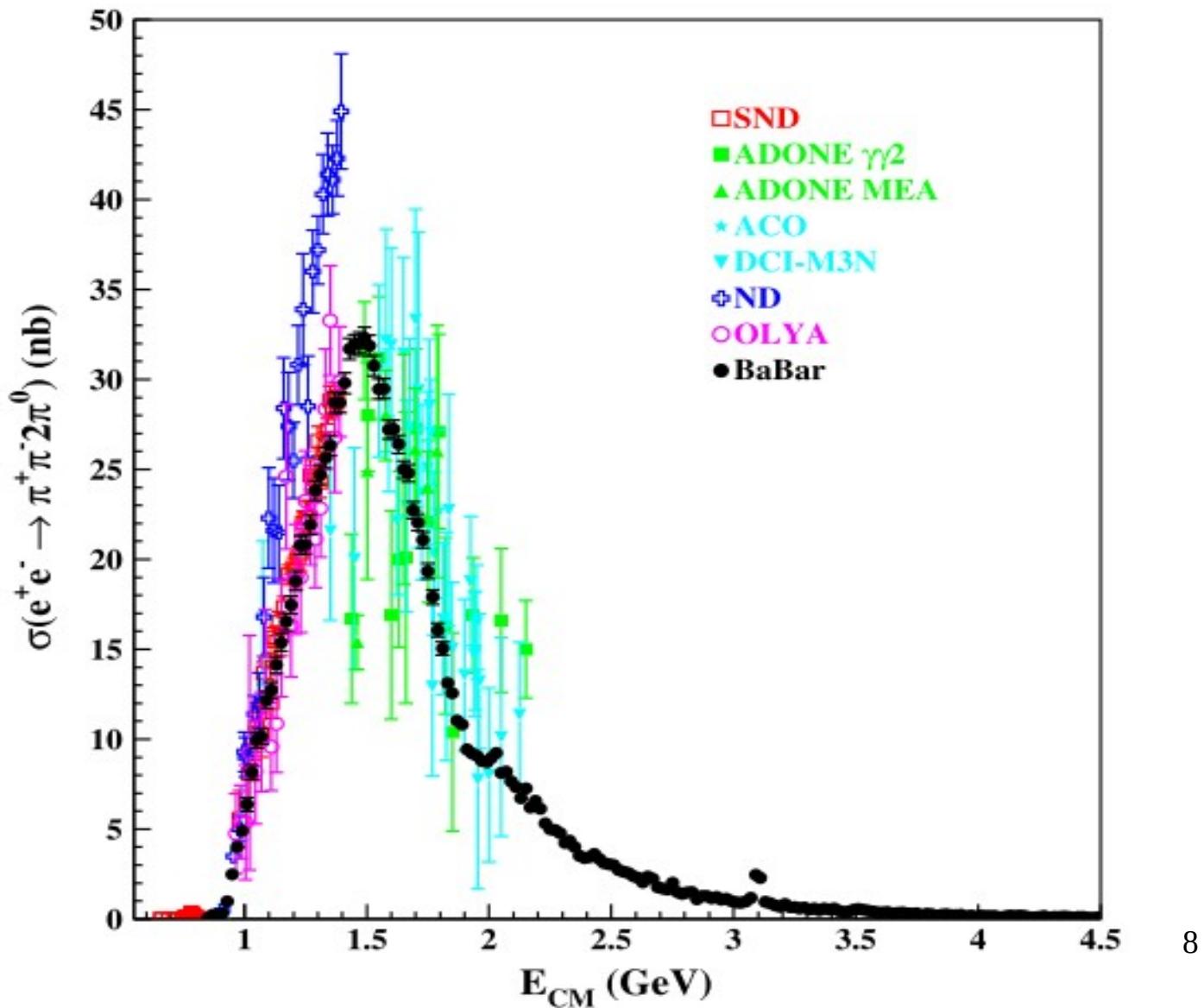


The evidence of $\rho f_0(980)$ in the process $e^+e^- \rightarrow 2\pi^0\pi^-\pi^+$ with BaBar



The evidence of $\rho f_2(1270)$ in the process $e^+e^- \rightarrow 2\pi^+\pi^-$ with BaBar

Измерения
 $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ на
других
экспериментов



Общая стратегия амплитудного анализа

Signal selection

$(ee \rightarrow \pi^+\pi^- 2\pi^0)$

Signal selection

$(ee \rightarrow 2\pi^+ 2\pi^-)$

Building of the amplitudes formalism

**The definition and minimization
of likelihood function (L)**

Model vs Experiment comparison

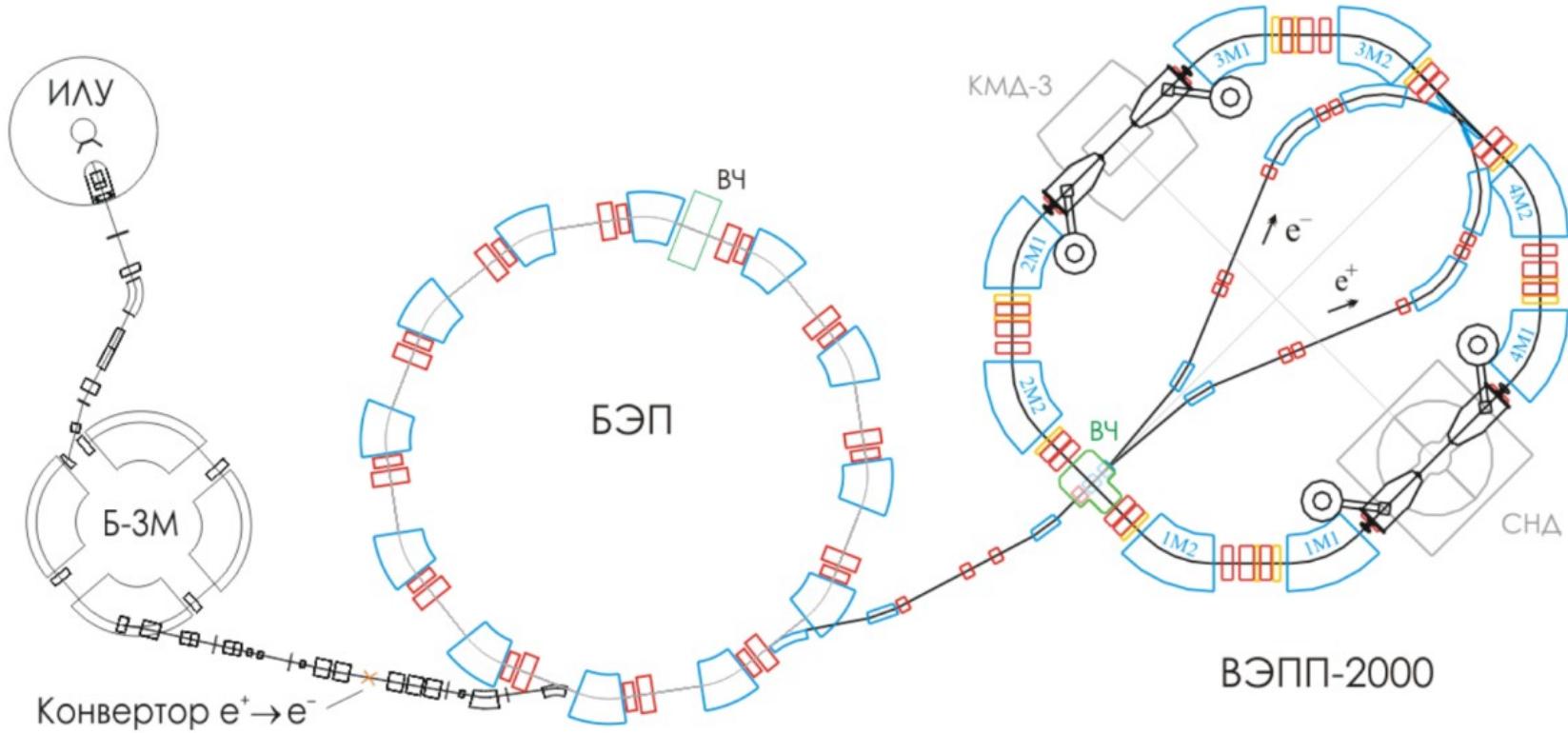
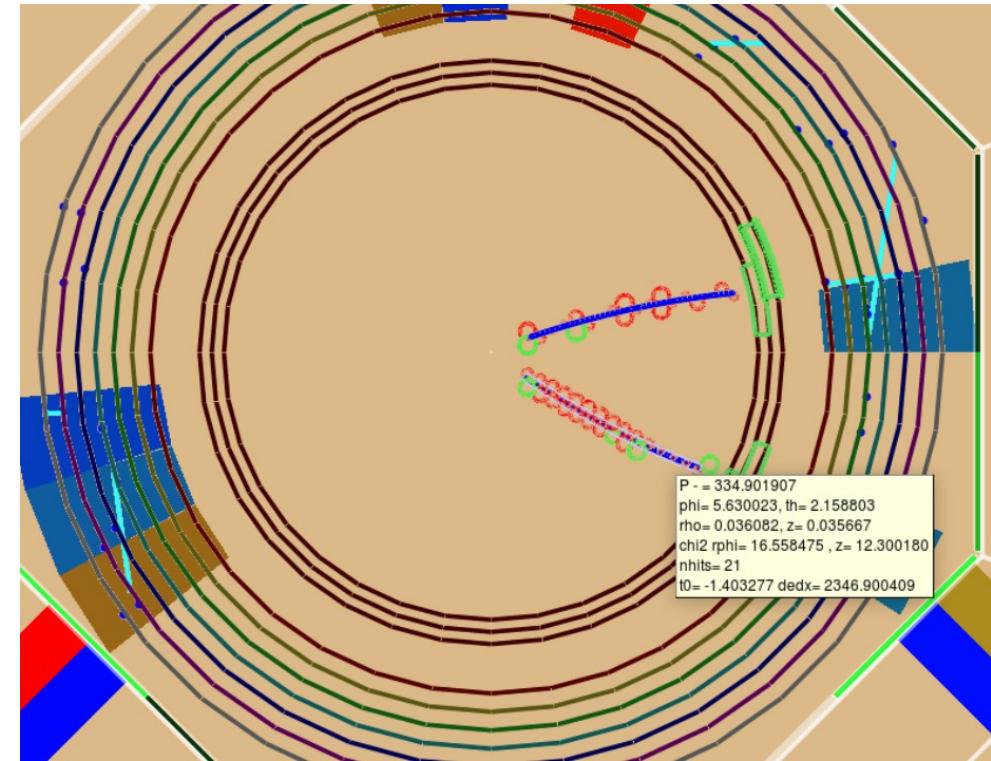
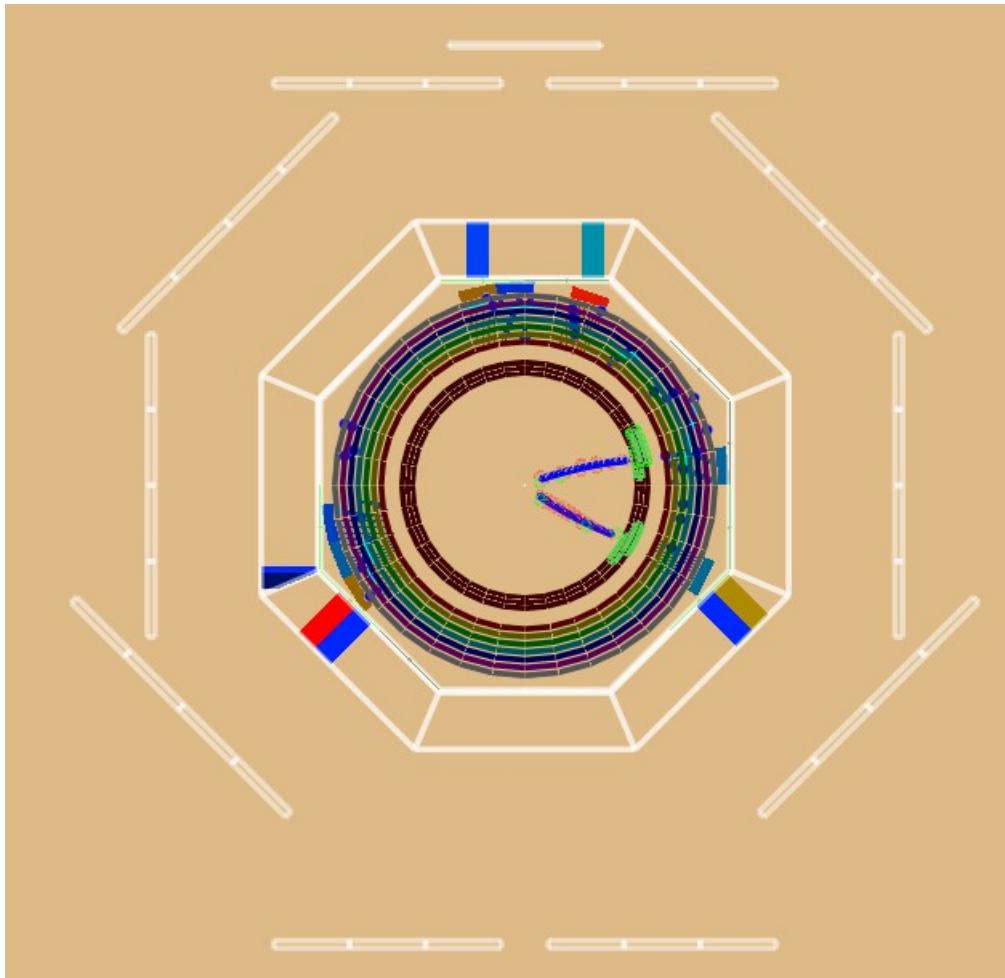


Схема ускорительного комплекса ВЭПП-2000

Сезон	Светимость, pb^{-1}
scan2011	20.16
scan2012	13.34

scan2012_omphi	0.68
scan2013_rho	11.59
scan2013_omphi	
scan2017	6.07

Отбор событий $ee \rightarrow \pi^+\pi^-2\pi^0$

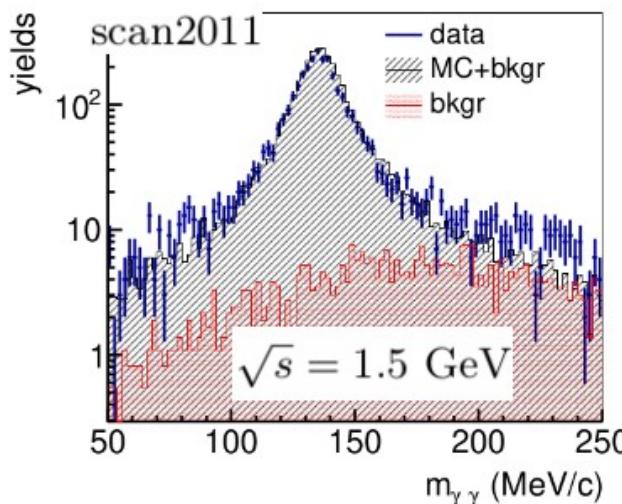


Визуализация типичного события $e^+e^- \rightarrow 2\pi^0\pi^+\pi^-$

Отбор событий

$(ee \rightarrow \pi^+\pi^- 2\pi^0)$ 64 kevents

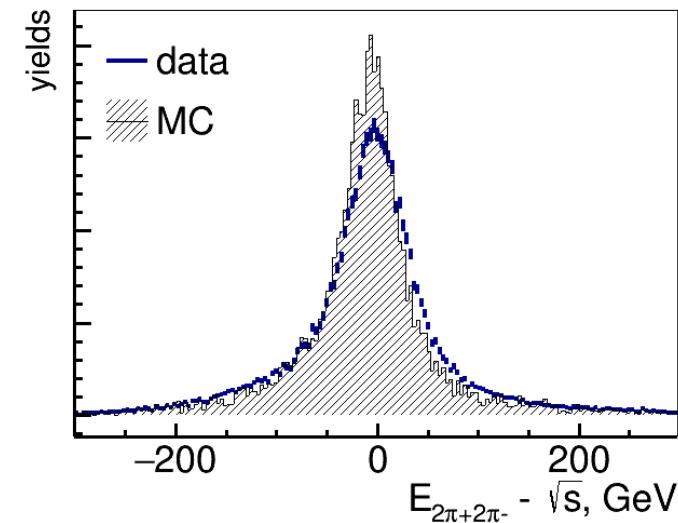
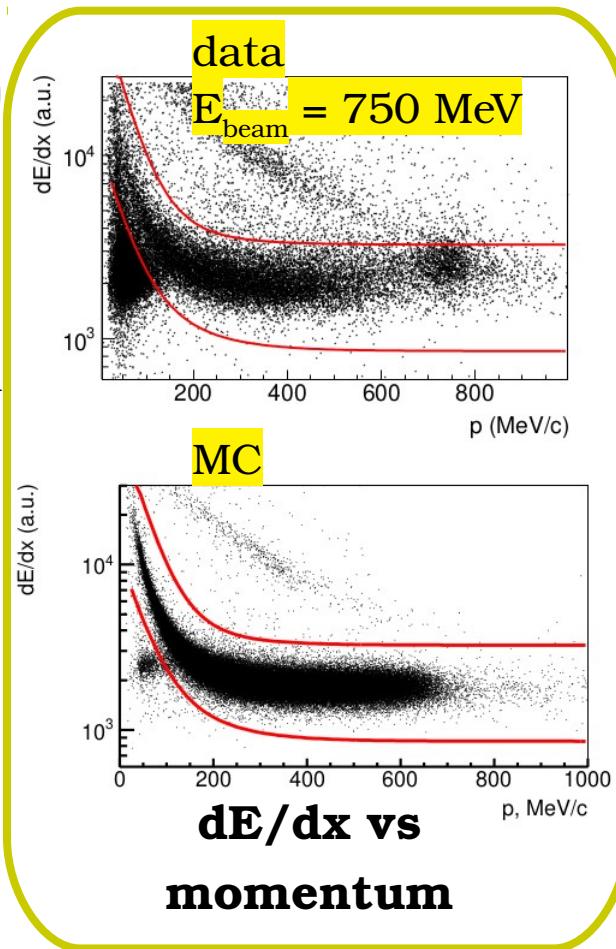
- $0.7 < \theta_{\pi,\gamma} < \pi - 0.7$ rad
- Total ($E - \sqrt{s}$) and $P < 150$ MeV/(c)
- Two candidates for π^0
- 5C kinematic fit
- The invariant mass spectrum of 3rd and 4th photons is used for the estimation of the contribution of background:



Отбор событий

$(ee \rightarrow 2\pi^+ 2\pi^-)$ 72 kevents

- $0.7 < \theta_{\pi} < \pi - 0.7$ rad
- Total ($E - \sqrt{s}$) and $P < 150$ MeV/(c)
- 4C kinematic fit
- The spectrum of total energy of four tracks ($E - \sqrt{s}$) is used for the estimation of the contribution of background:



Функция правдоподобия

The production of 4π system can proceed via a list of intermediate states:

- $\omega[1^{--}]\pi^0[0^{-+}]$ (only $2\pi^0 2\pi^\pm$)
- $a_1(1200)[1^+]\pi[0^-]$
- $\rho[1^{--}]f_0/\sigma[0^{++}]$
- $\rho f_2(1270)[2^{++}]$
- $\rho^+ \rho^-$ (only $2\pi^0 2\pi^\pm$)
- $a_2(1320)[2^{++}]\pi$
- $h_1(1170)[1^{+-}]\pi^0$ (only $2\pi^0 2\pi^\pm$)
- $\pi'(1300)(0^{-+})\pi$

The relative number of events I at a particular point Ω in phase space can be represented as

$$I(\Omega) = |V_\alpha A_\alpha(\Omega)|^2$$

where the sum runs over all intermediate states, V_α - the complex production amplitude (the free parameter) and $A_\alpha(\Omega)$ - the amplitude at a particular point in phase space.

Функция правдоподобия

The likelihood for model under test is

$$L = -\log \prod_{i=signal} \frac{p_i^+ p_i^- 2 p_i^0}{\int \varepsilon I(\Omega) d\Omega} - \log \prod_{i=background} \frac{2 p_i^+ 2 p_i^-}{\int \varepsilon I(\Omega) d\Omega}$$

The limited acceptance and efficiency of the detector is taken into account by summing only over simulated events that pass the reconstruction and analysis cuts.

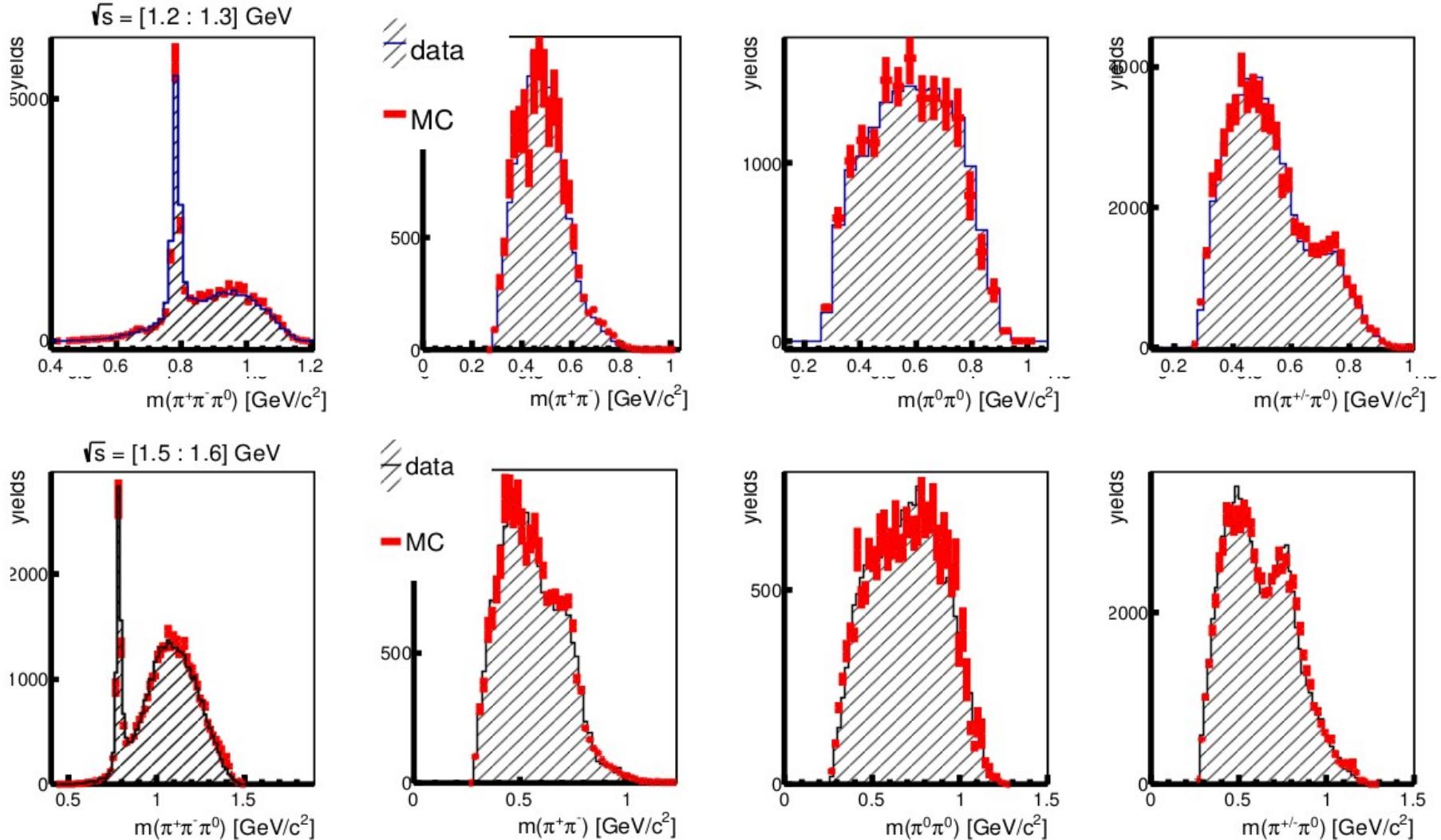
$$\int \varepsilon I(\Omega) d\Omega = \frac{1}{N_{MC}^{gen}} \sum_{rec}^{phase\ space\ MC} |V_\alpha A_\alpha(\Omega)|^2$$

- An amplitude is normalized to 1: $\int |A_\alpha(\Omega)|^2 d\Omega = 1$;
- The $\omega\pi^0$ amplitude is clearly seen at all energies, so $A_{\omega\pi^0}$ fixes at 1;

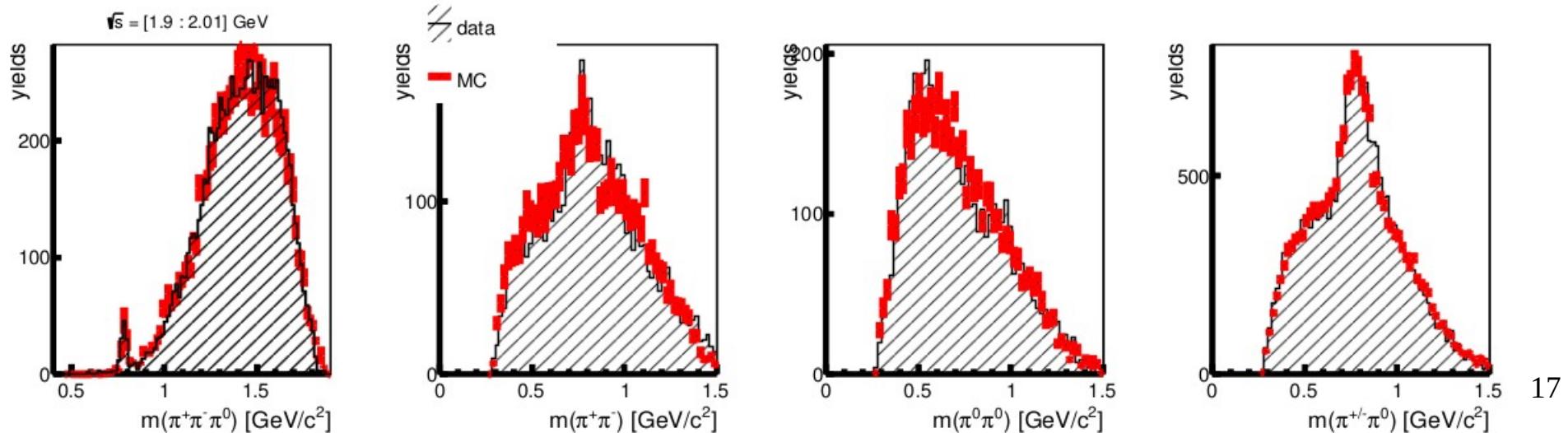
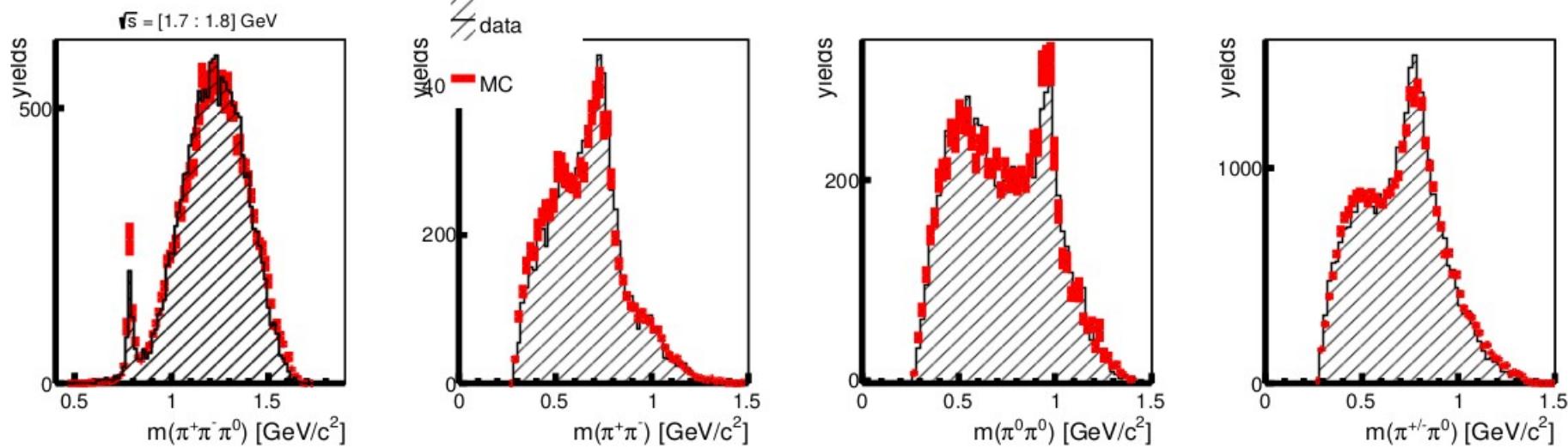
Функция правдоподобия

- The amplitudes is symmetric (anti-symmetric) with respect to the interchange of the momenta of neutral (charged) mesons according to Bose symmetry and C-parity conservation.
- $$L(\omega\rho\pi) = g_{\omega\rho\pi} \cdot \epsilon_{\mu\nu\rho\sigma} \cdot \delta^{ab} \cdot \omega_\mu \cdot d_\nu \pi^{*a} \cdot (d_\rho \rho_\sigma^{*b} - d_\sigma \rho_\rho^{*b}),$$
$$L(a_1\rho\pi) = g_{a_1\rho\pi} \cdot \epsilon^{abc} \cdot a_{1\mu}^a \cdot d_\nu \pi^{*b} \cdot (d_\mu \rho_\nu^{*c} - d_\nu \rho_\mu^{*c}),$$
$$L(a_1\sigma\pi) = g_{a_1\sigma\pi} \cdot \delta^{ab} \cdot (d_\mu a_{1\nu}^a - d_\nu a_{1\mu}^a) \cdot d_\mu \phi^*(\sigma) \cdot d_\nu \phi^*(\pi),$$
$$L(\rho' \rho f_0) = g_{\rho' \rho f_0} \cdot \delta^{ab} \cdot (d_\mu \rho_\nu'^a - d_\nu \rho_\mu'^a) (d_\mu \rho_\nu^{*b} - d_\nu \rho_\mu^{*b}) \cdot \phi_{f0}^*,$$
$$L(\rho' \rho^+ \rho^-) = g_{\rho' \rho^+ \rho^-} \cdot \epsilon^{abc} (d_\mu \rho_\nu'^a - d_\nu \rho_\mu'^a) \cdot (d_\alpha \rho_\nu^{*b} - d_\nu \rho_\alpha^{*b}) \cdot (d_\mu \rho_\alpha^{*c} - d_\alpha \rho_\mu^{*c}).$$
$$L(\rho' h_1 \pi^0) = g_{\rho' h_1 \pi^0} \cdot \delta^{ab} (d_\mu \rho_\nu'^a - d_\nu \rho_\mu'^a) \cdot (d_\mu h_{1\nu}^{*b} - d_\nu h_{1\mu}^{*b}) \phi_\pi^*,$$
- Masses and central values of widths of resonances are fixed according to PDG.
- Flatt'e distribution is used for the propagator of (980).

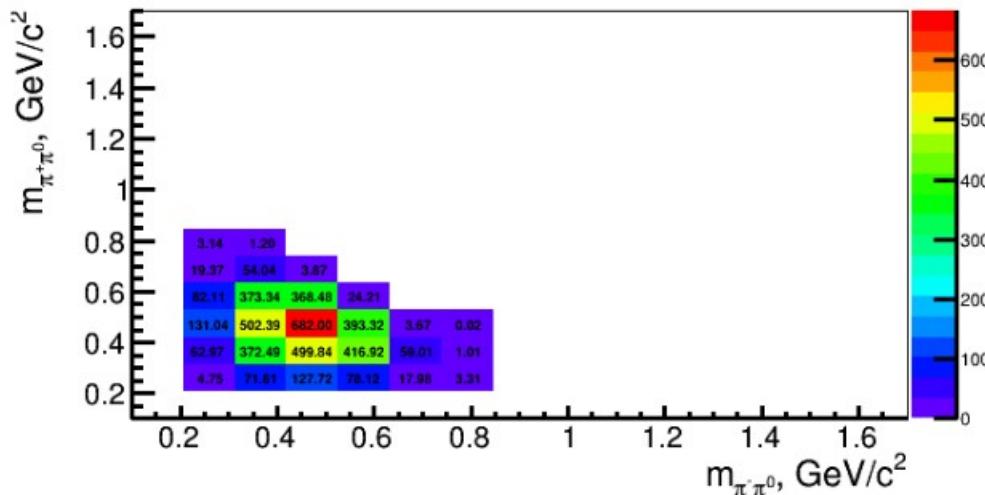
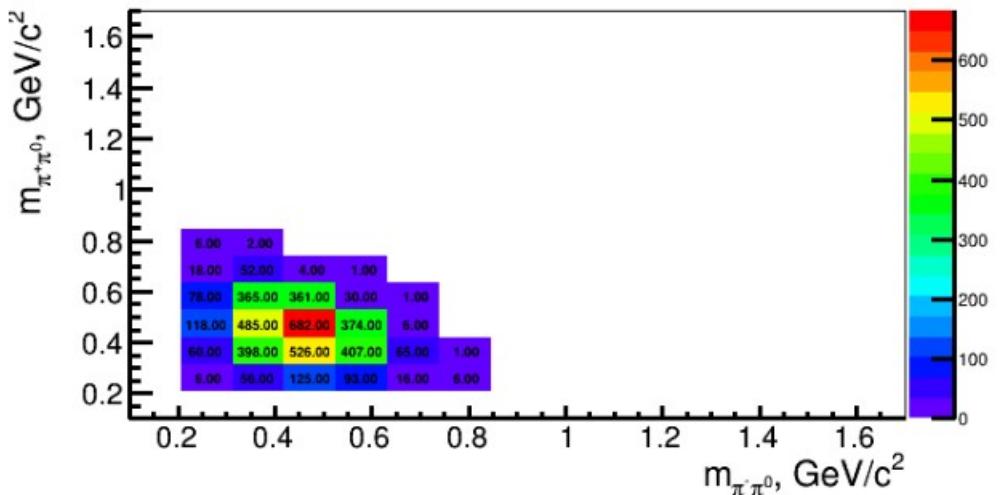
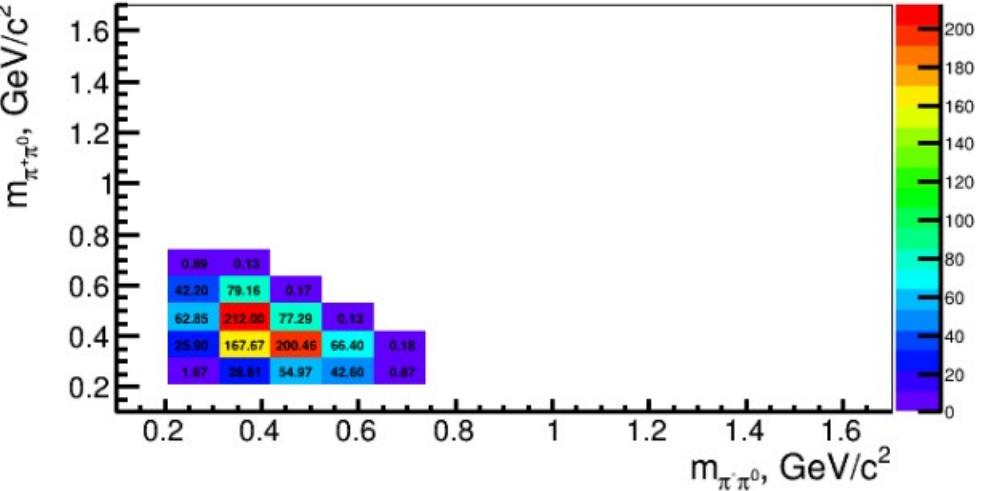
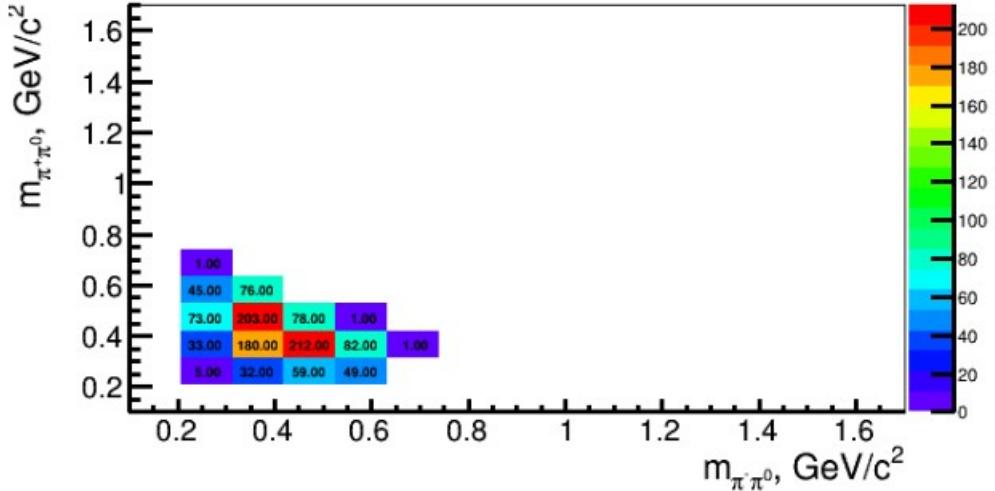
(ee \rightarrow pi⁺pi⁻2pi⁰)



(ee \rightarrow pi⁺pi⁻2pi⁰)



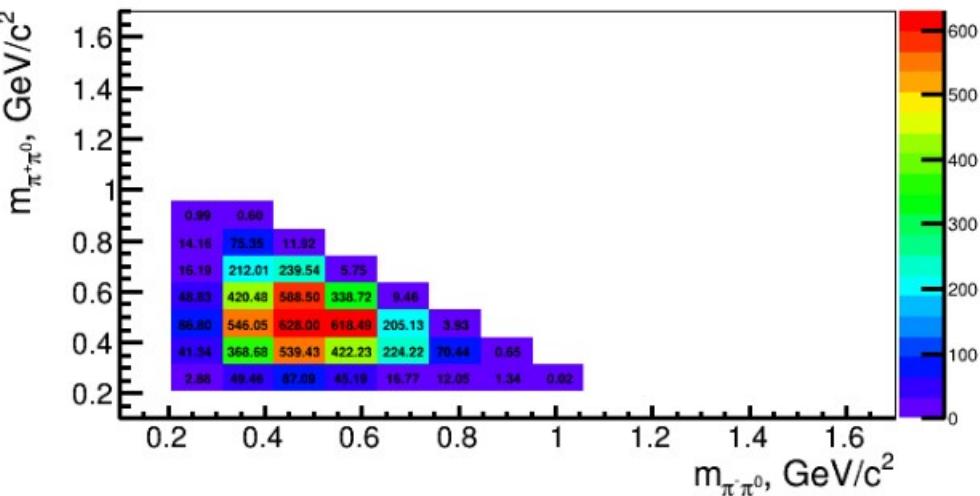
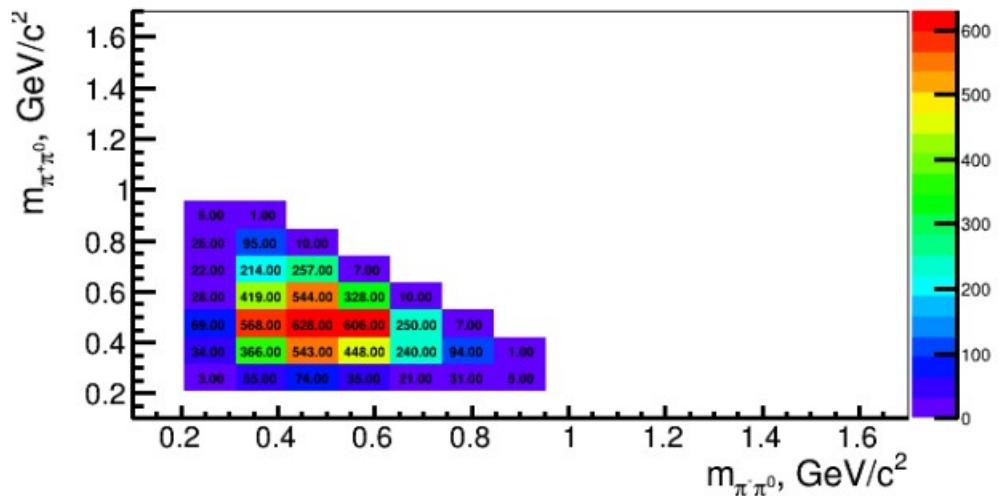
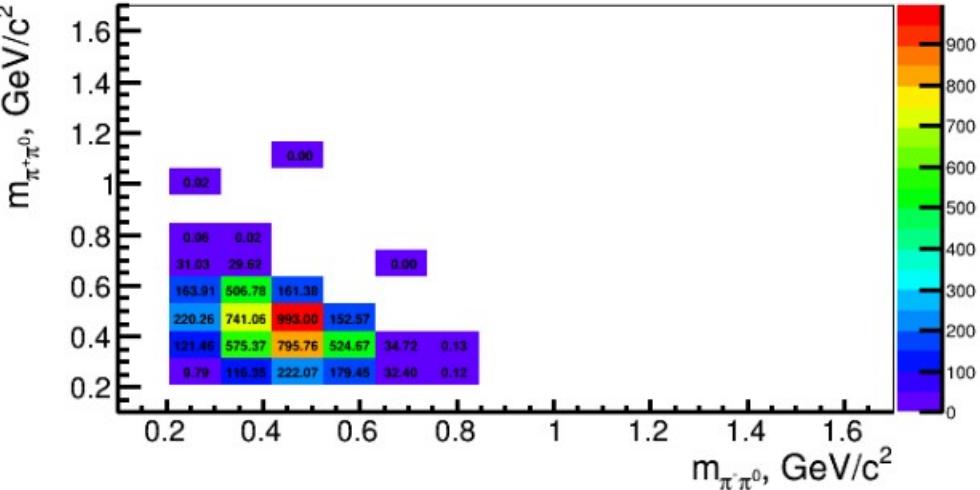
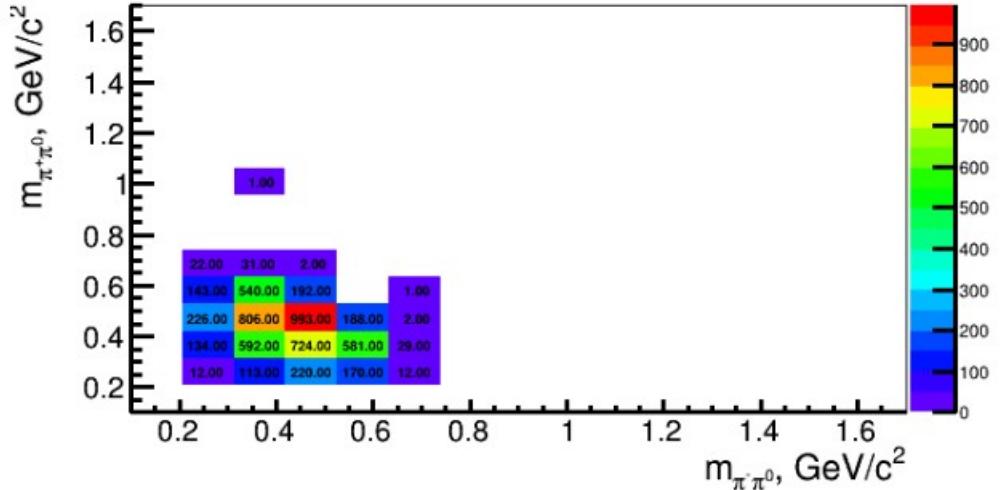
(ee \rightarrow pi⁺pi⁻2pi⁰)



1.03-1.1 GeV

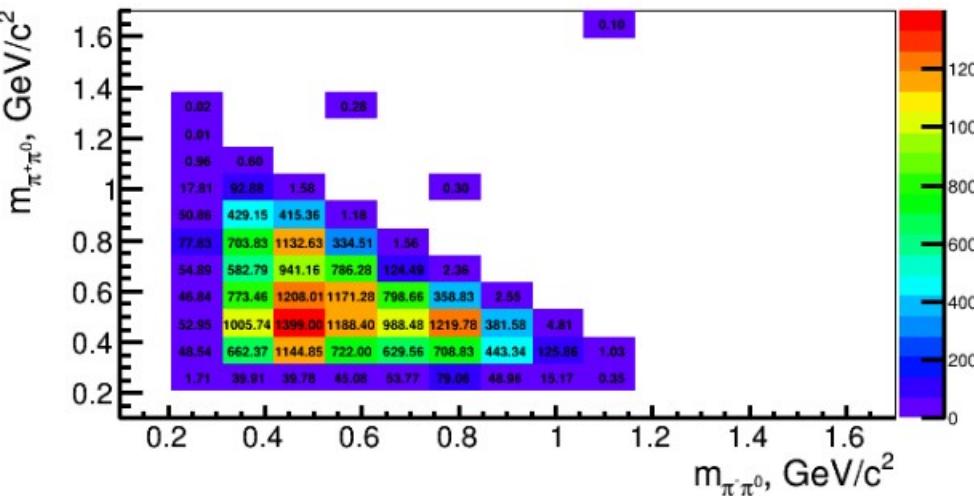
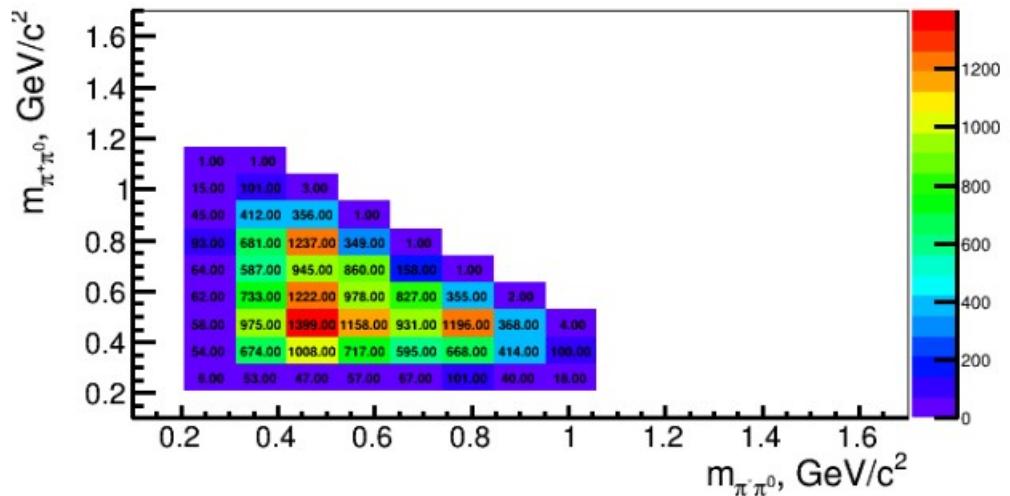
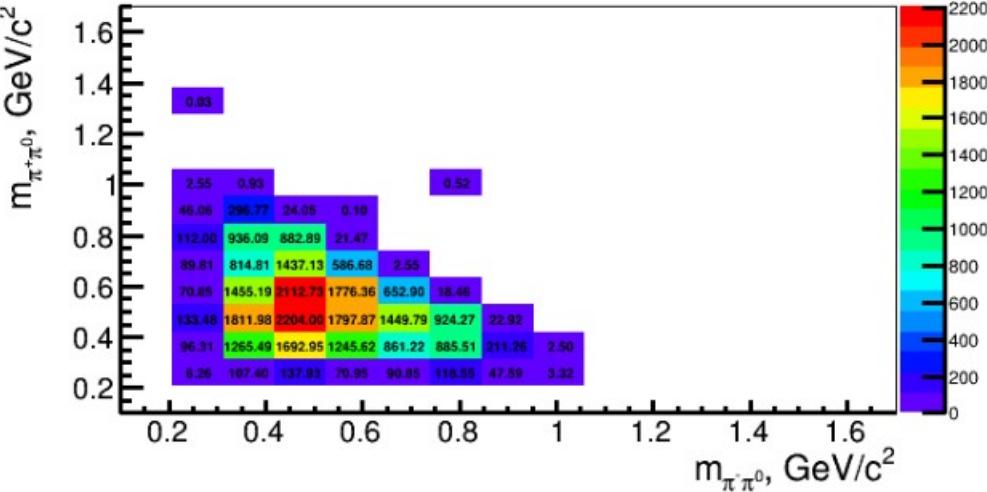
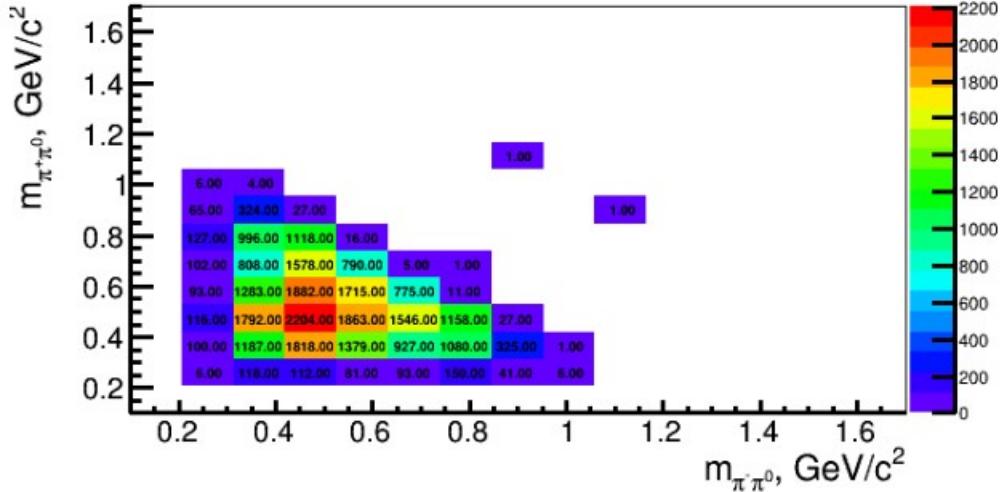
(ee \rightarrow pi⁺pi⁻2pi⁰)

1-1.03 GeV



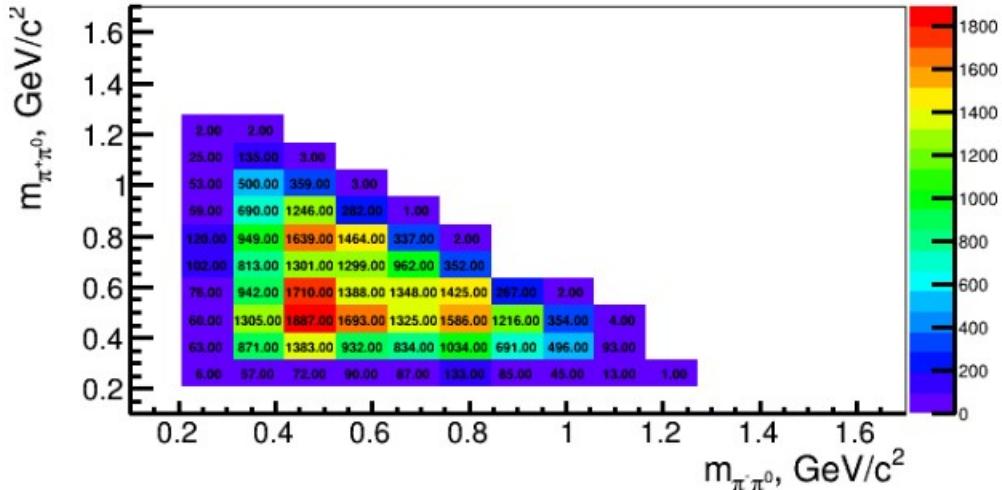
(ee \rightarrow pi⁺pi⁻2pi⁰)

1.2-1.3 GeV



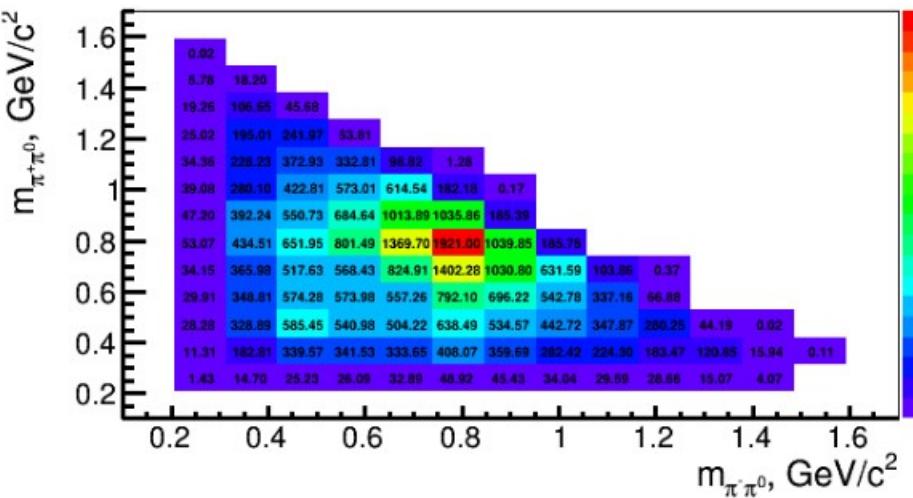
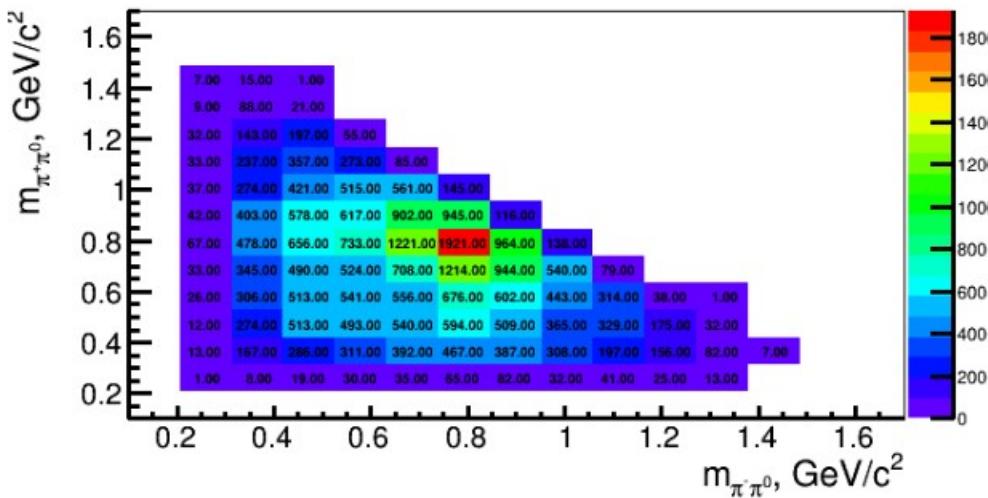
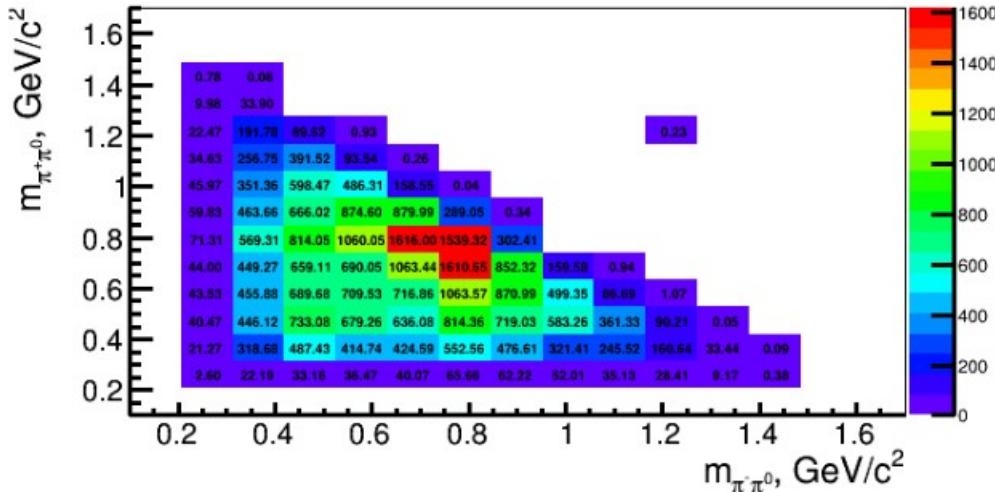
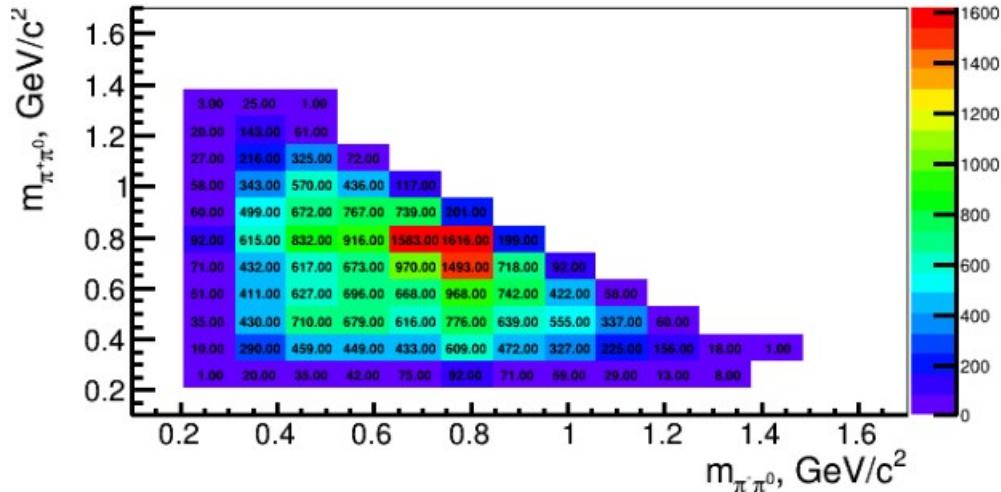
(ee \rightarrow pi⁺pi⁻2pi⁰)

1.4-1.5 GeV



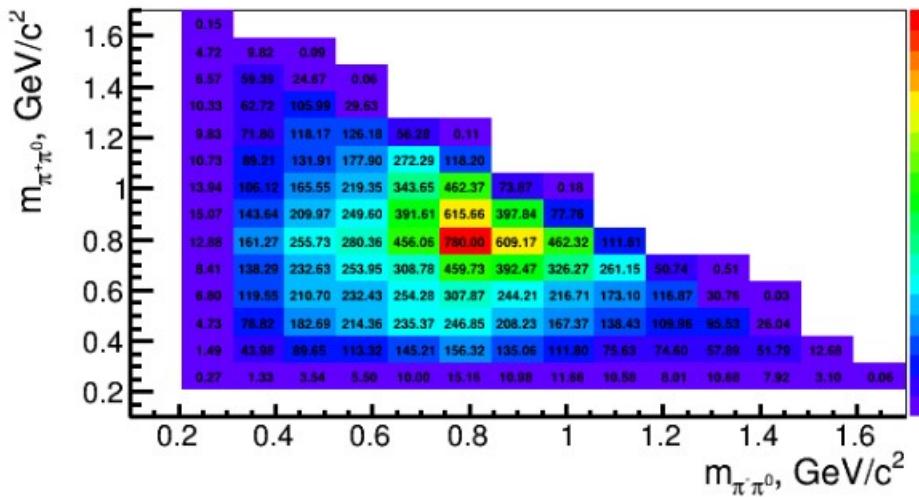
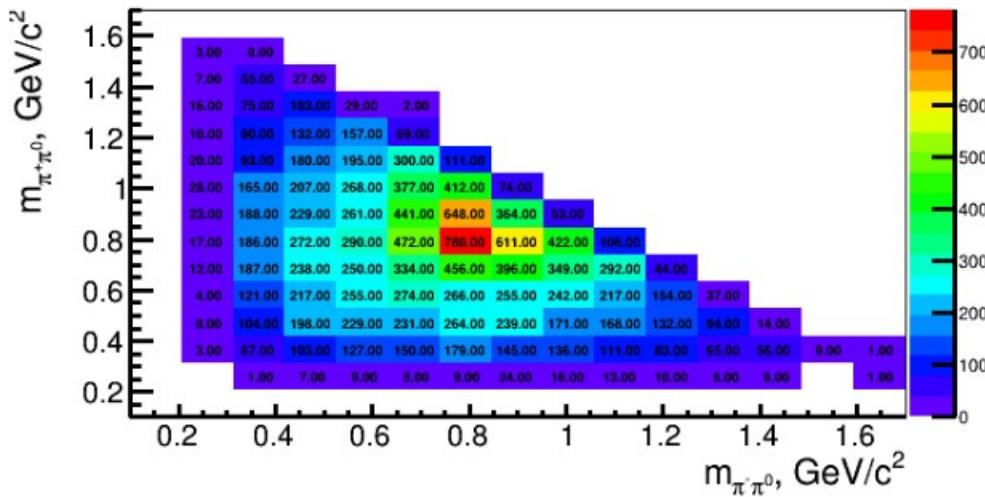
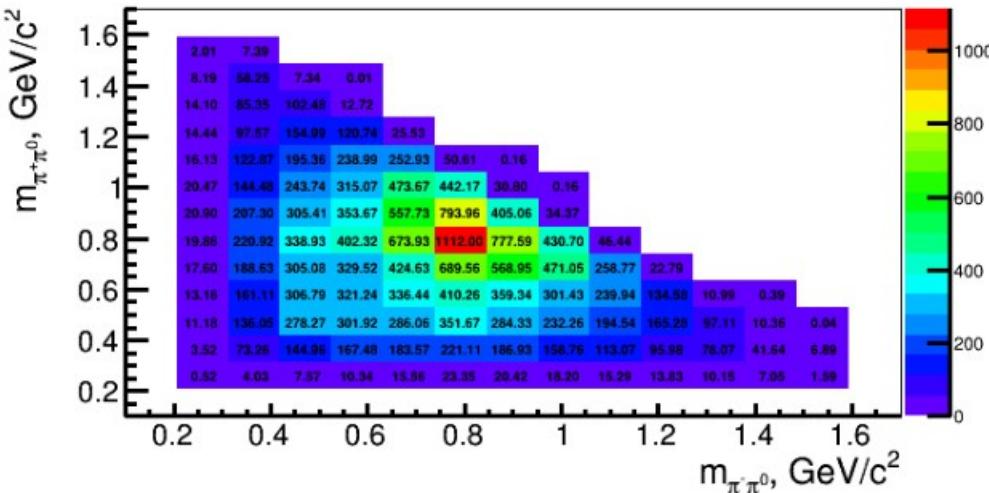
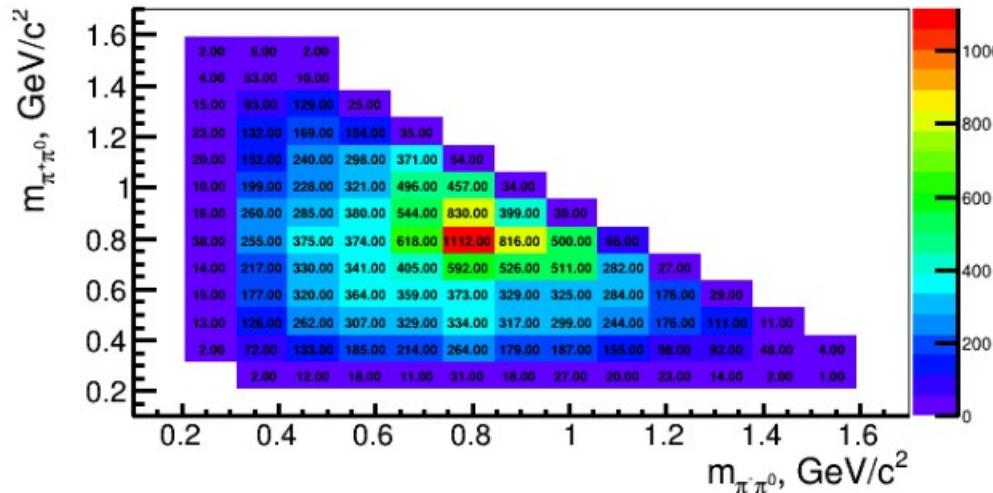
(ee \rightarrow pi⁺pi⁻2pi⁰)

1.6-1.7 GeV



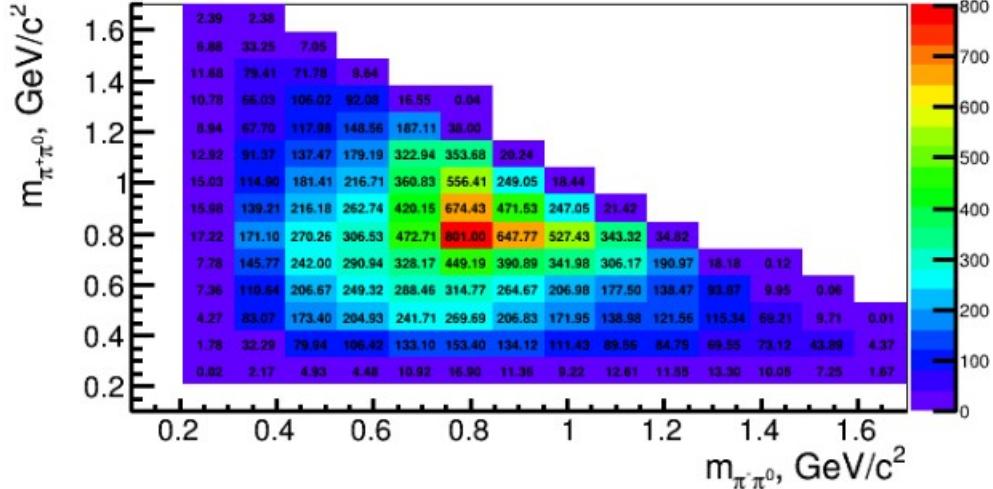
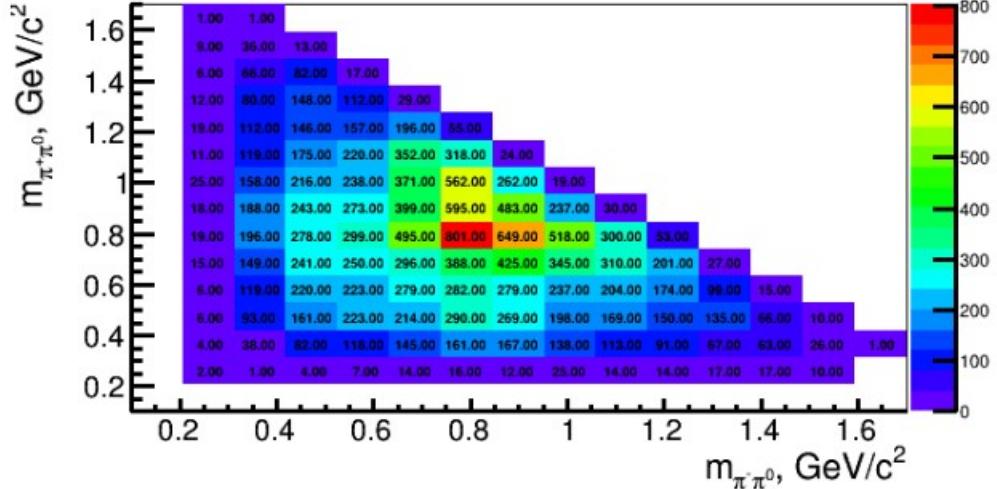
(ee \rightarrow pi⁺pi⁻2pi⁰)

1.8-1.87654 GeV

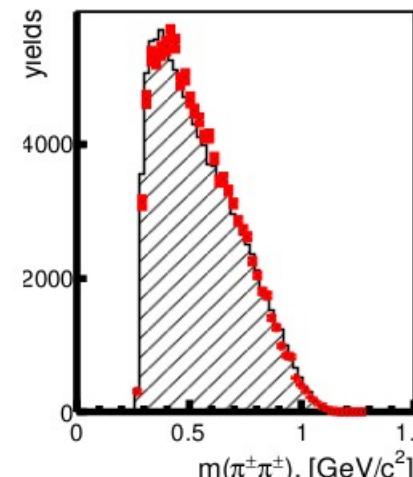
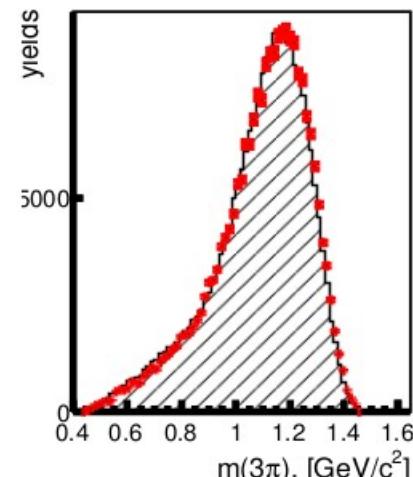
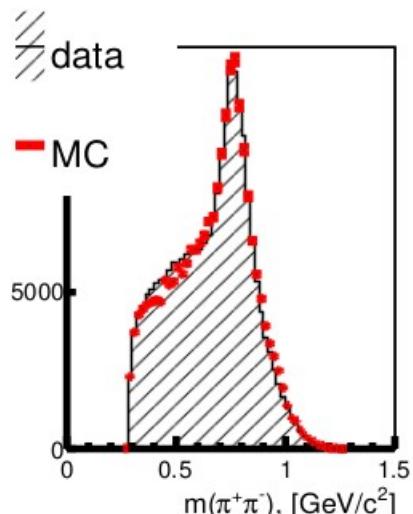
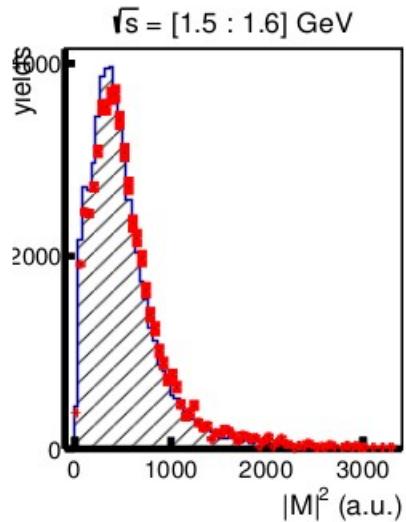
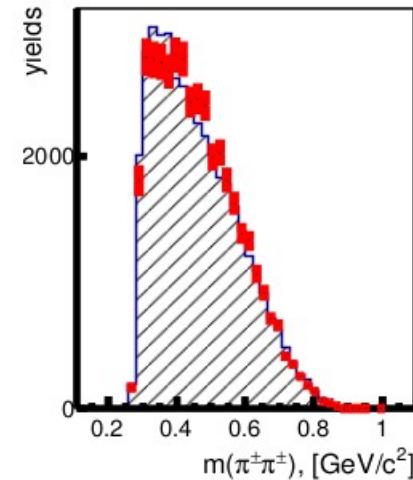
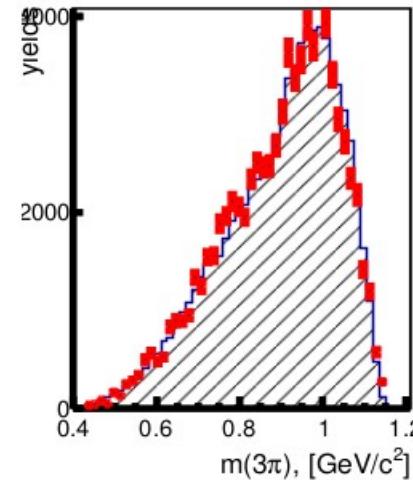
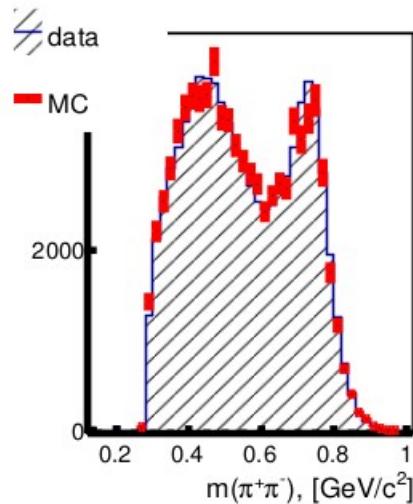
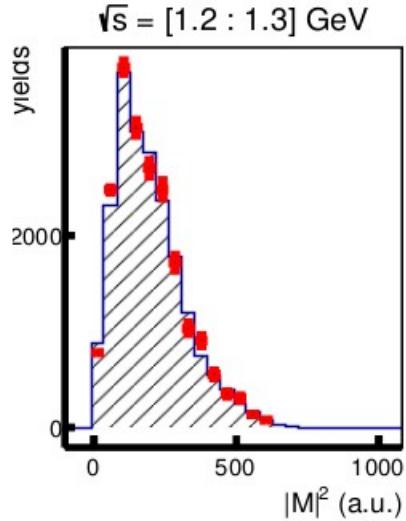


(ee \rightarrow pi⁺pi⁻2pi⁰)

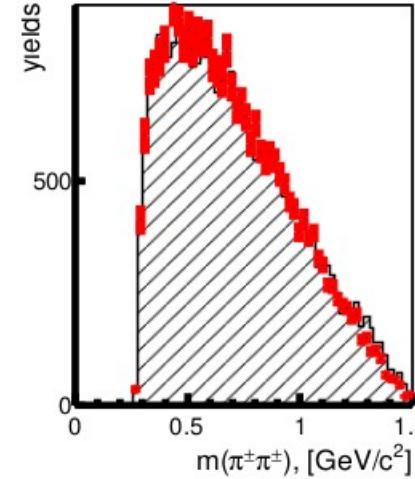
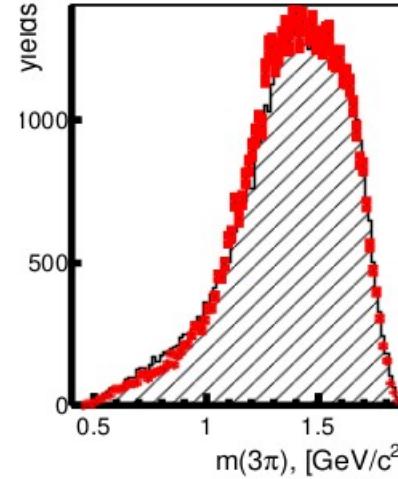
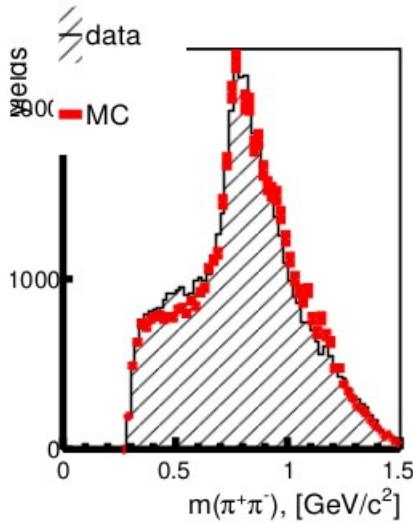
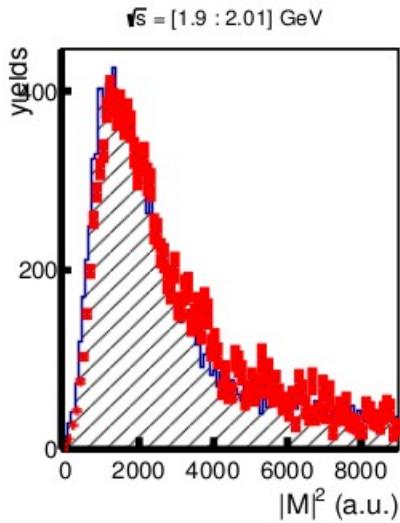
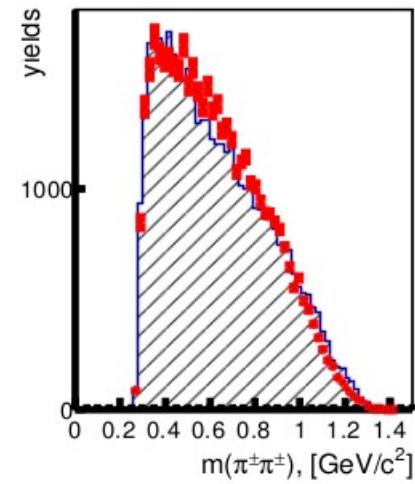
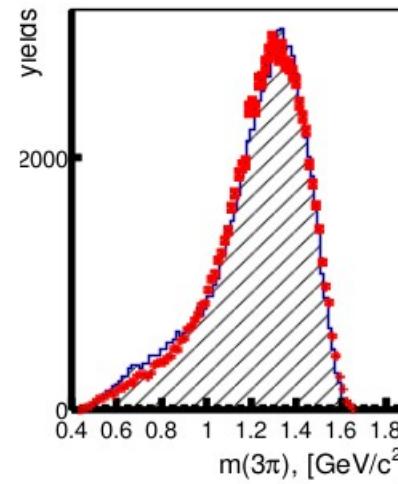
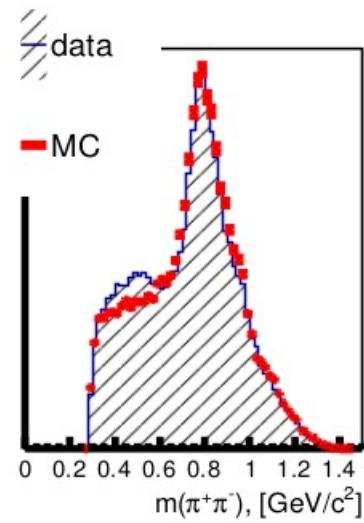
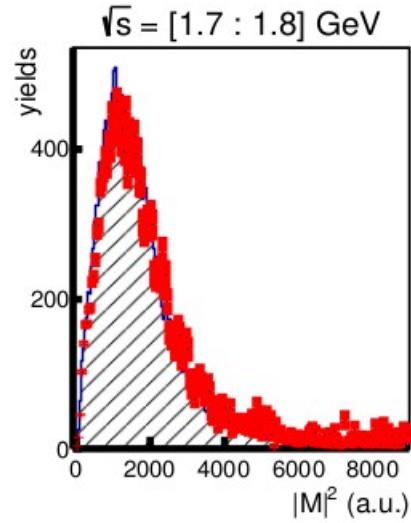
1.9-2.01 GeV



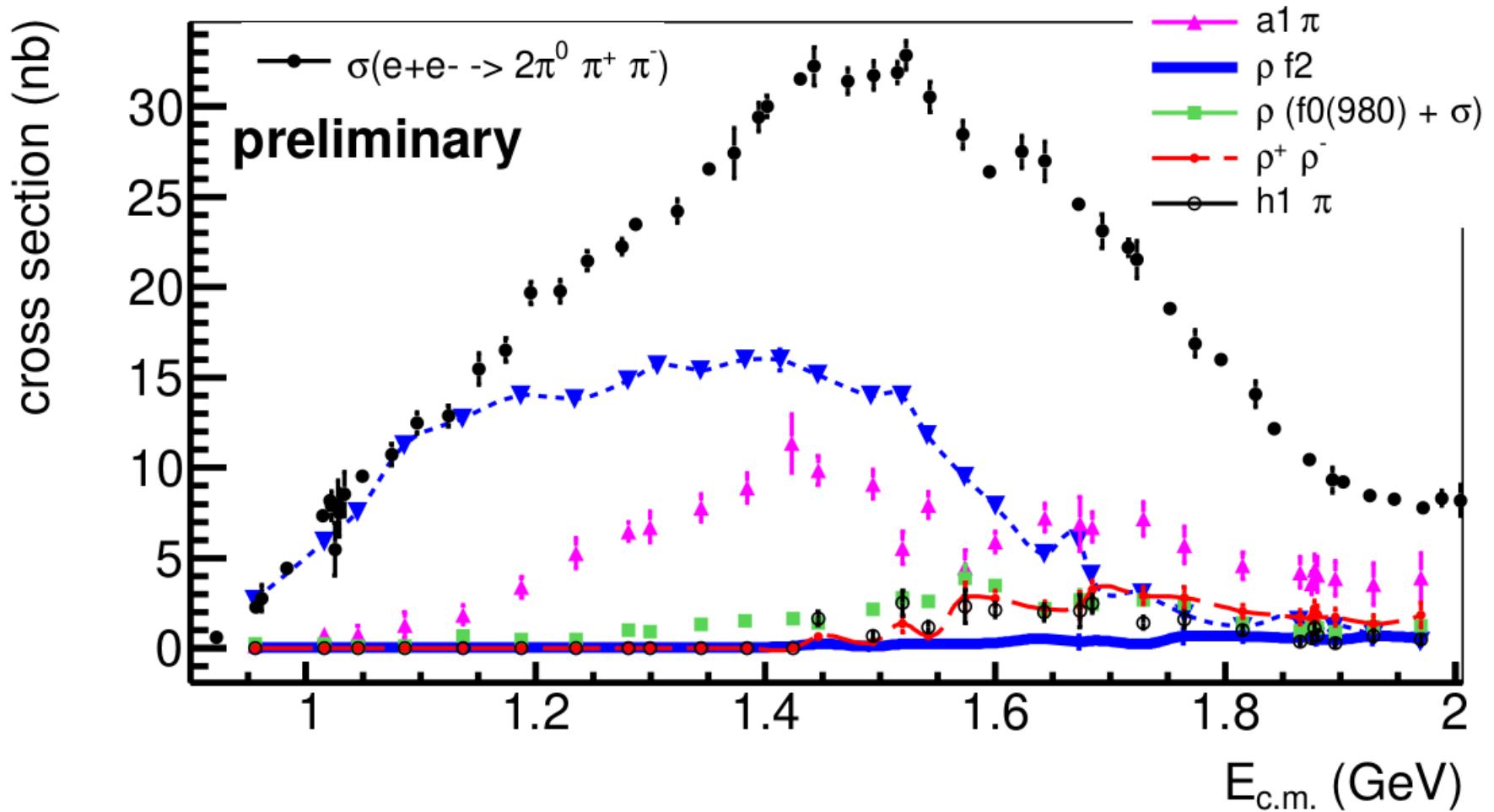
(ee \rightarrow 2pi⁺2pi⁻)



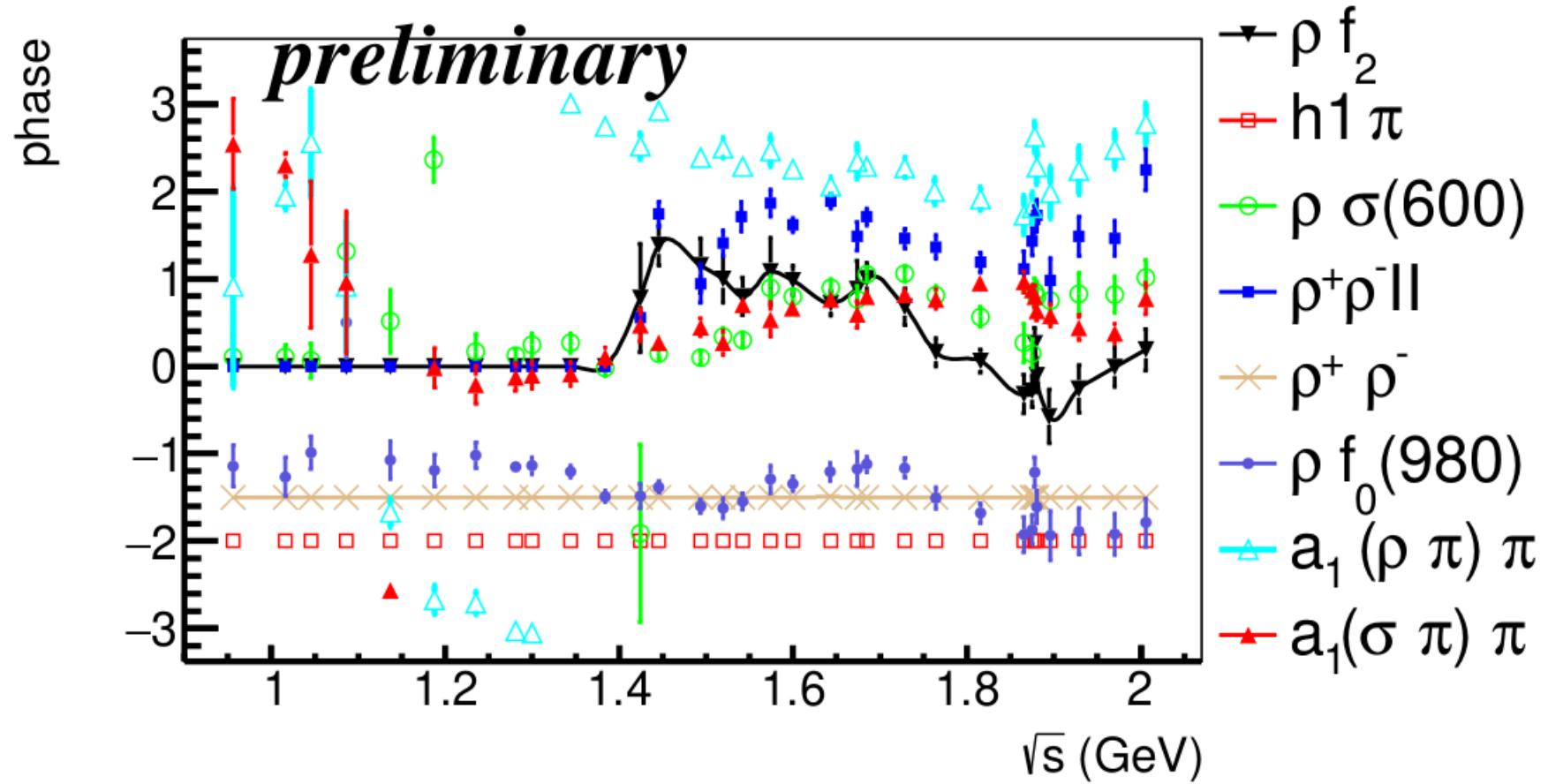
(ee \rightarrow 2pi $^+$ 2pi $^-$)

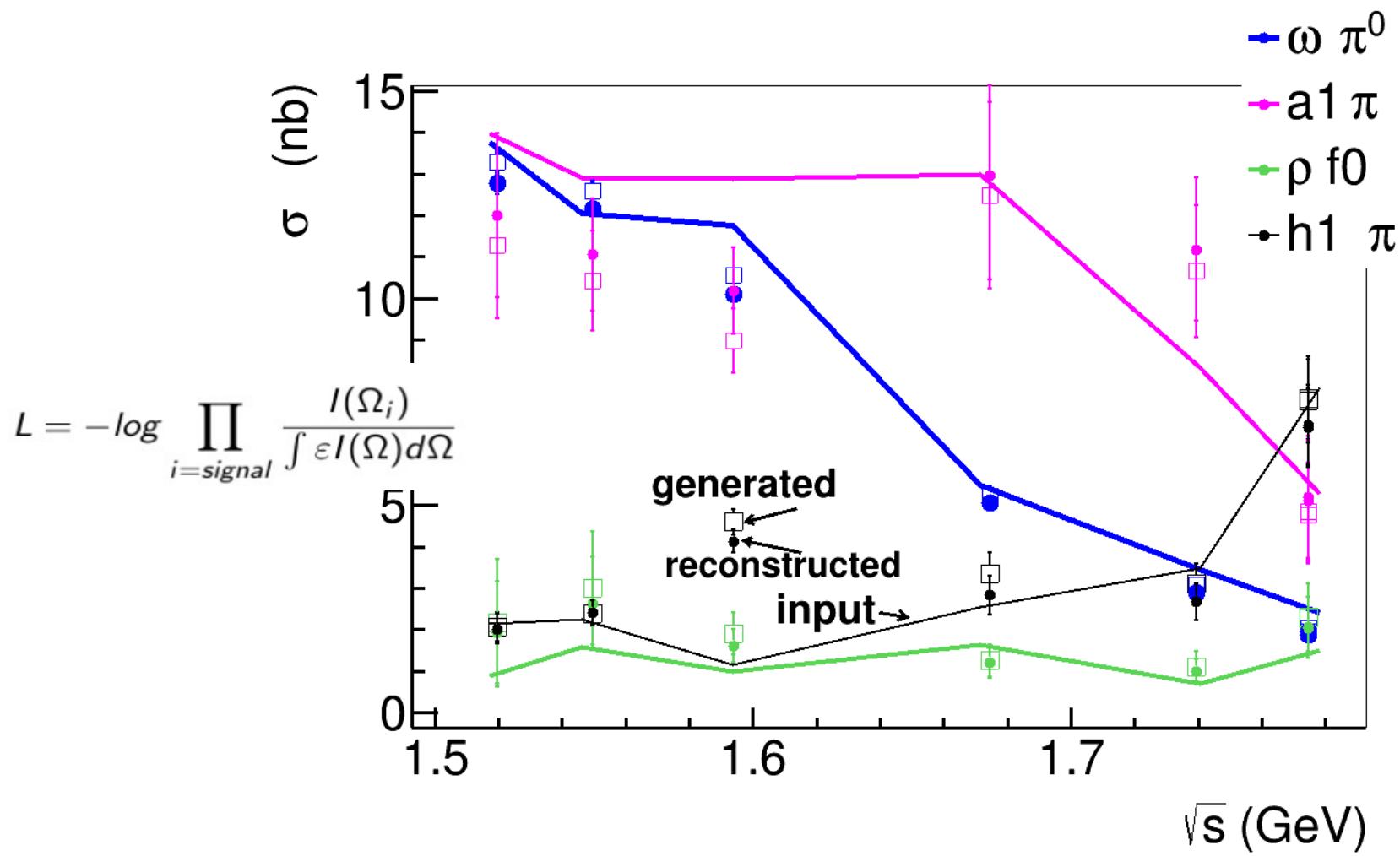


$$R_i(s) = \frac{\int |V_i \cdot \mathbf{H}_{i\perp}(\Omega)|^2 d\Phi}{\int |\sum_{\beta} V_{\beta} \mathbf{H}_{\beta\perp}(\Omega)|^2 d\Phi} \cdot \sigma_{2\pi^0\pi^+\pi^-}(s)$$



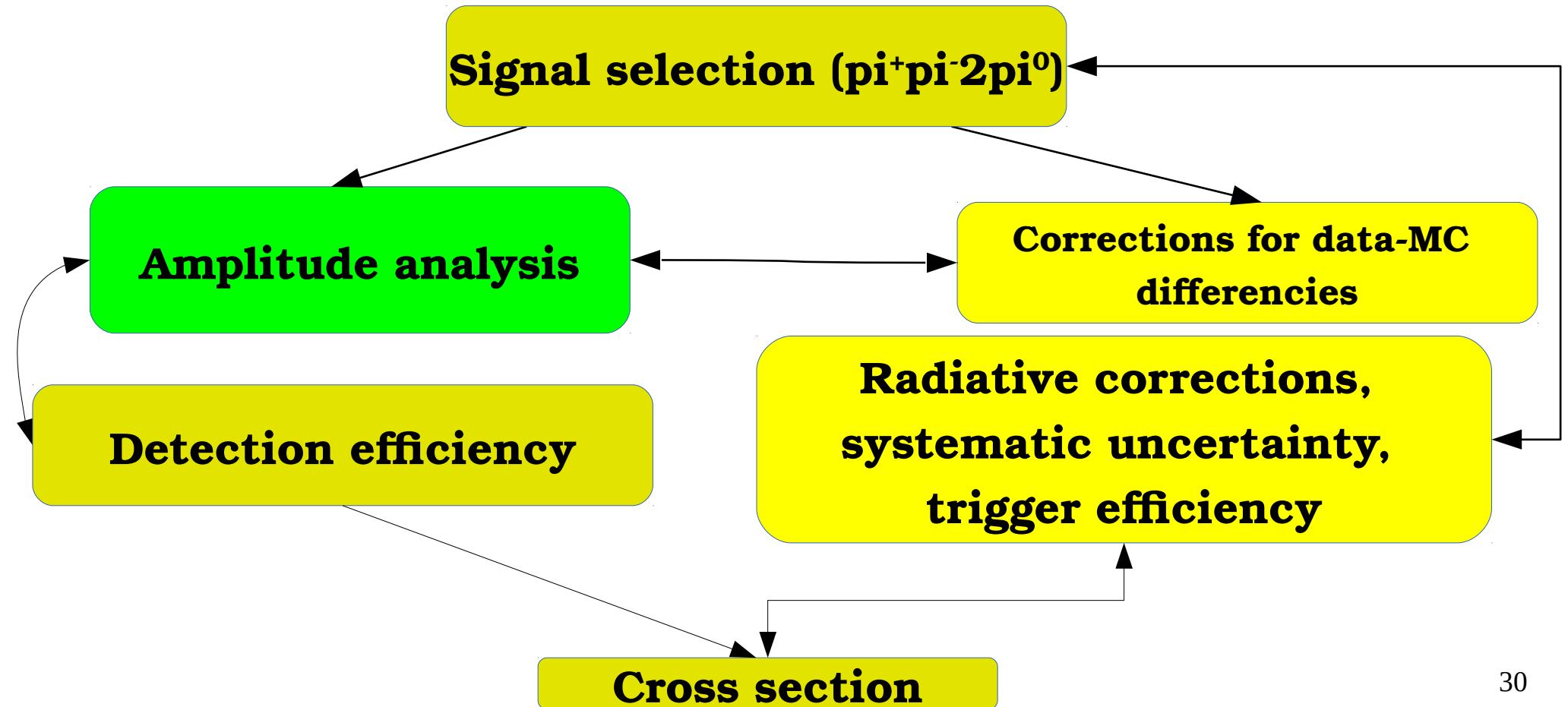
Значения фаз различных компонент матричного элемента





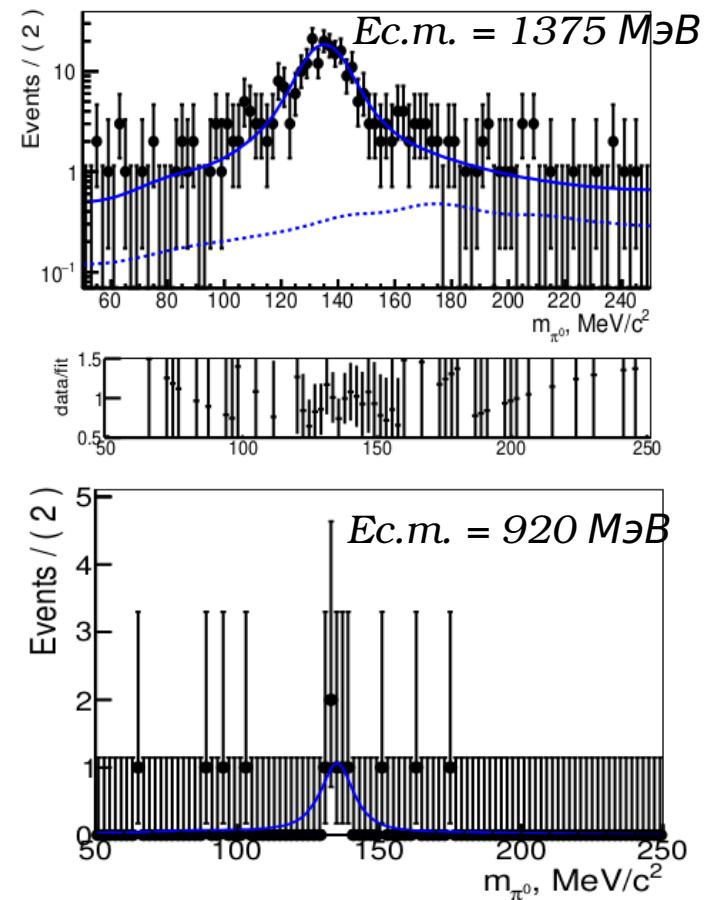
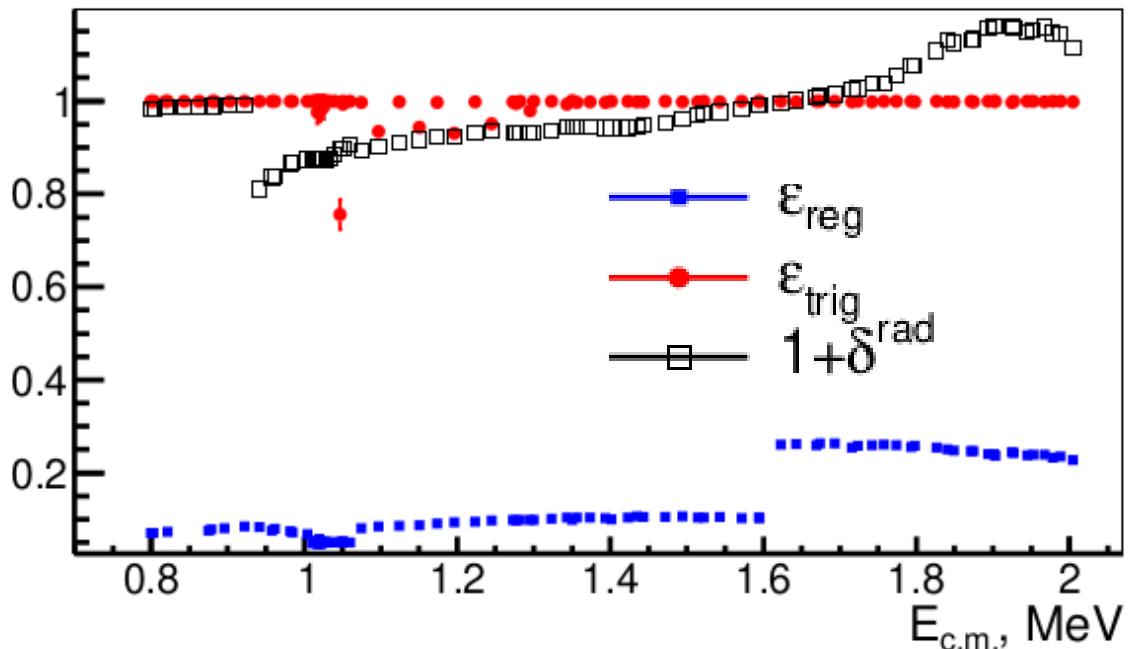
- Ошибка методики $\sim 3\text{-}7\%$

Общая стратегия измерения сечения

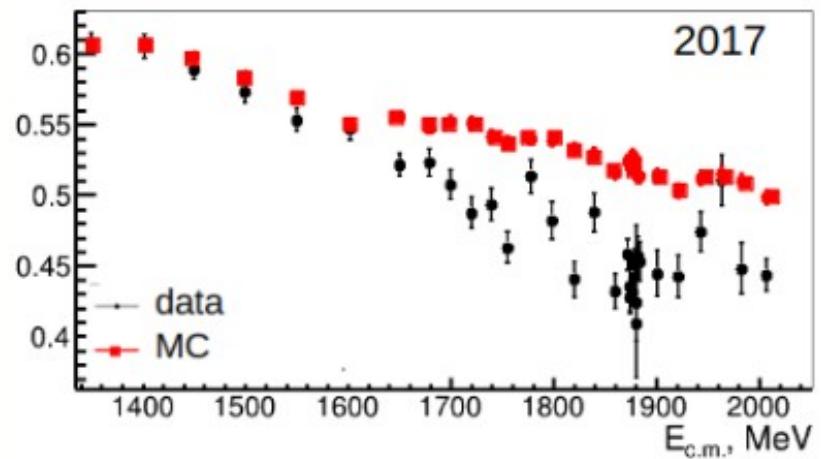
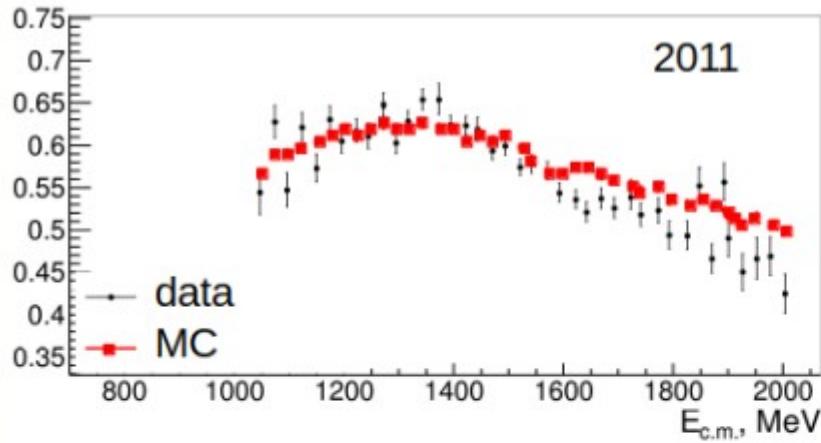


$$\sigma^{Born} = \frac{N_{exp}}{\epsilon_{reg} \epsilon_{trig} L (1 + \delta^{rad.})}$$

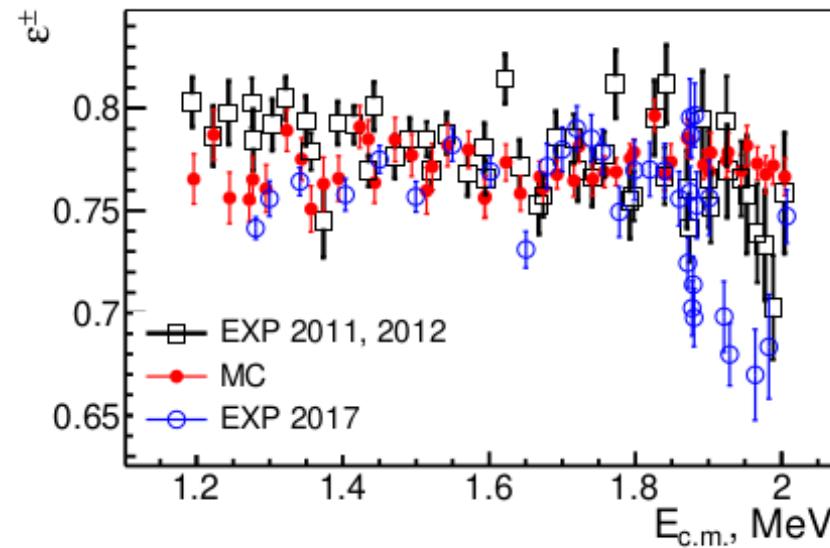
$$1 + \delta^{rad}(s) = \frac{\int F(x,s) \sigma^{Born}(s(1-x)) dx}{\sigma^{Born}(s)}$$



Пример аппроксимации спектра инвариантной массы 3-го и 4-го фотонов



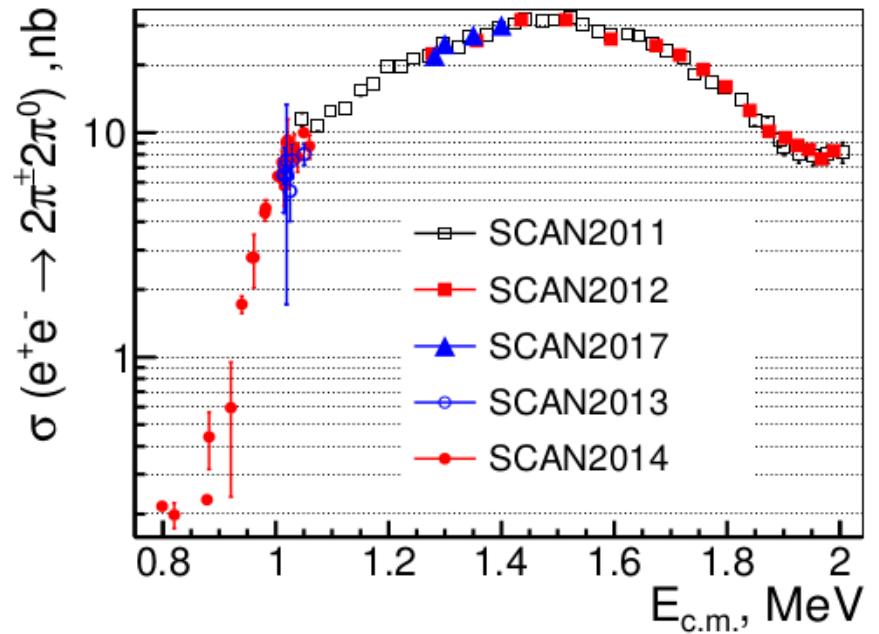
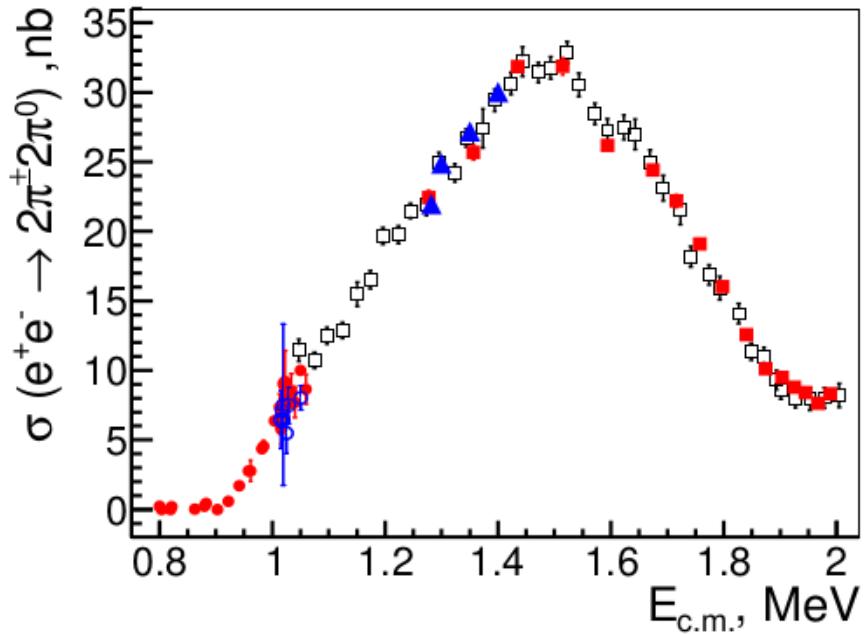
Эффективность реконструкции
нейтральных пионов



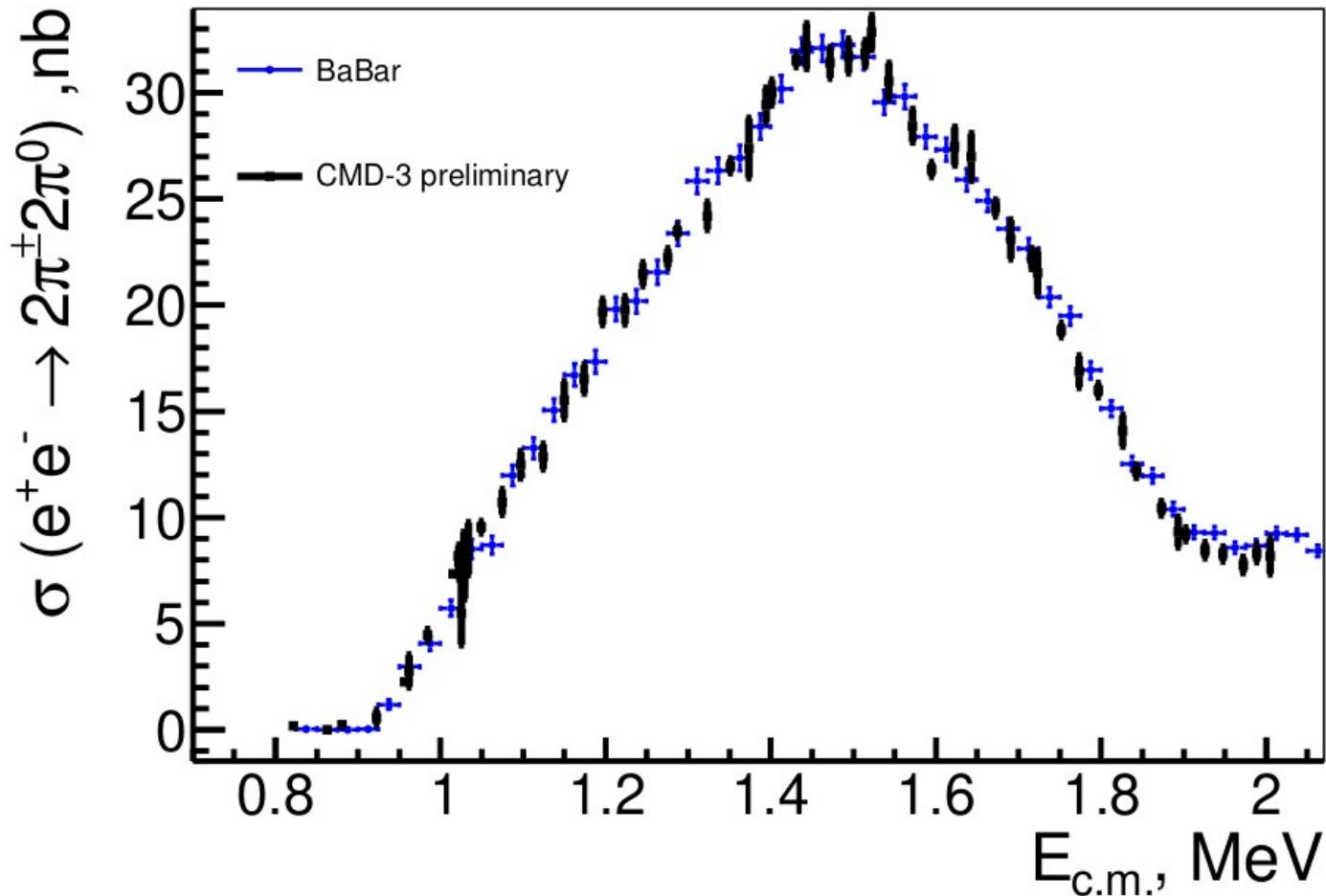
Эффективность реконструкции
заряженных пионов

Источник ошибки	$E_{\text{c.m.}} < 1.06$ ГэВ (%)	$1.06 < E_{\text{c.m.}} < 1.6$ ГэВ (%)	$1.6 < E_{\text{c.m.}}$ ГэВ (%)
Вычитание фона	3	1	4
Модельная ошибка	0.5	0.8	0.8
Условия отбора	4	4	2
Реконструкция заряженных пионов	2	2	2
Реконструкция нейтральных пионов	8	8	8
Эффективность триггера	0.3	0.3	0.3
Радиационная поправка	1.5	0.9	0.9
Светимость	1.0	1.0	1.0
Total	10	9	10

Источники систематической ошибки в сечении процесса $e^+e^- \rightarrow 2\pi^0\pi^+\pi^-$



Зависимость полученных значений сечений
 $e^+e^- \rightarrow 2\pi^0\pi^+\pi^-$ энергии для разных сезонов набора
данных.

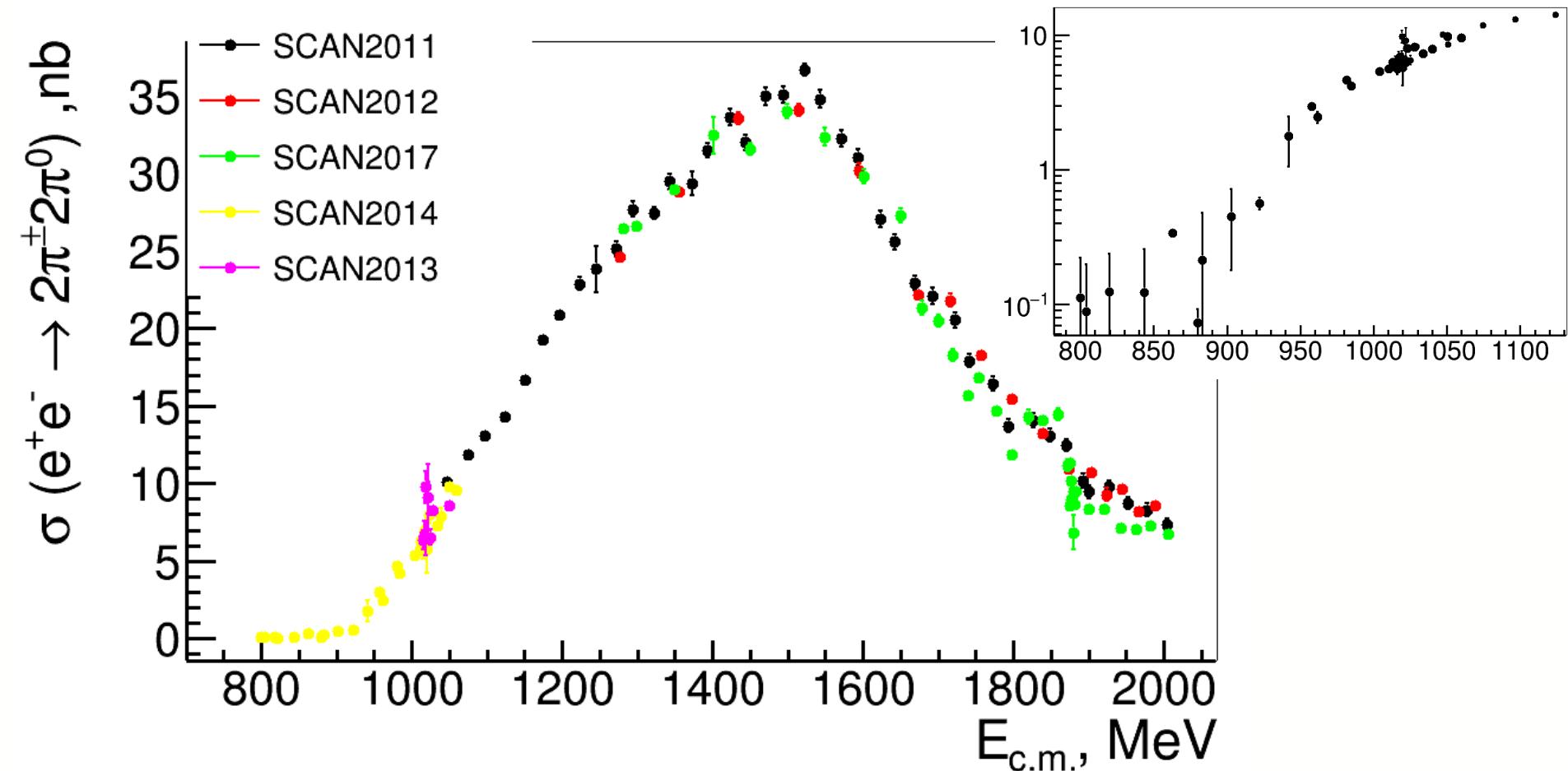


Сравнение полученных усредненных значений сечений $e^+e^- \rightarrow 2\pi^0\pi^0$ от энергии с результатом на детекторе *BaBar*. ³⁵

Заключение

- Выполнено предварительное изучение промежуточной динамики процесса $e^+e^- \rightarrow 4\pi$ в диапазоне энергий $\sqrt{s} = [0.95-2.01] \text{ ГэВ}$
- Подтверждена доминантность амплитуд $\omega\pi$ и $a_1\pi$ при $\sqrt{s} < 2 \text{ ГэВ}$
- Канал $\omega\pi$ является доминирующим в процессе $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ при $\sqrt{s} < 1.5 \text{ ГэВ}$
- Определены вклады амплитуд $\rho\sigma$, ρf_0 , $\rho^-\rho^+$, ρf_2 , $h_1\pi$
- Измерено сечение процесса $e^+e^- \rightarrow 2\pi^0\pi^+\pi^-$ с систематической точностью около 10% в области энергий $E = 0.8 \div 2 \text{ ГэВ}$.

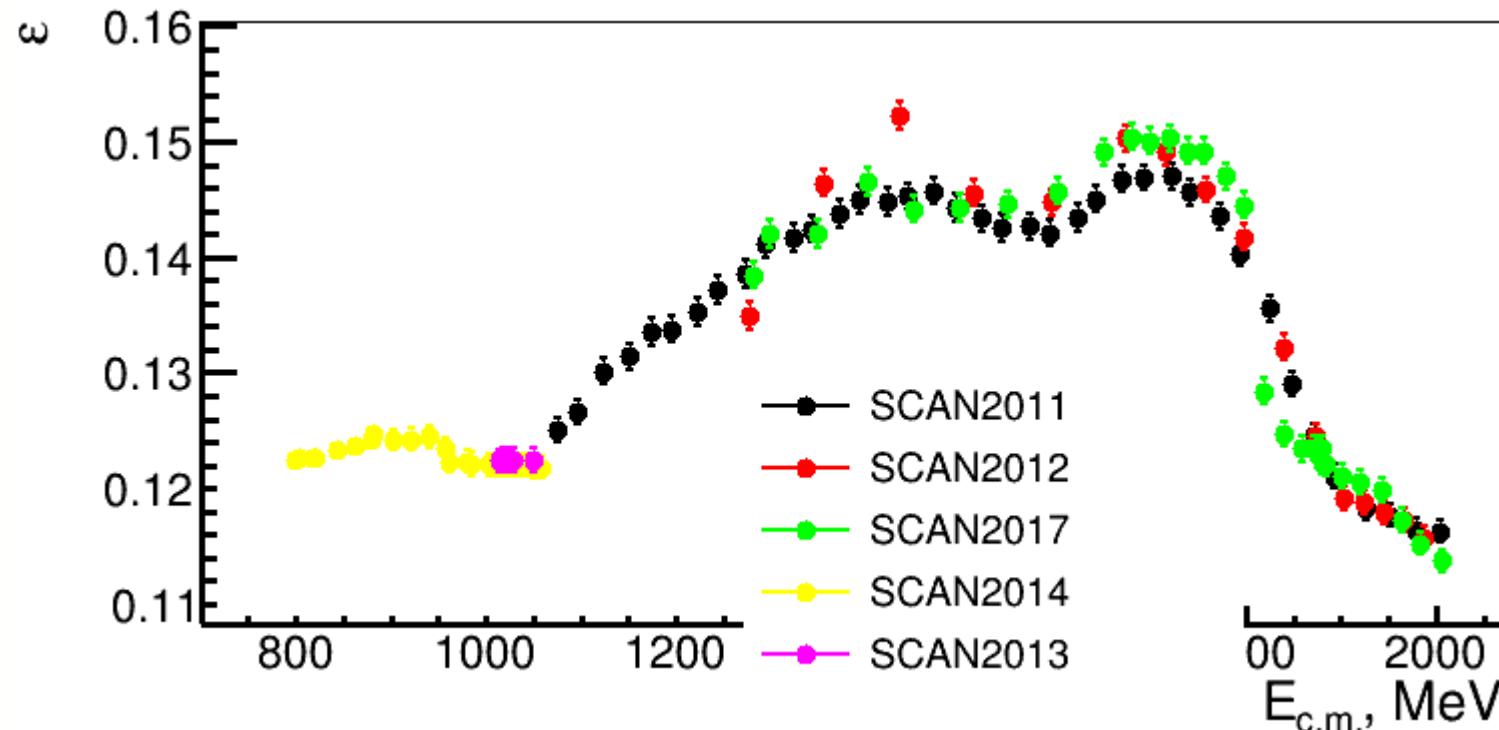
back-up



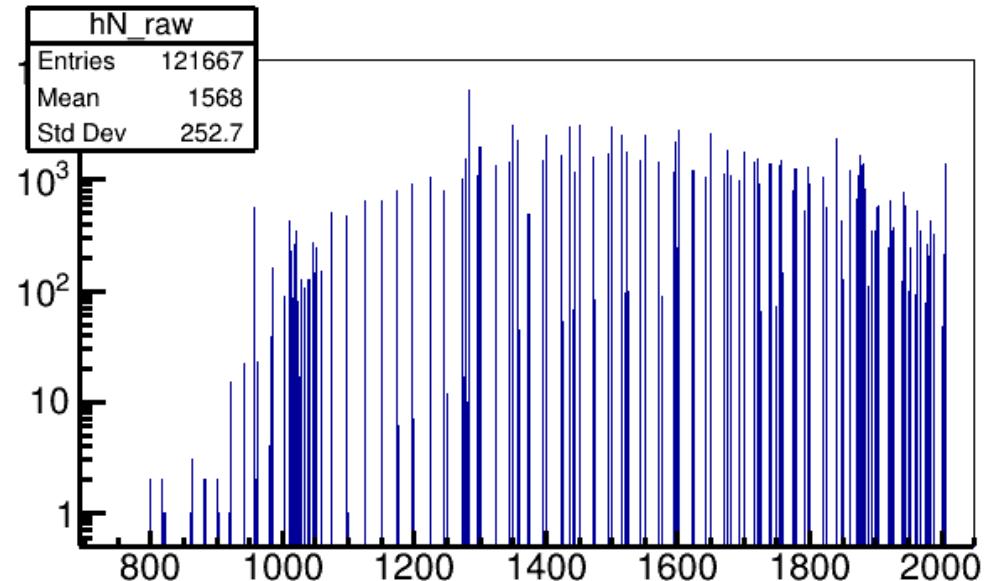
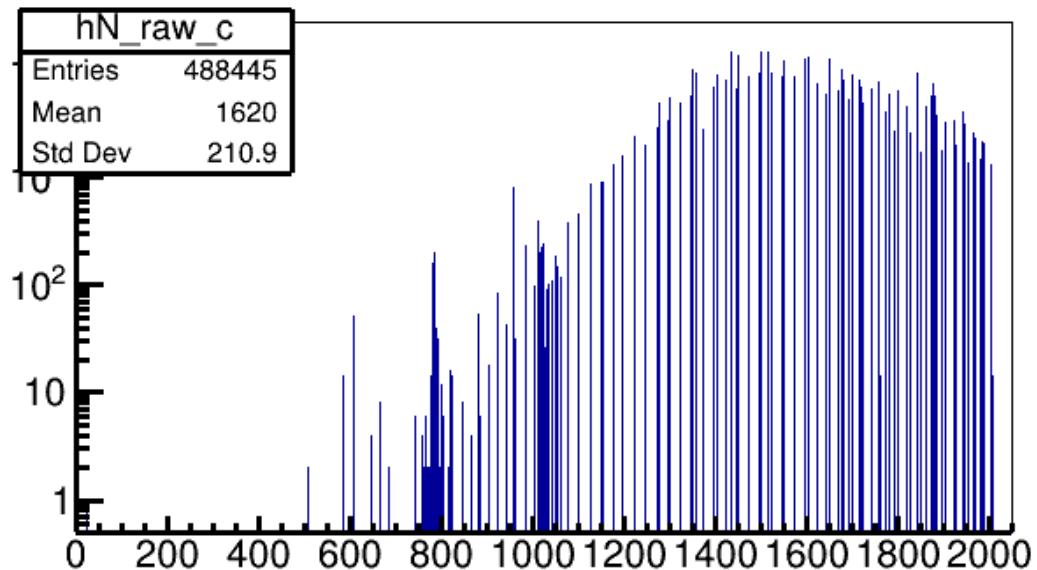
The cross-section of $e^+e^- \rightarrow 2\pi^0\pi^+\pi^-$ vs $E_{\text{c.m.}}$ for different scans
Soon we will publish the cross section of the cross section

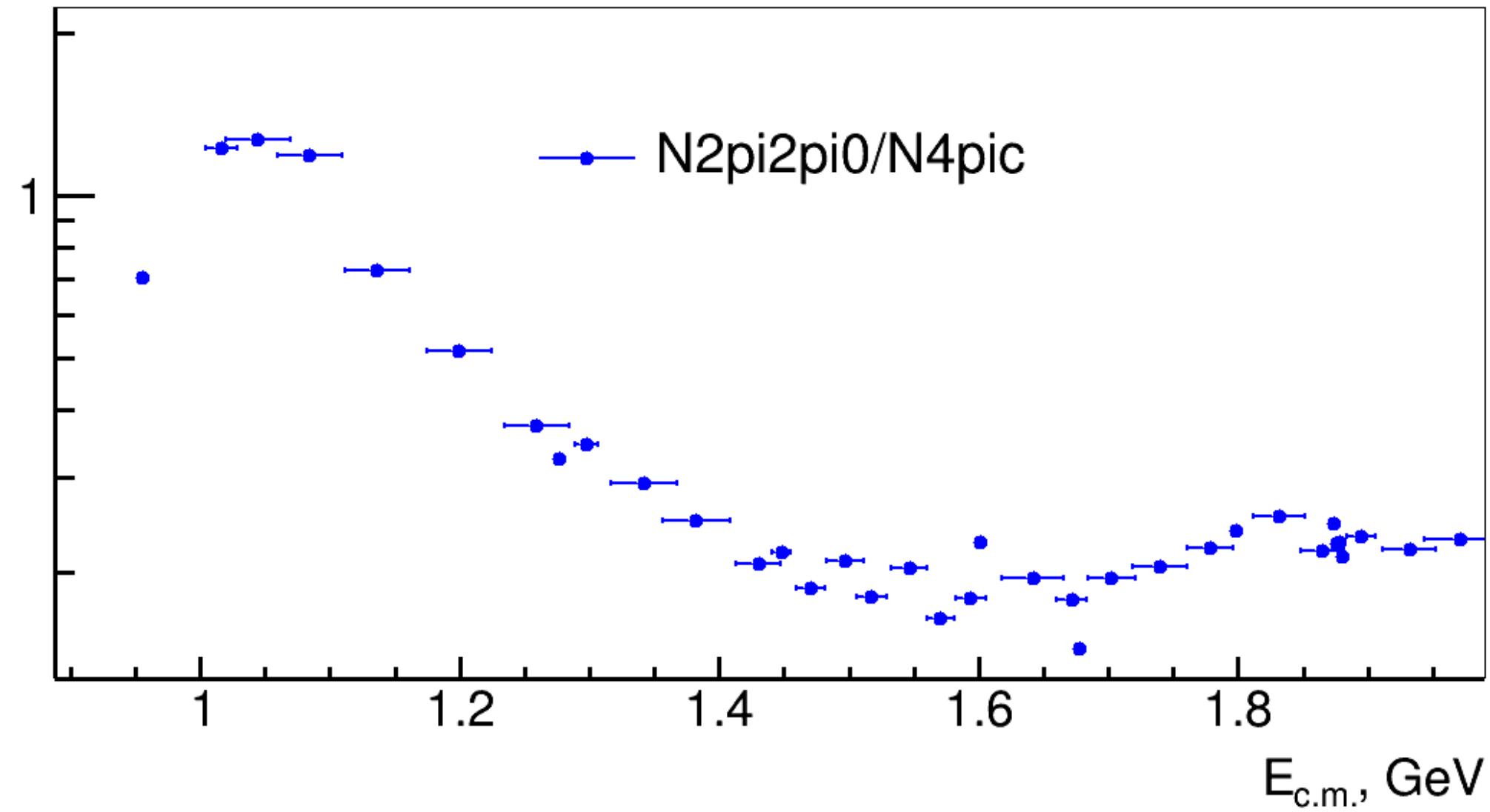
$$\varepsilon_{4\pi} = \frac{\sum_{det}^{ph.sp.} (|M(p_{\pi^0}, p_{\pi^0}, p_{\pi^+}, p_{\pi^-})|^2)}{\sum_{gen}^{ph.sp.} (|M(p_{\pi^0}, p_{\pi^0}, p_{\pi^+}, p_{\pi^-})|^2)}$$

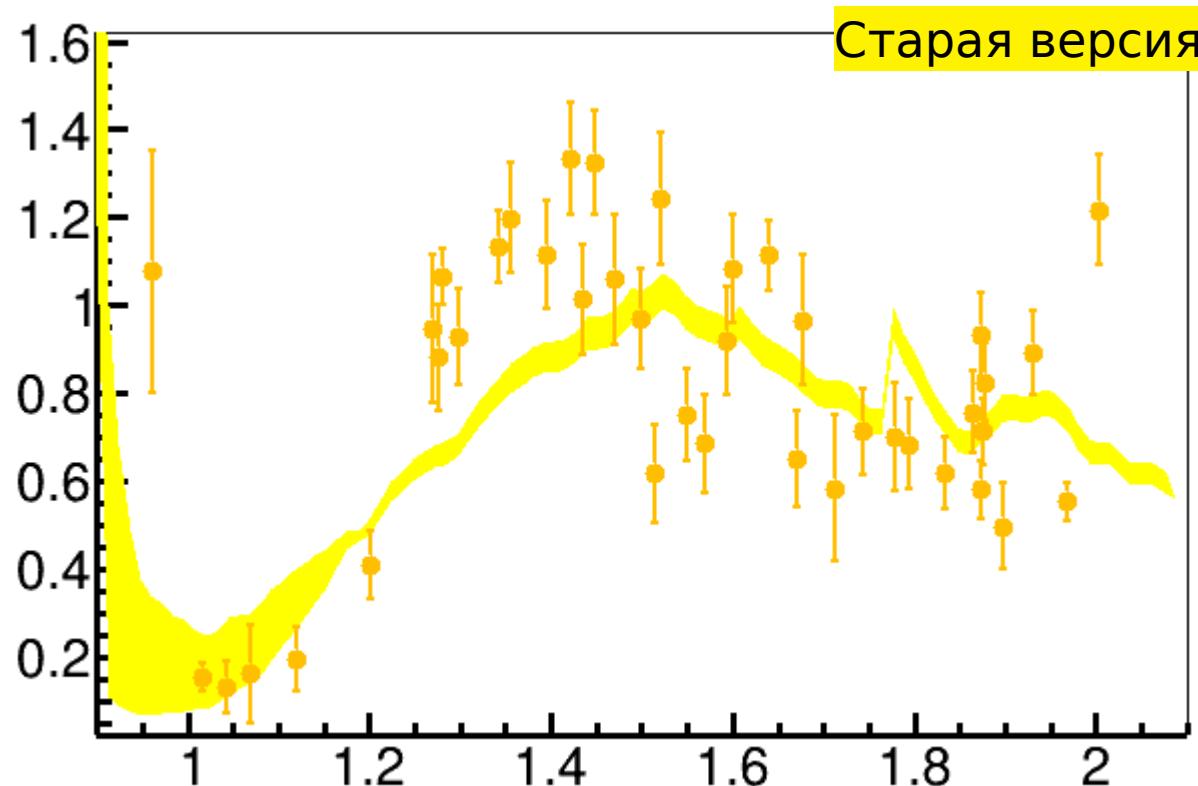
The max energy of simulated ISR is 0.2 Gev.



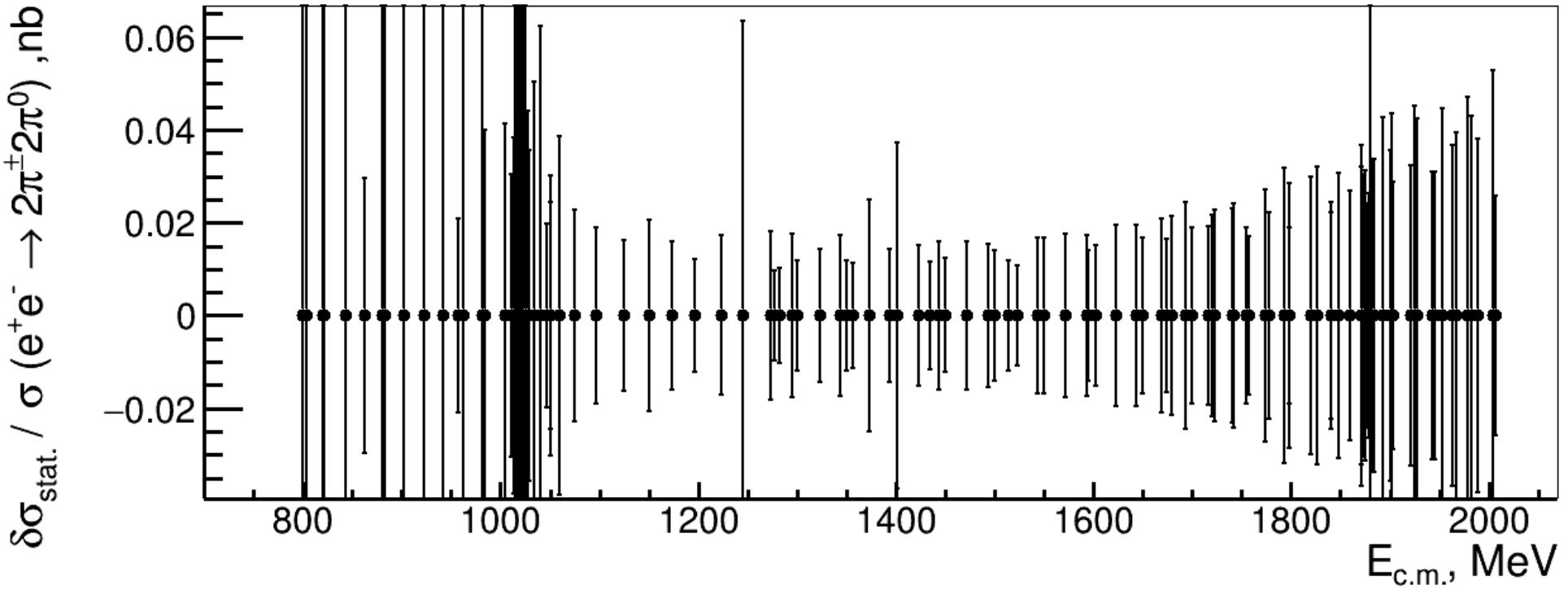
Detection efficiency vs. c.m. energy.







Отношение сечений $e^+e^- \rightarrow 2\pi^+2\pi^-$ к $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$.
Закрашенная область — эксперимент (из др. работ).
Точки с ошибками — результат амплитудного анализа (из данной работы).



Statistical precision is about 2-3% in average