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Изучение процесса электрон-позитронной аннигиляции в четыре пиона

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20.06.2019

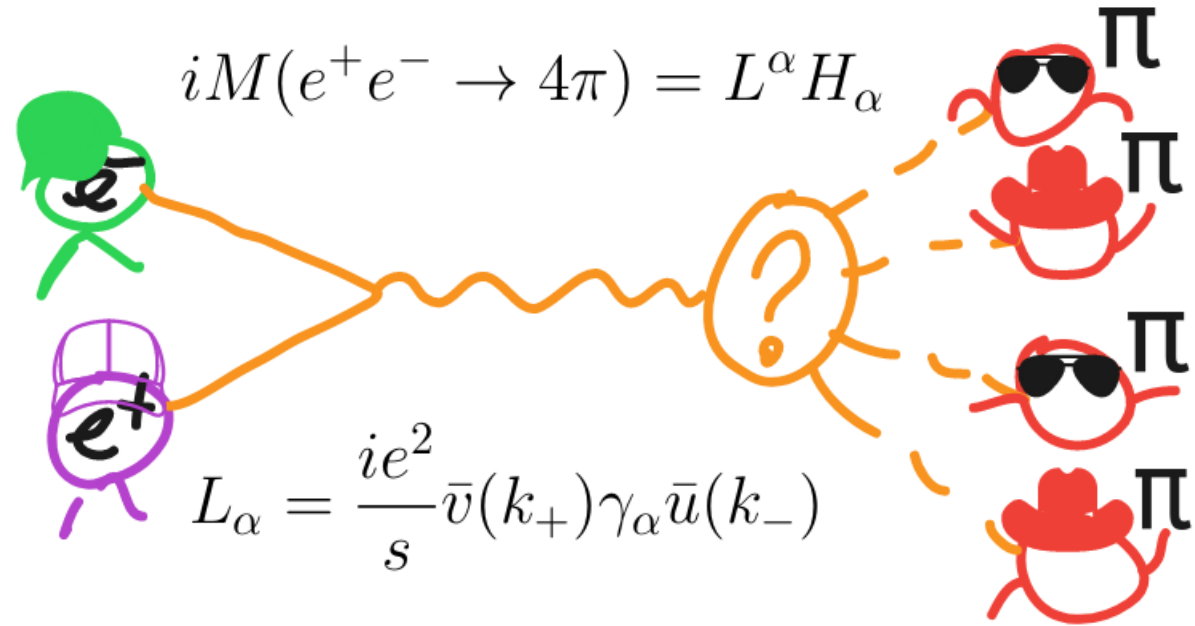
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Солодов Евгений Петрович

Оглавление

- Введение, цель работы
- История данного исследования
- Амплитудный анализ (*unbinned likelihood amplitude analysis*)
- Сравнение с экспериментом
- Сечение процесса
- Заключение

Цель работы

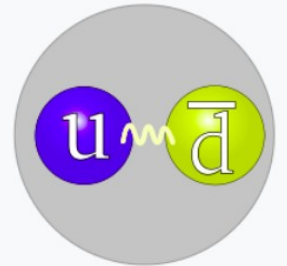
- Построить модель промежуточной динамики

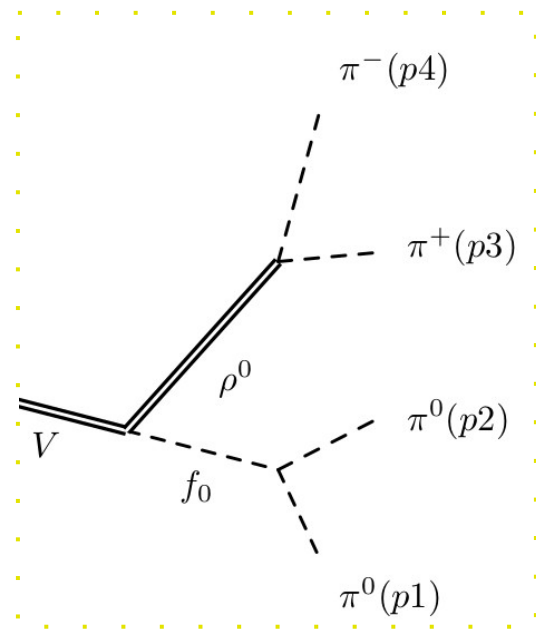
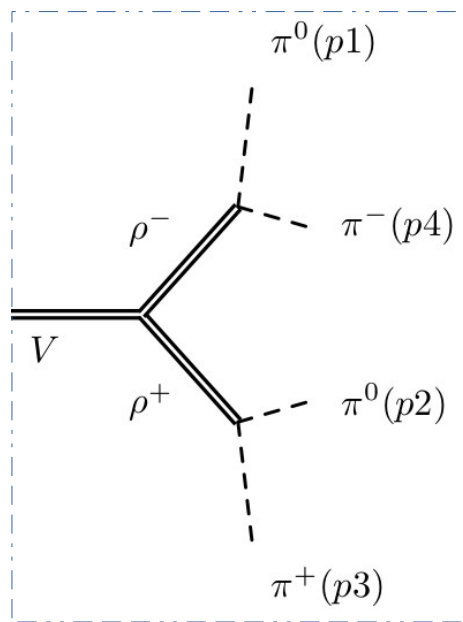
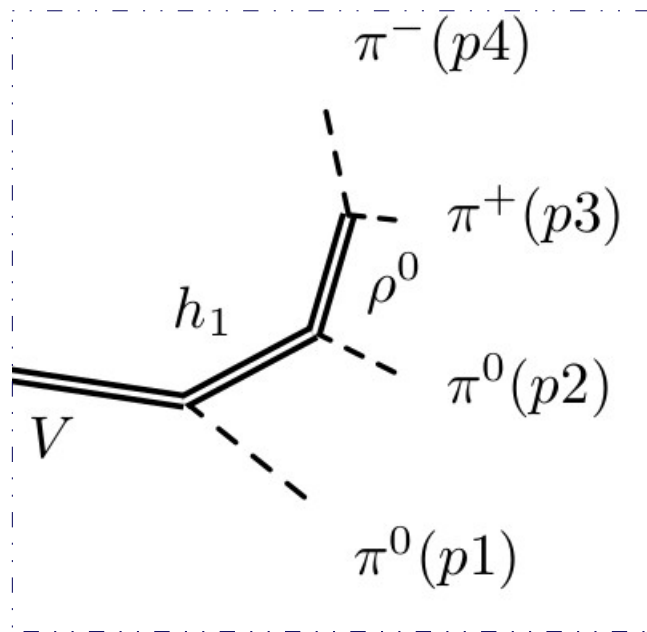
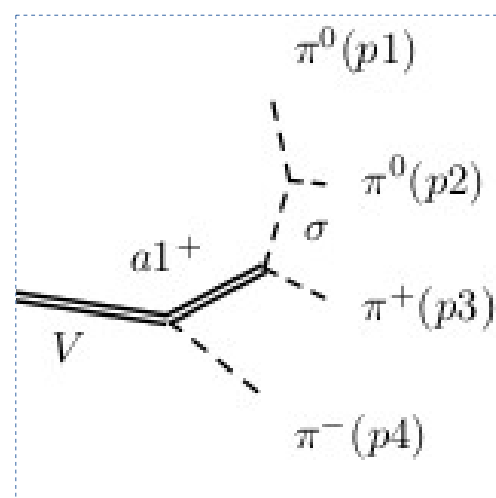
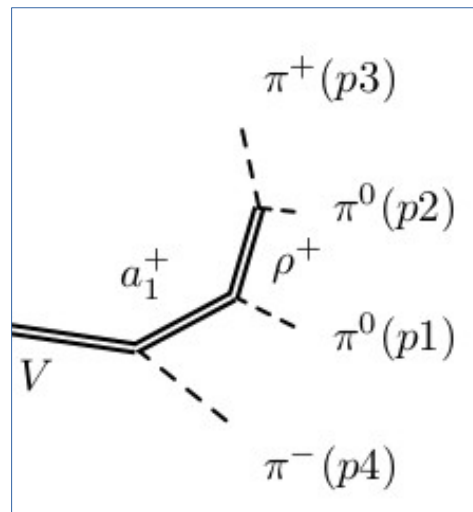
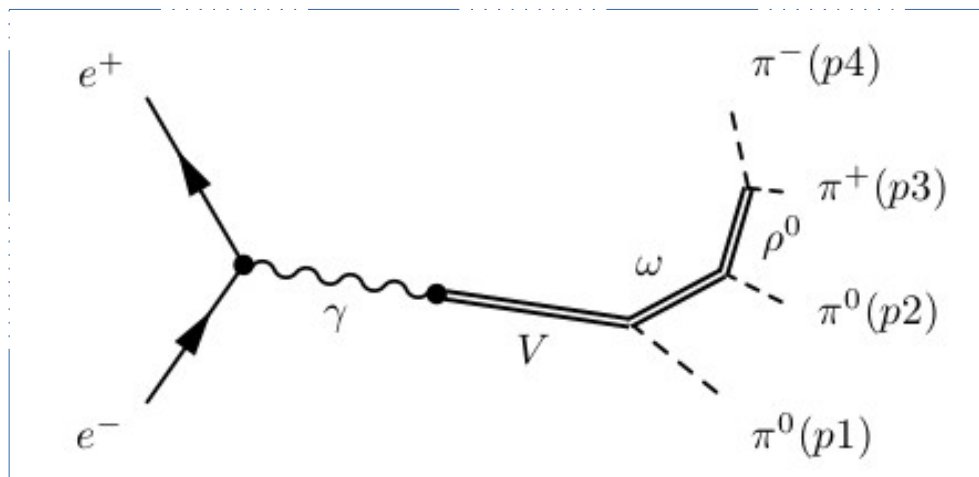


- Померить сечение

$$\sigma = \frac{N}{L} - ?$$

Пион (π^\pm (π^0))





Амплитудный анализ с детектором КМД-2 (5.8 pb⁻¹)

- The data in the $ee \rightarrow \pi^+\pi^-2\pi^0$ (22128 events) with $\sqrt{s} = [1.05-1.38]$ GeV is used

- The **dominance of the $\omega\pi$ and $a_1\pi$** is proved

- The data in $ee \rightarrow \pi^+\pi^-2\pi^0$ and $ee \rightarrow 2\pi^+2\pi^-$ (28552) is used for the estimation:
 $B(a_1 \rightarrow \sigma\pi)/B(a_1 \rightarrow \rho\pi) \sim 0.3$

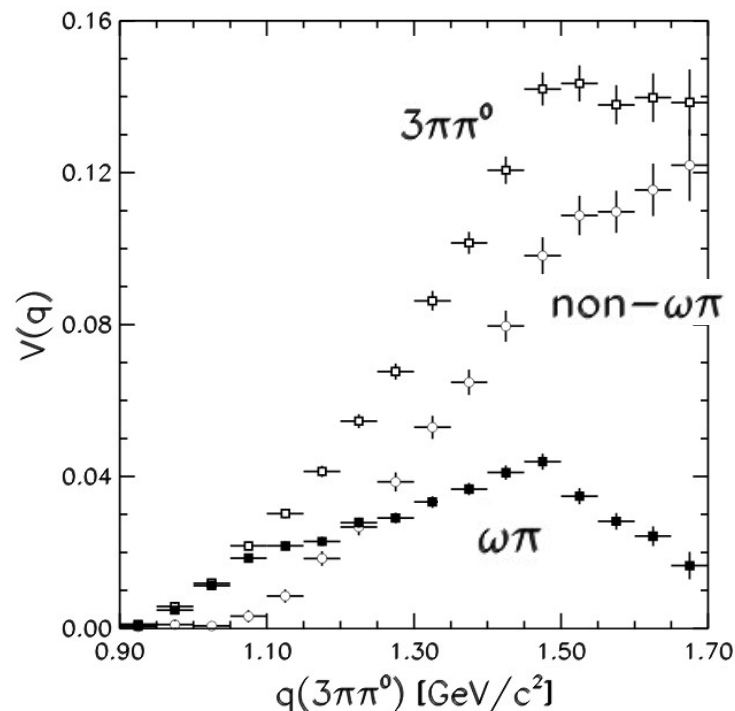
- The measured cross section are systematically shifted from other measurement.

The results of a search for the admixture of other possible states

Model	L_{\min}	r_X [%]	U.L. [%]
$\omega\pi^0 + a_1\pi$	1264	—	—
$\omega\pi^0 + a_1\pi + \rho\sigma$	1256	$2.1^{+1.2}_{-0.9}$	4.3
$\omega\pi^0 + a_1\pi + h_1\pi$	1263	$0.1^{+0.2}_{-0.1}$	0.4
$\omega\pi^0 + a_1\pi + a_2\pi$	1263	$0.2^{+0.4}_{-0.2}$	0.8
$\omega\pi^0 + a_1\pi + \pi'\pi$	1250	$9.5^{+3.2}_{-2.8}$	15.
$\omega\pi^0 + a_1\pi + \rho^+\rho^-$	1246	$4.7^{+2.0}_{-1.6}$	7.7

*R.R. Akhmetshin et al., Physics Letters B **466**, 392–402 (1999)*

Амплитудный анализ процесса $\tau \rightarrow 3\pi\pi^0\nu_\tau$ с детектором CLEO (1999)



Spectral functions

$$\frac{\Gamma(\tau^- \rightarrow \nu_\tau 2\pi^- \pi^+ \pi^0)}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = \frac{3 \cos^2 \theta_c}{2\pi \alpha^2 m_\tau^8} \int_0^{m_\tau^2} dQ^2 Q^2 (m_\tau^2 - Q^2)^2 (m_\tau^2 + 2Q^2) \cdot \left[\frac{1}{2} \sigma_{e^+e^- \rightarrow 2\pi^- 2\pi^+}(Q^2) + \sigma_{e^+e^- \rightarrow \pi^+ \pi^- 2\pi^0}(Q^2) \right]$$

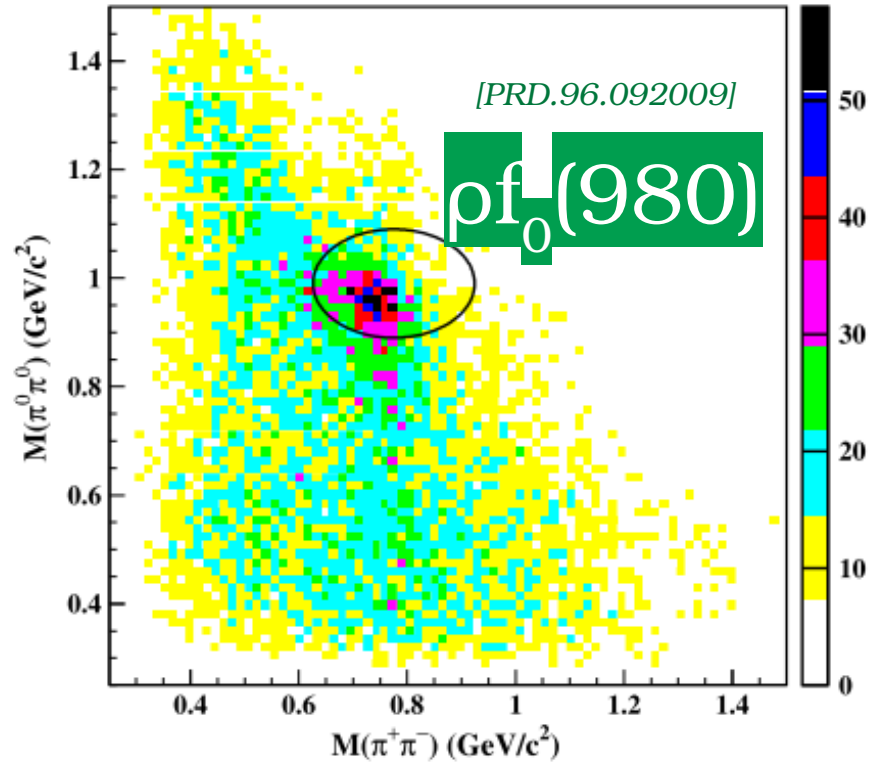
Model	Integrated amplitudes	Goodness-of-fit
Model 2	$R_{\omega\pi} = 0.38 \pm 0.02 \pm 0.02$ $R_{a_1\pi} = 0.43 \pm 0.02 \pm 0.02$	< 5%
Model 3	$R_{\omega\pi} = 0.38 \pm 0.02 \pm 0.01$ $R_{a_1\pi} = 0.49 \pm 0.02 \pm 0.02$ $R_{\sigma\rho} = 0.01 \pm 0.02 \pm 0.01$ $R_{f_0\rho} = 0.01 \pm 0.01 \pm 0.01$	20%

Fit results for various models

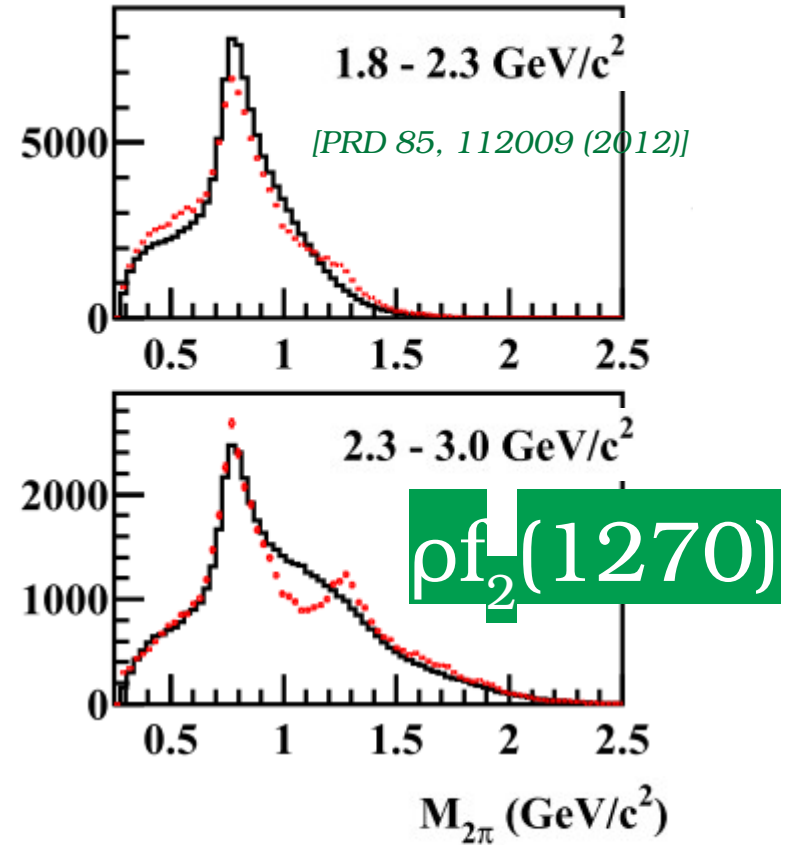
- Model with $\omega\pi$ and $a_1\pi$ and $\rho\sigma$, ρf_0 provides the best description of the data.

Physical Review D - Particles, Fields, Gravitation and Cosmology, **61**, 1-16 (2000).

Наблюдение состояний ρf_0 и ρf_2 с детектором BaBar при $E_{\text{с.м.}} > 1.8$ ГэВ

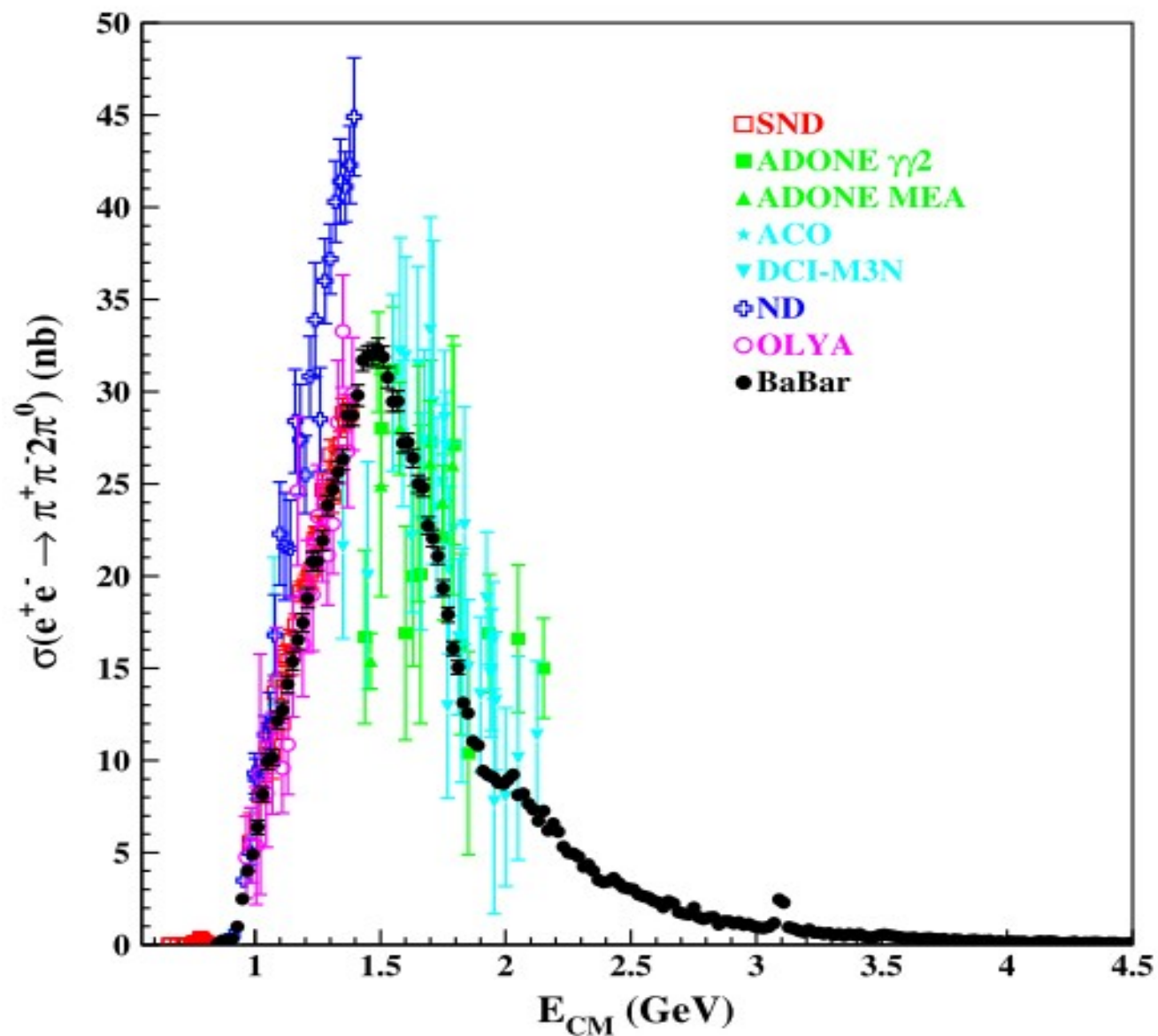


The evidence of $\rho f_0(980)$ in the process $e^+e^- \rightarrow 2\pi^0\pi^+\pi^-$ with BaBar



The evidence of $\rho f_2(1270)$ in the process $e^+e^- \rightarrow 2\pi^+2\pi^-$ with BaBar ⁷

Измерения
 $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ на
других
экспериментах



Общая стратегия амплитудного анализа

Signal selection
($ee \rightarrow \pi^+\pi^-2\pi^0$)

Signal selection
($ee \rightarrow 2\pi^+2\pi^-$)

Building of the amplitudes formalism

**The definition and minimization
of likelihood function (L)**

Model vs Experiment comparison

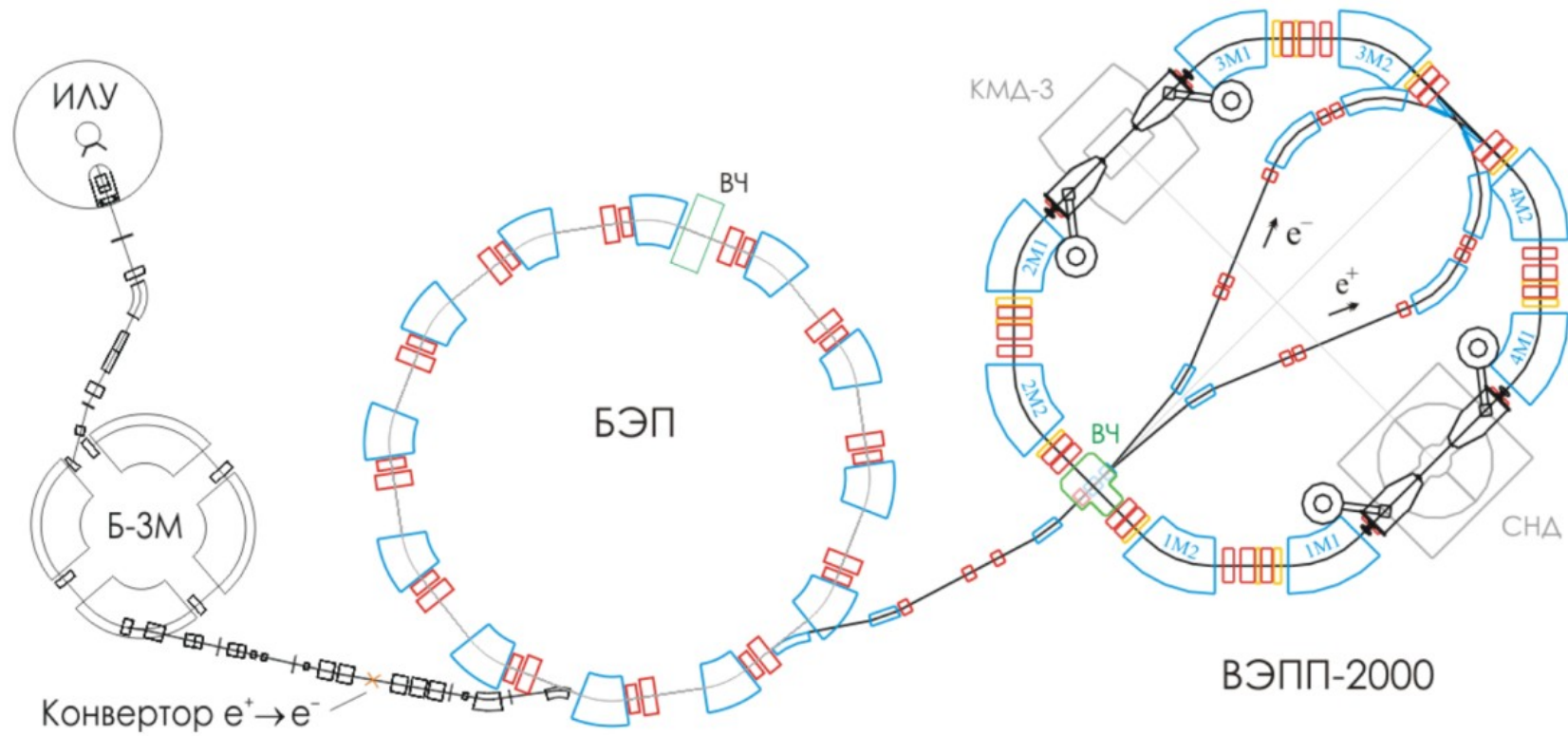
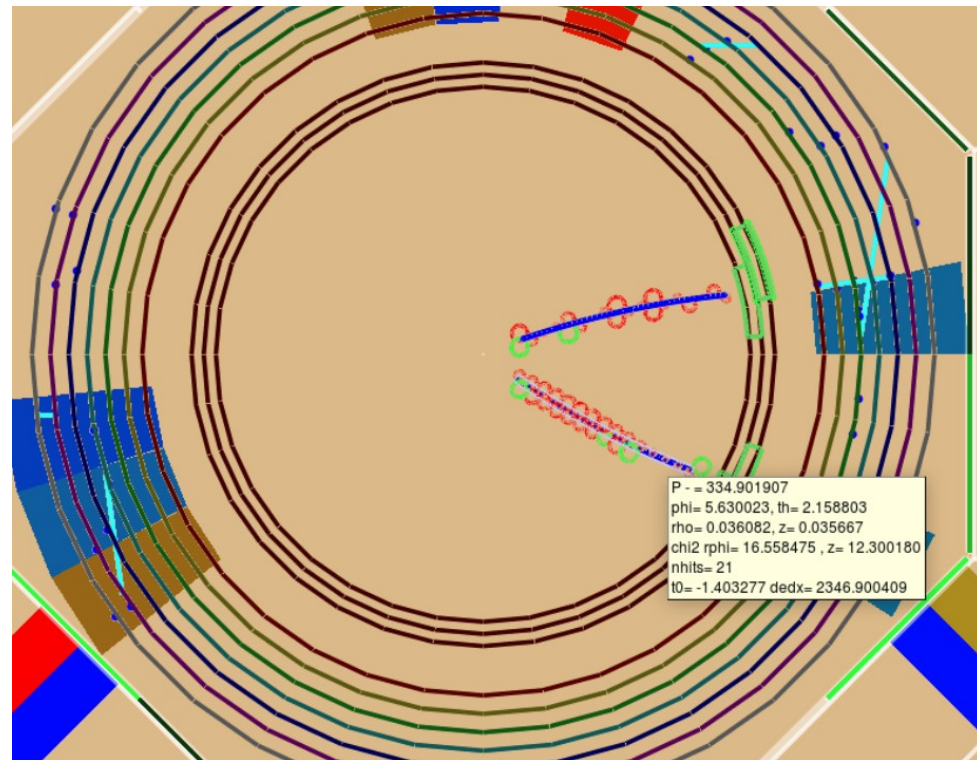
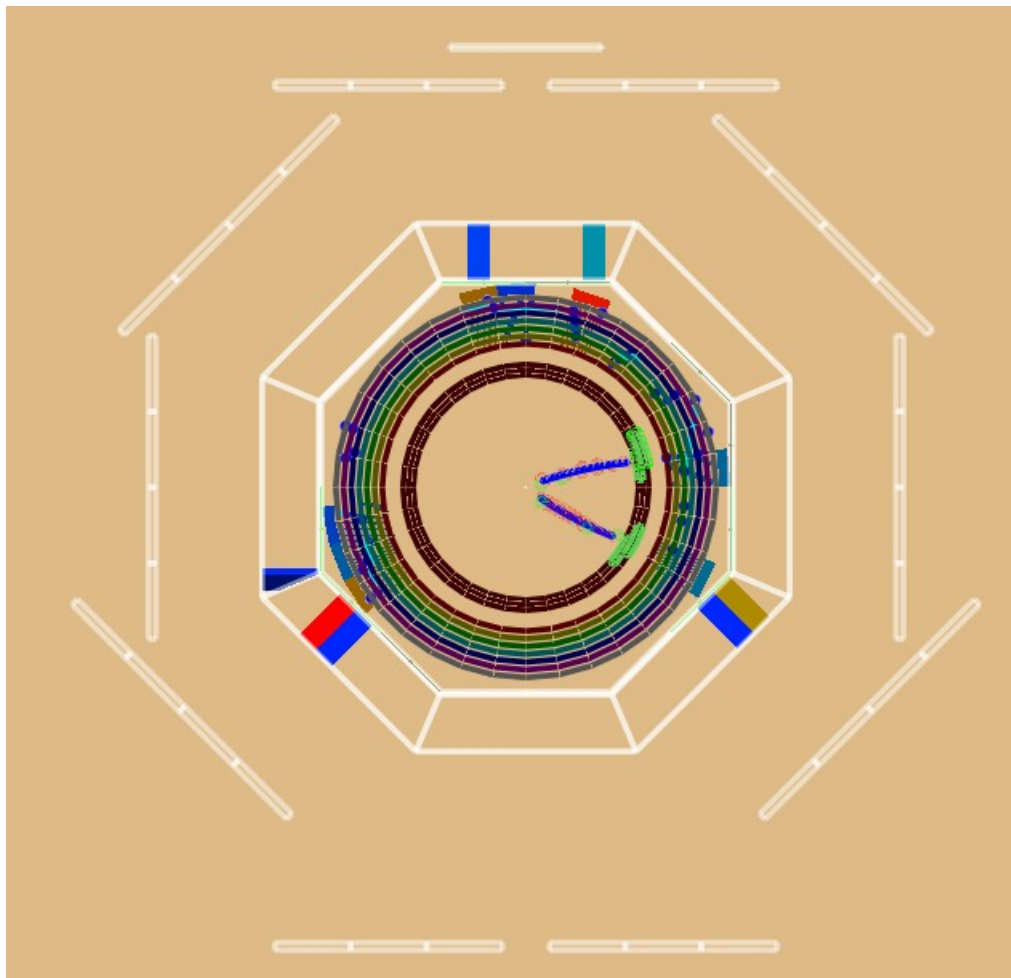


Схема ускорительного комплекса ВЭПП-2000

Сезон	Светимость, пб^{-1}
scan2011	20.16
scan2012	13.34

scan2012_omphi	0.68
scan2013_rho	11.59
scan2013_omphi	
scan2017	6.07

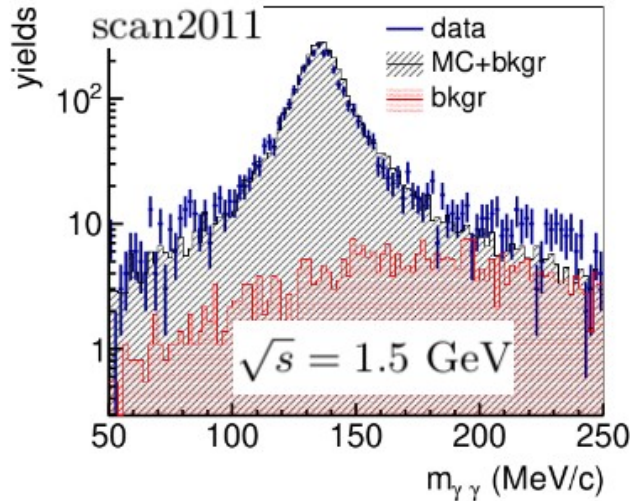
Отбор событий $ee \rightarrow \pi^+\pi^2\pi^0$



Визуализация типичного события $e^+e^- \rightarrow 2\pi^0\pi^+\pi^-$

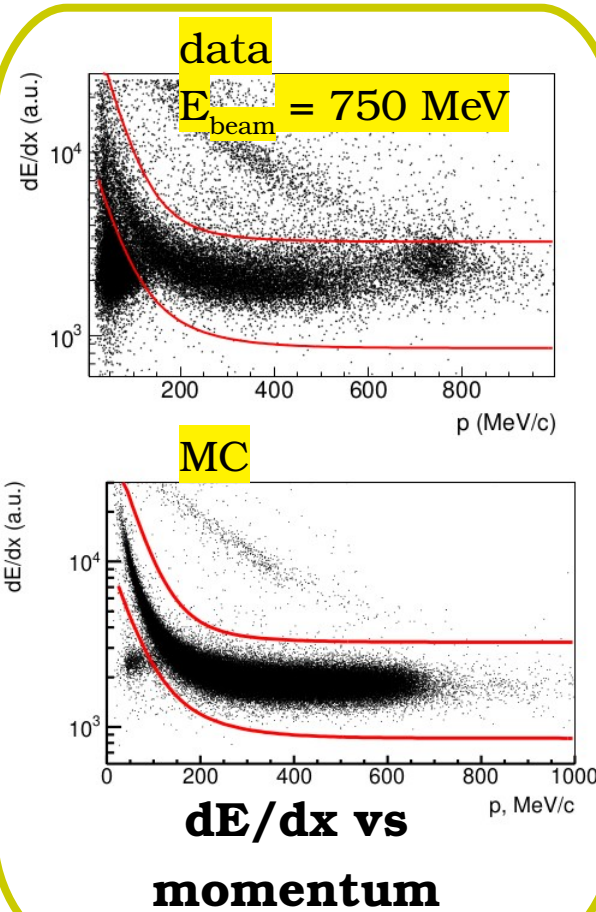
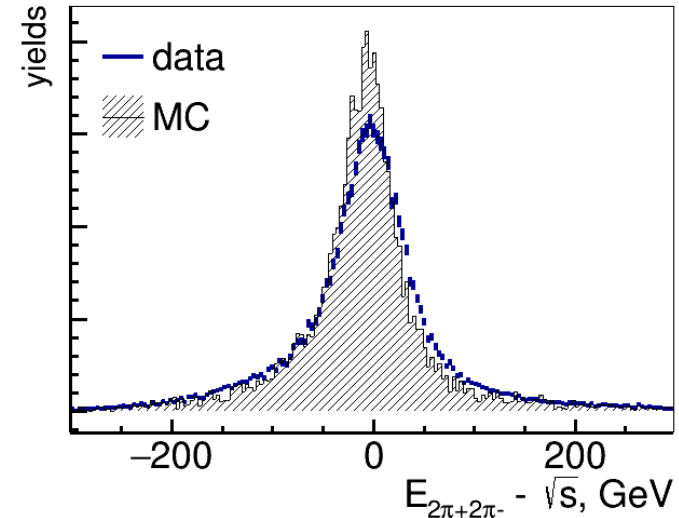
Отбор событий ($ee \rightarrow \pi^+\pi^-2\pi^0$) 64 kevents

- $0.7 < \theta_{\pi,\gamma} < \pi - 0.7$ rad
- Total ($E-\sqrt{s}$) and $P < 150$ MeV/(c)
- Two candidates for π^0
- 5C kinematic fit
- The **invariant mass** spectrum of 3rd and 4th **photons** is used for the estimation of the contribution of background:



Отбор событий ($ee \rightarrow 2\pi^+2\pi^-$) 72 kevents

- $0.7 < \theta_{\pi} < \pi - 0.7$ rad
- Total ($E-\sqrt{s}$) and $P < 150$ MeV/(c)
- 4C kinematic fit
- The spectrum of **total energy of four tracks** ($E-\sqrt{s}$) is used for the estimation of the contribution of background:



Функция правдоподобия

The production of 4π system can proceed via a list of intermediate states:

- $\omega[1^{--}]\pi^0[0^{-+}]$ (only $2\pi^0 2\pi^\pm$)
- $a_1(1200)[1^+]\pi[0^-]$
- $\rho[1^{--}]f_0/\sigma[0^{++}]$
- $\rho f_2(1270)[2^{++}]$
- $\rho^+\rho^-$ (only $2\pi^0 2\pi^\pm$)
- $a_2(1320)[2^{++}]\pi$
- $h_1(1170)[1^{+-}]\pi^0$ (only $2\pi^0 2\pi^\pm$)
- $\pi'(1300)(0^{-+})\pi$

The relative number of events I at a particular point Ω in phase space can be represented as

$$I(\Omega) = |V_\alpha A_\alpha(\Omega)|^2$$

where the sum runs over all intermediate states, V_α - the complex production amplitude (the free parameter) and $A_\alpha(\Omega)$ - the amplitude at a particular point in phase space.

Функция правдоподобия

The likelihood for model under test is

$$L = -\log \prod_{i=signal}^{p\bar{i}^+ p\bar{i}^- 2p\bar{i}^0} \frac{I(\Omega_i)}{\int \varepsilon I(\Omega) d\Omega} - \log \prod_{i=signal}^{2p\bar{i}^+ 2p\bar{i}^-} \frac{I(\Omega_i)}{\int \varepsilon I(\Omega) d\Omega}$$

The limited acceptance and efficiency of the detector is taken into account by summing only over simulated events that pass the reconstruction and analysis cuts.

$$\int \varepsilon I(\Omega) d\Omega = \frac{1}{N_{MC}^{gen}} \sum_{rec}^{phase\ space\ MC} |V_\alpha A_\alpha(\Omega)|^2$$

- An amplitude is normalized to 1: $\int |A_\alpha(\Omega)|^2 d\Omega = 1$;
- The $\omega\pi^0$ amplitude is clearly seen at all energies, so $A_{\omega\pi^0}$ fixes at 1;

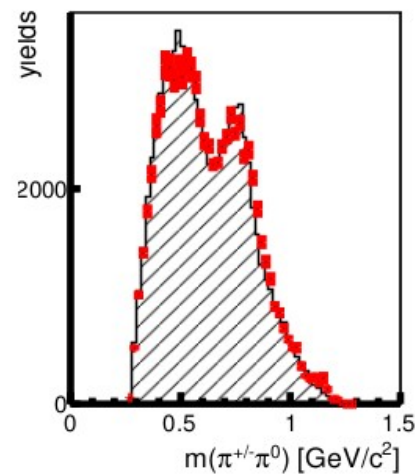
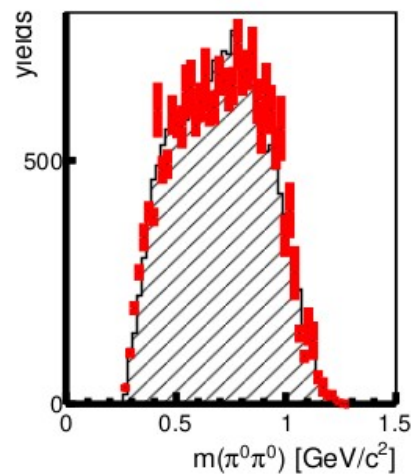
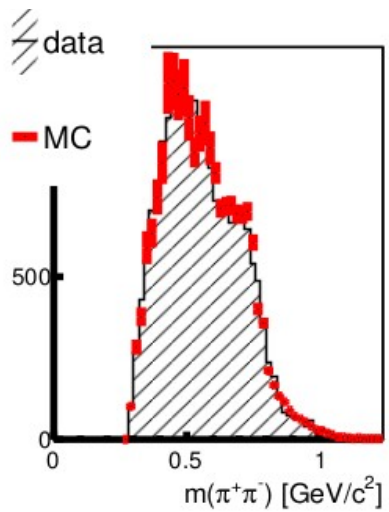
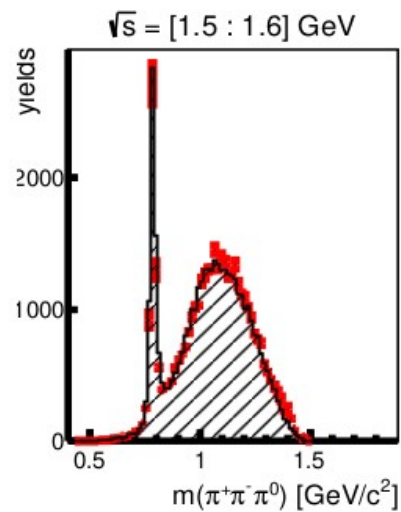
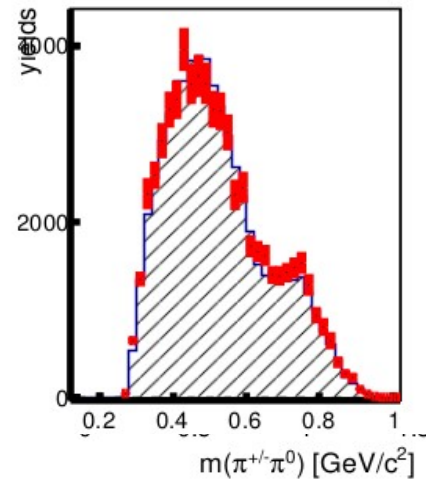
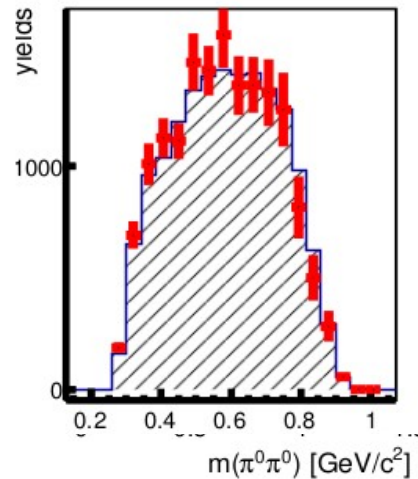
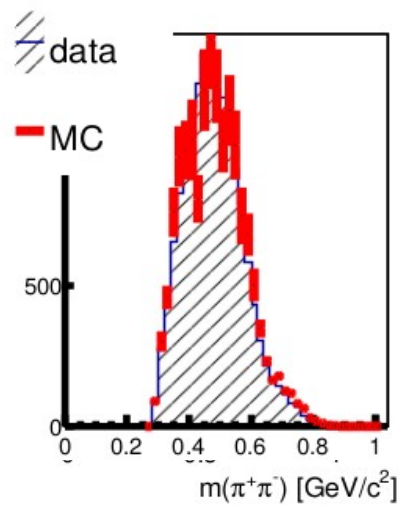
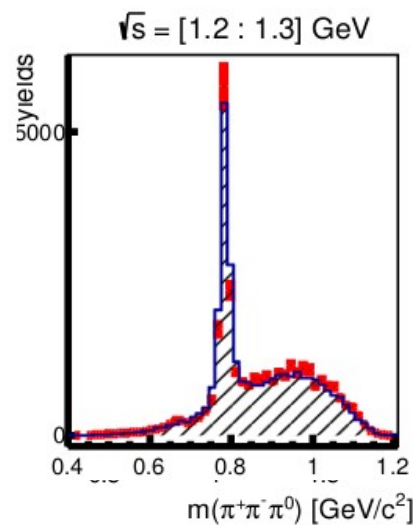
Функция правдоподобия

- The amplitudes is symmetric (anti-symmetric) with respect to the interchange of the momenta of neutral (charged) mesons according to Bose symmetry and C-parity conservation.

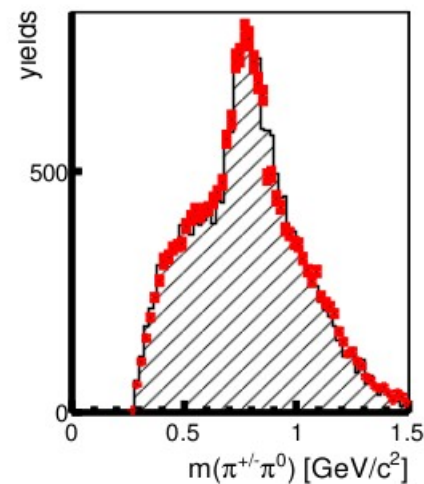
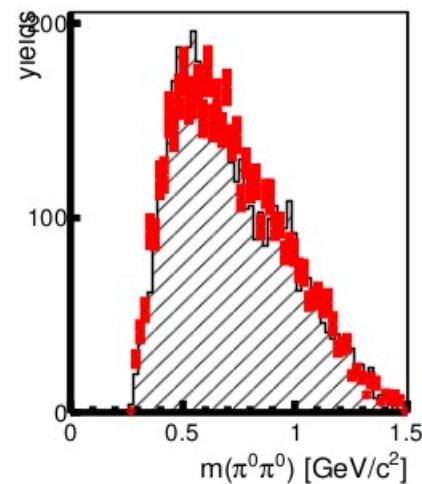
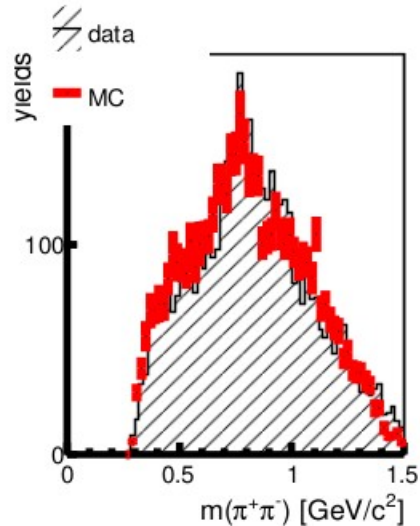
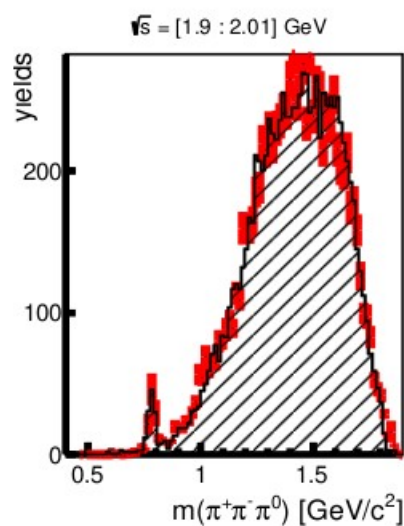
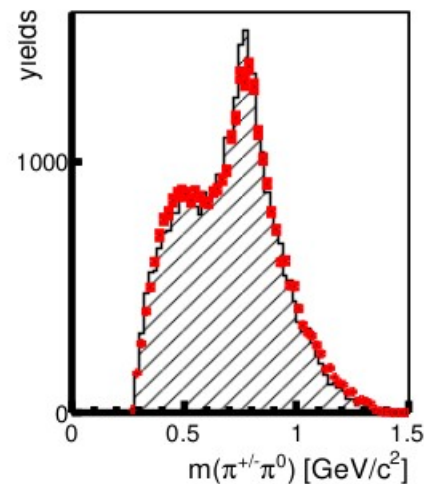
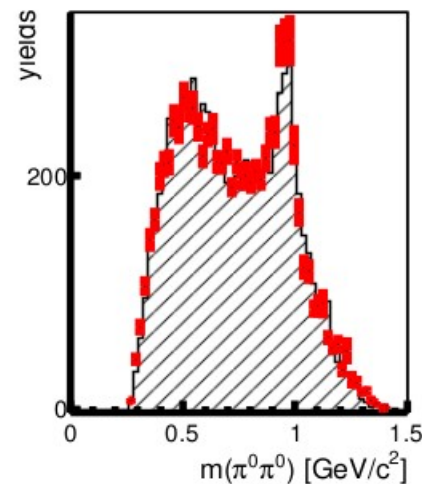
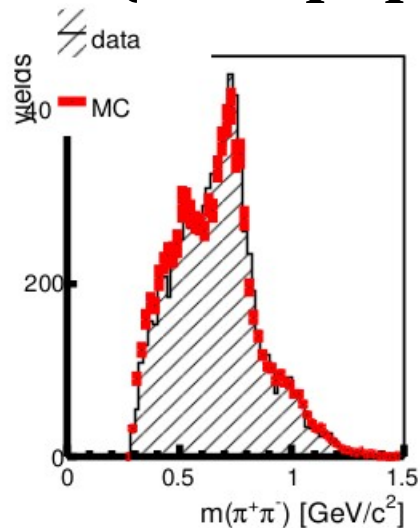
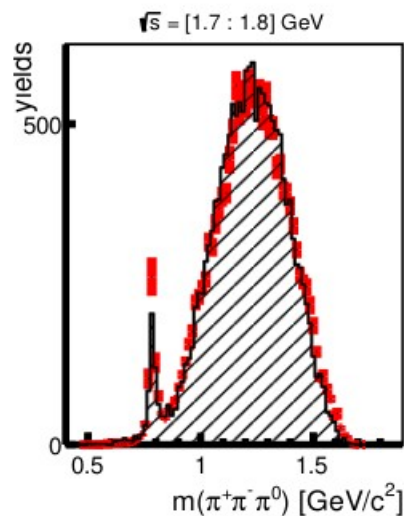
$$\begin{aligned}
 L(\omega\rho\pi) &= g_{\omega\rho\pi} \cdot \epsilon_{\mu\nu\rho\sigma} \cdot \delta^{ab} \cdot \omega_\mu \cdot d_\nu \pi^{*a} \cdot (d_\rho \rho_\sigma^{*b} - d_\sigma \rho_\rho^{*b}), \\
 L(a_1\rho\pi) &= g_{a_1\rho\pi} \cdot \epsilon^{abc} \cdot a_{1\mu}^a \cdot d_\nu \pi^{*b} \cdot (d_\mu \rho_\nu^{*c} - d_\nu \rho_\mu^{*c}), \\
 L(a_1\sigma\pi) &= g_{a_1\sigma\pi} \cdot \delta^{ab} \cdot (d_\mu a_{1\nu}^a - d_\nu a_{1\mu}^a) \cdot d_\mu \phi^*(\sigma) \cdot d_\nu \phi^{*b}(\pi), \\
 L(\rho'\rho f_0) &= g_{\rho'\rho f_0} \cdot \delta^{ab} \cdot (d_\mu \rho_\nu'^a - d_\nu \rho_\mu'^a) (d_\mu \rho_\nu^{*b} - d_\nu \rho_\mu^{*b}) \cdot \phi_{f_0}^*, \\
 L(\rho'\rho^+\rho^-) &= g_{\rho'\rho^+\rho^-} \cdot \epsilon^{abc} (d_\mu \rho_\nu'^a - d_\nu \rho_\mu'^a) \cdot (d_\alpha \rho_\nu^{*b} - d_\nu \rho_\alpha^{*b}) \cdot (d_\mu \rho_\alpha^{*c} - d_\alpha \rho_\mu^{*c}), \\
 L(\rho'h_1\pi^0) &= g_{\rho'h_1\pi^0} \cdot \delta^{ab} (d_\mu \rho_\nu'^a - d_\nu \rho_\mu'^a) \cdot (d_\mu h_{1\nu}^{*b} - d_\nu h_{1\mu}^{*b}) \phi_\pi^*,
 \end{aligned}$$

- Masses and central values of widths of resonances are fixed according to PDG.
- Flatté distribution is used for the propagator of ρ_0 (1980).

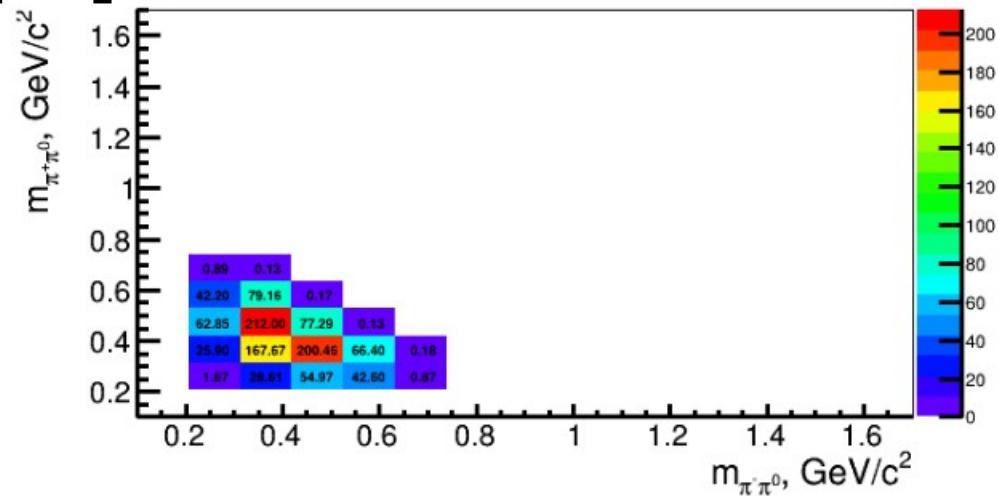
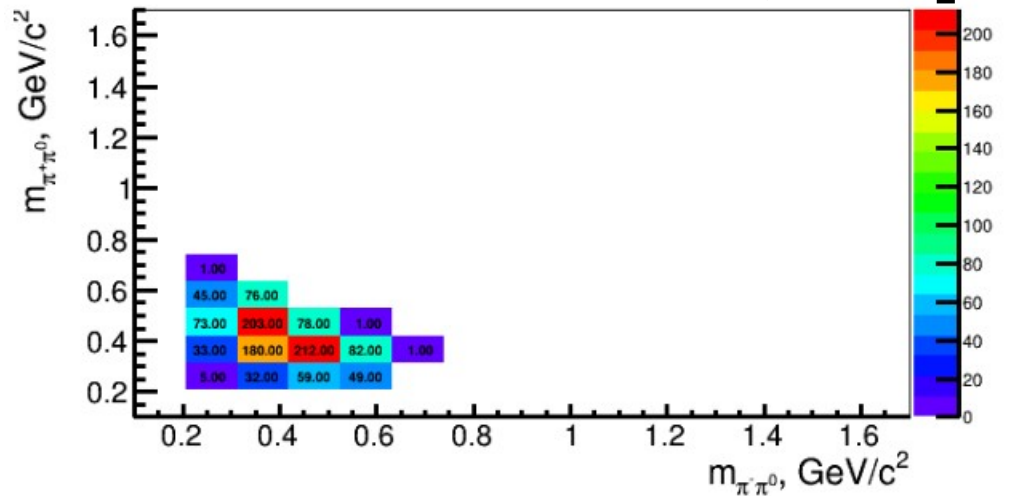
$(ee \rightarrow \pi^+\pi^-\pi^0)$



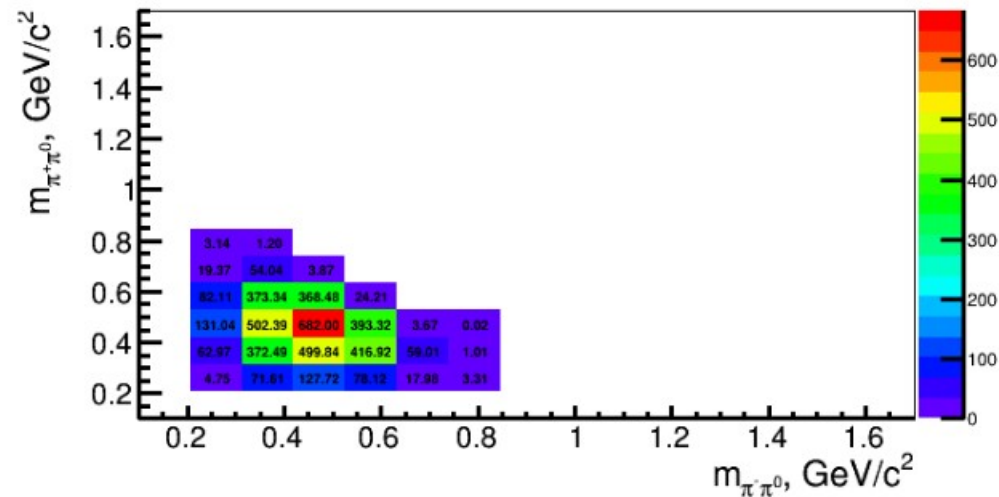
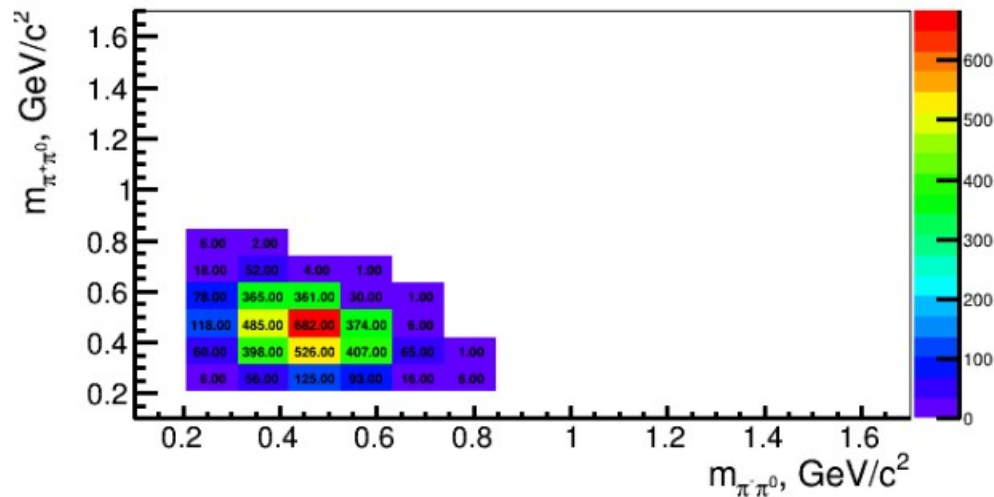
$(ee \rightarrow \pi^+\pi^-\pi^0)$



$(ee \rightarrow \pi^+\pi^-\pi^0)$

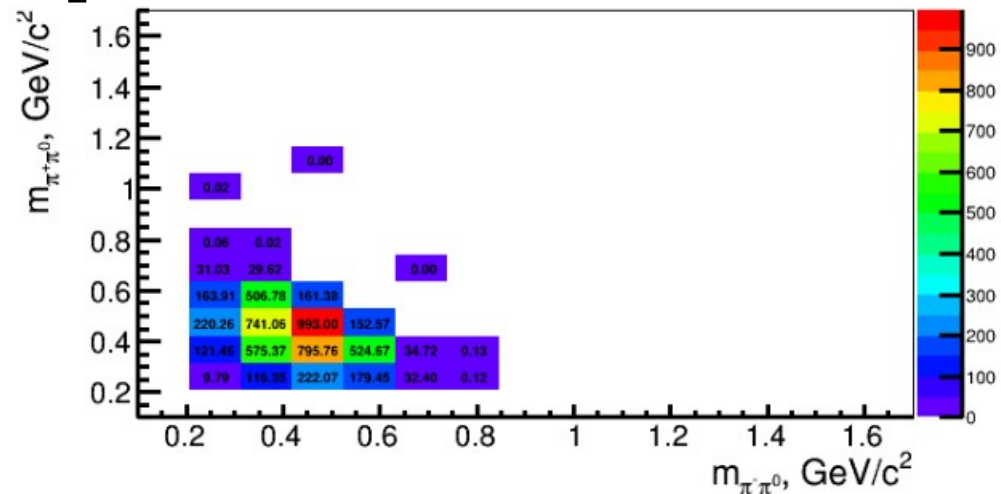
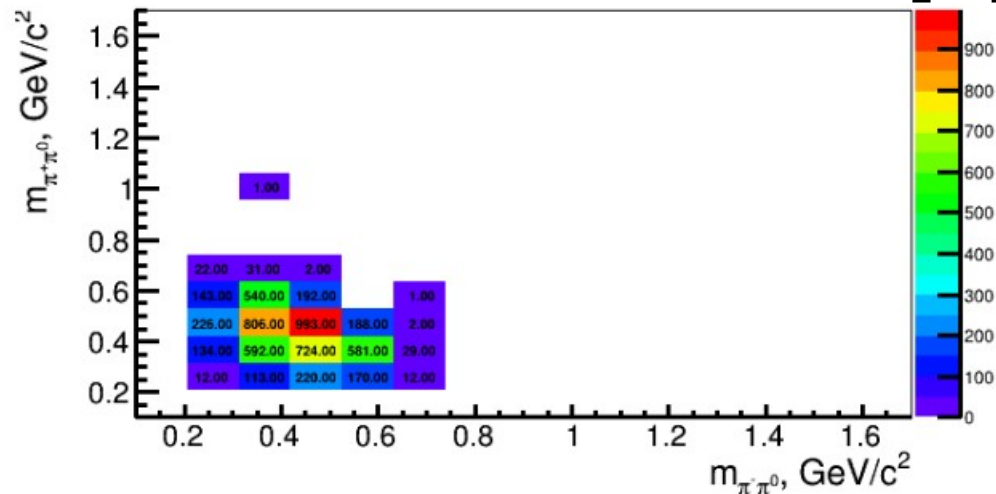


1.03-1.1 GeV

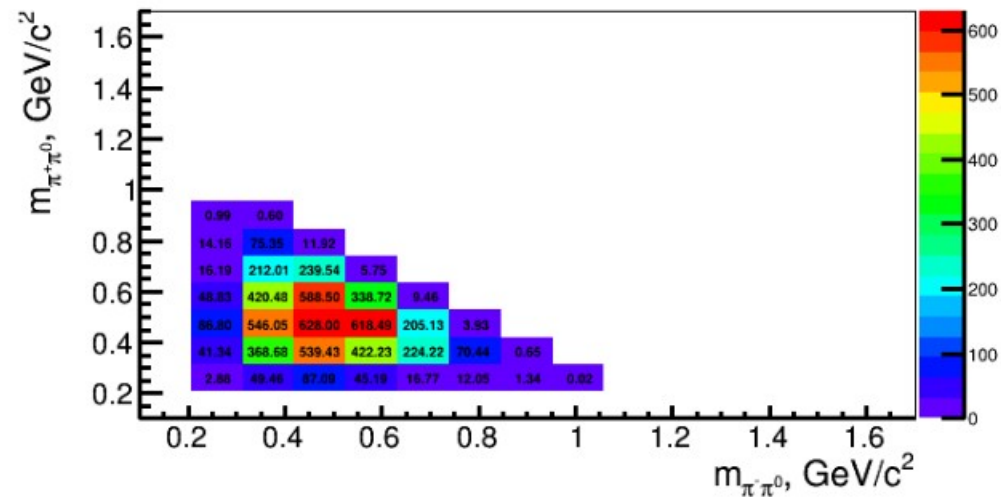
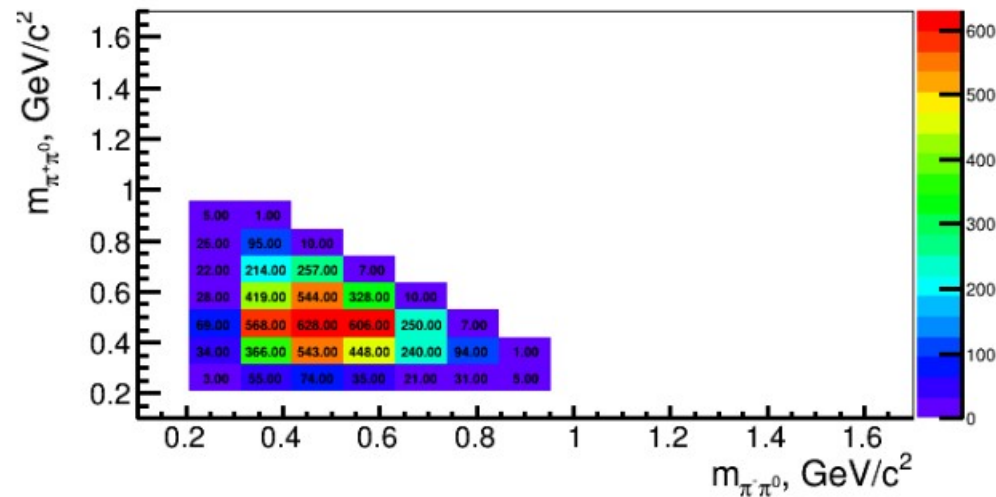


($ee \rightarrow \pi^+\pi^-\pi^0$)

1-1.03 GeV

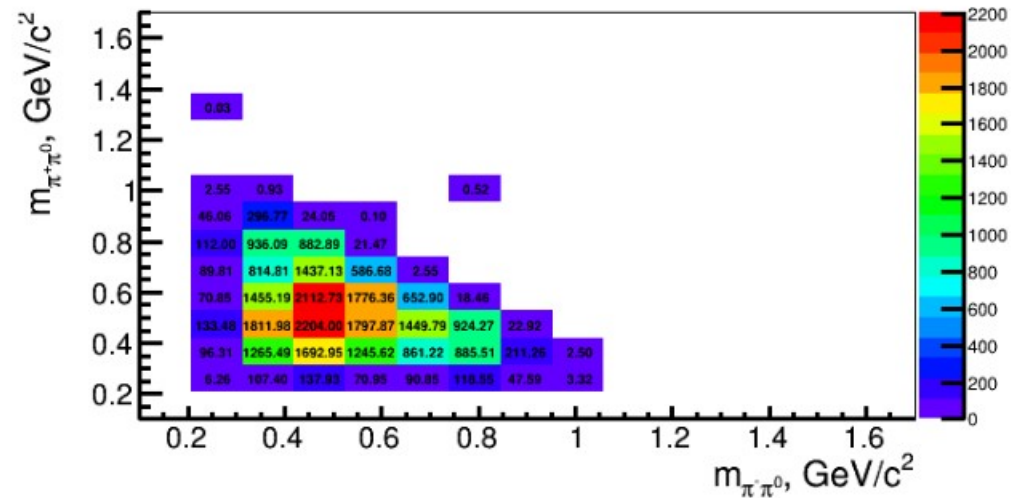
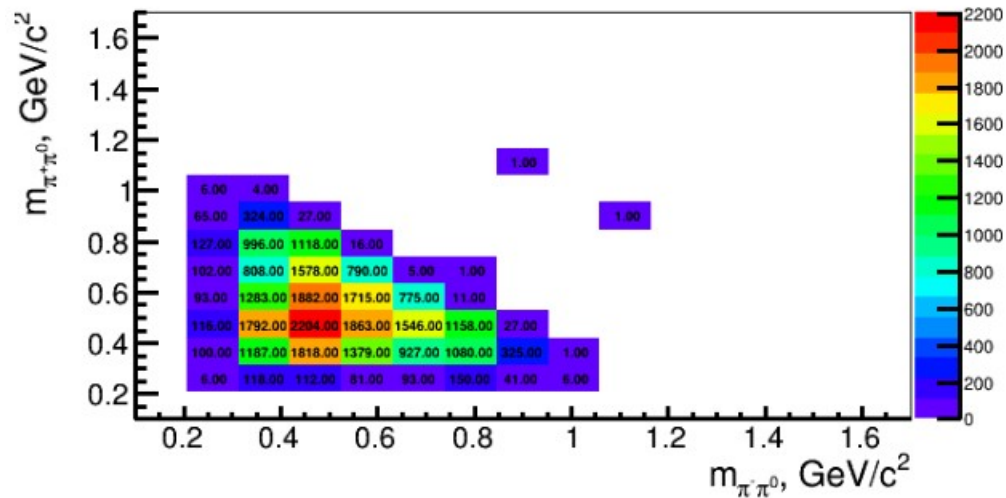


1.1-1.2 GeV

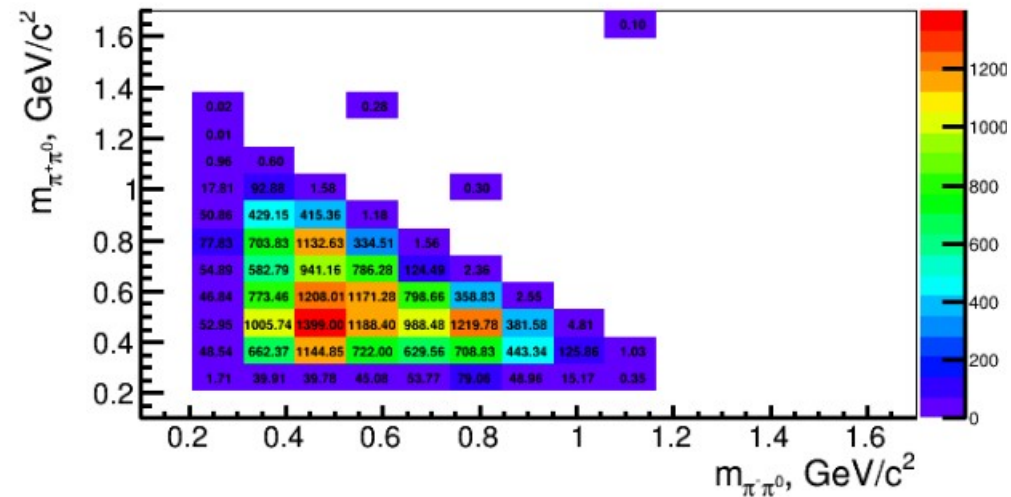
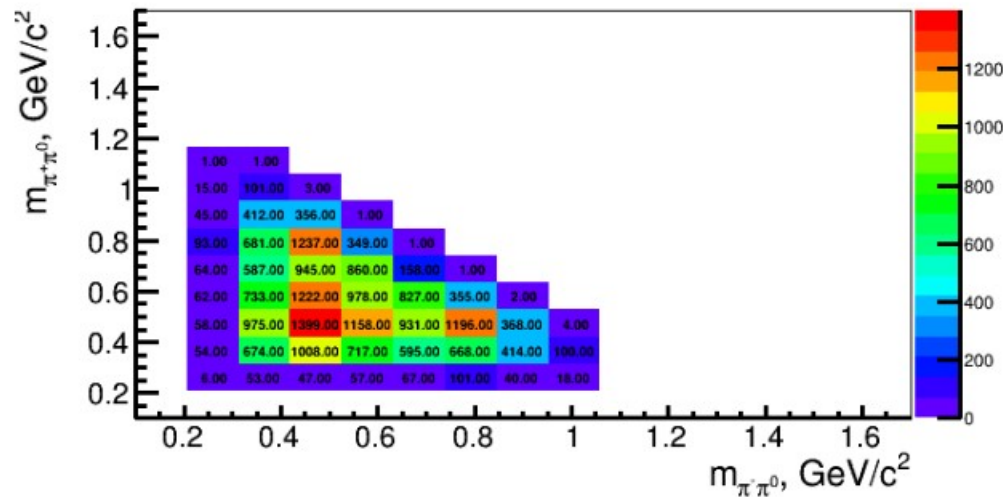


$(ee \rightarrow \pi^+\pi^-\pi^0)$

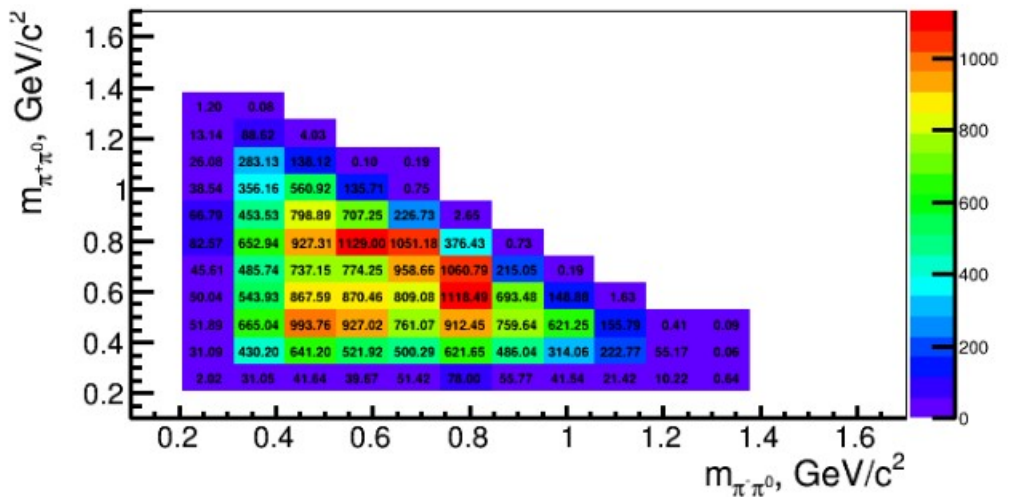
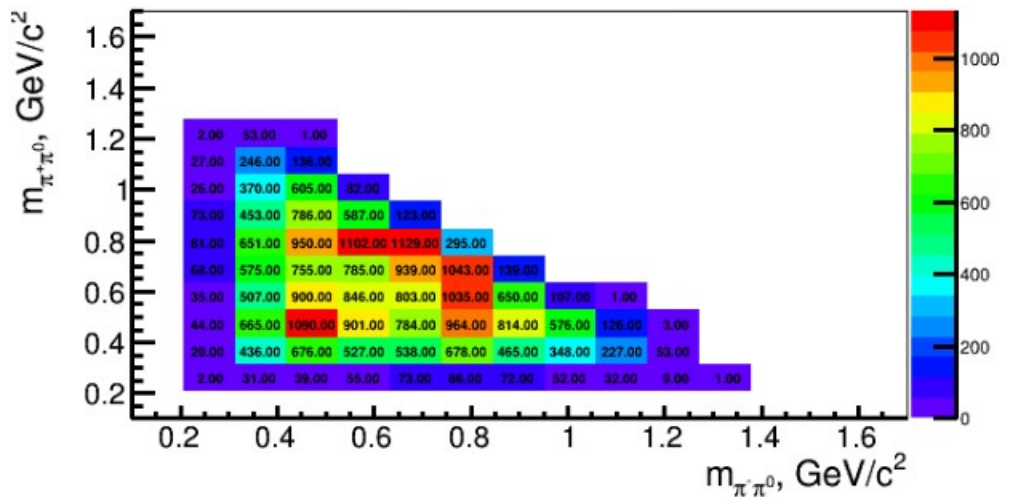
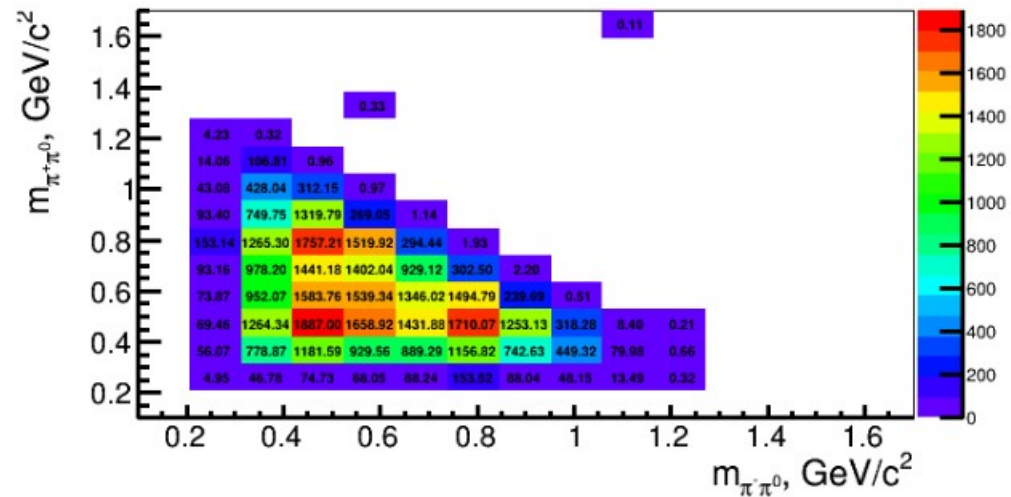
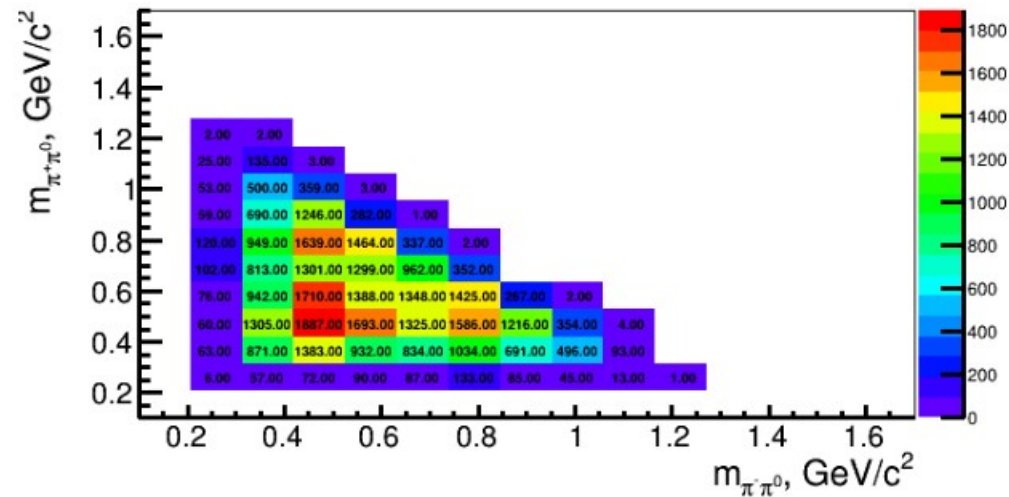
1.2-1.3 GeV



1.3-1.4 GeV

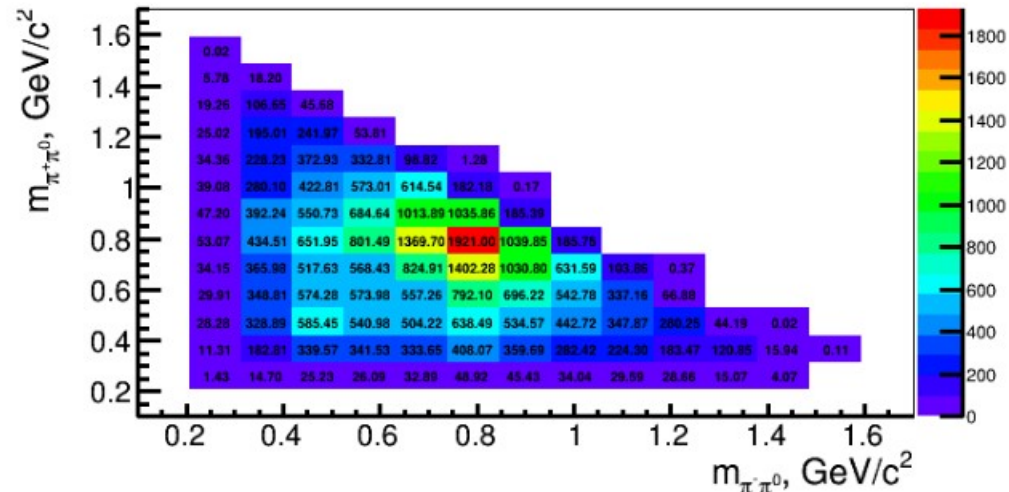
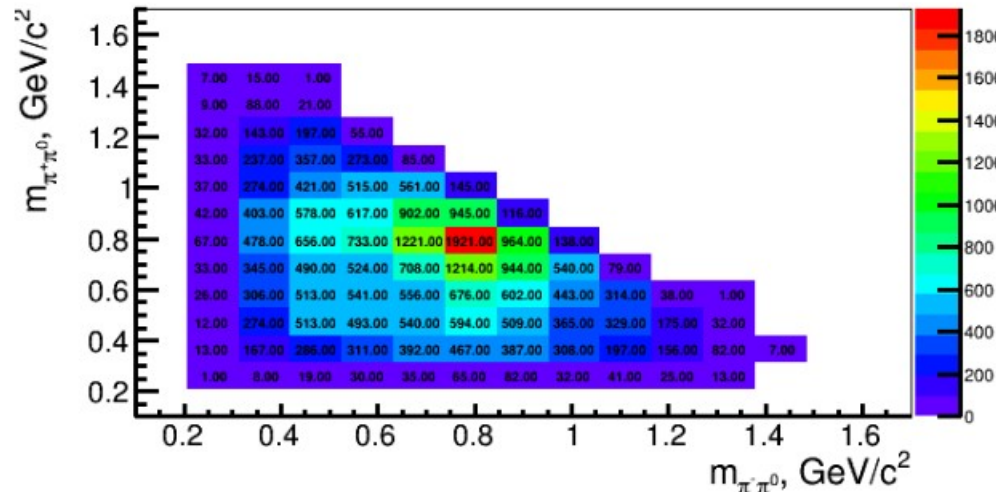
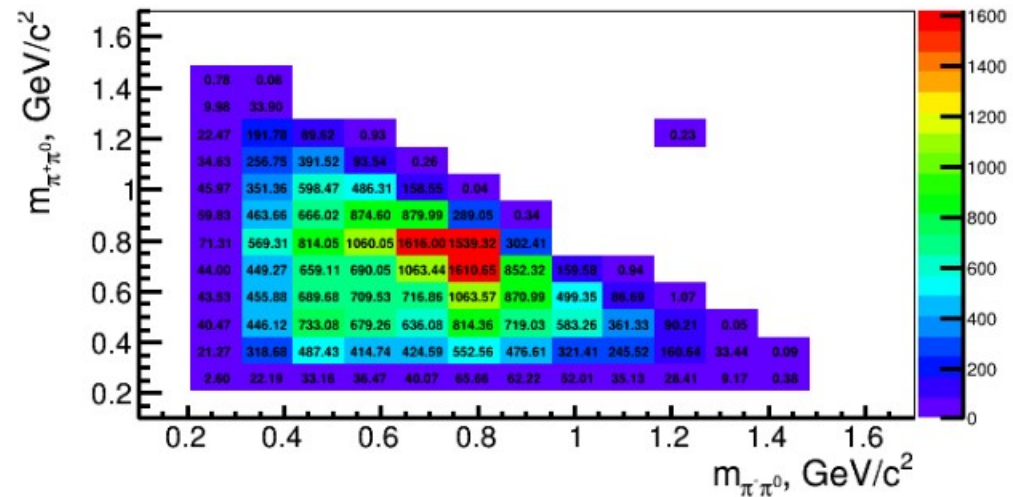
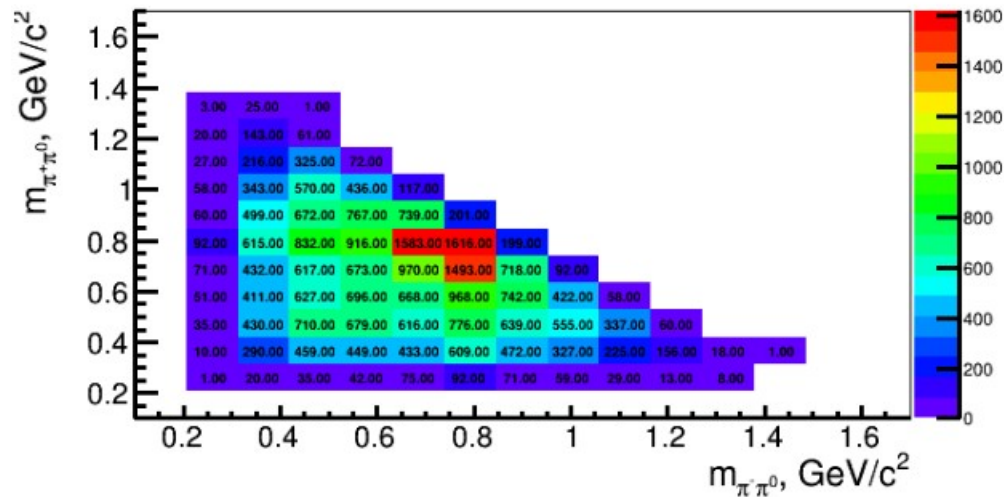


$(ee \rightarrow \pi^+\pi^-\pi^0)$



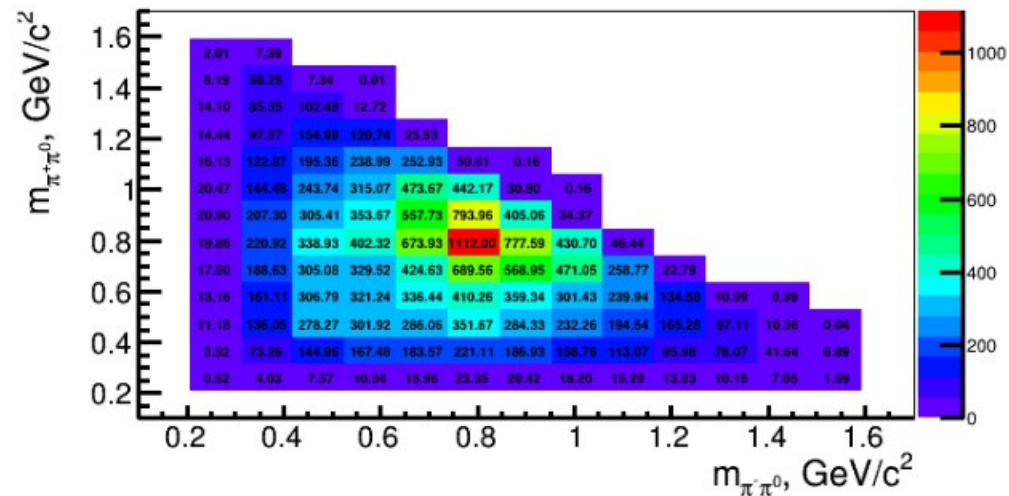
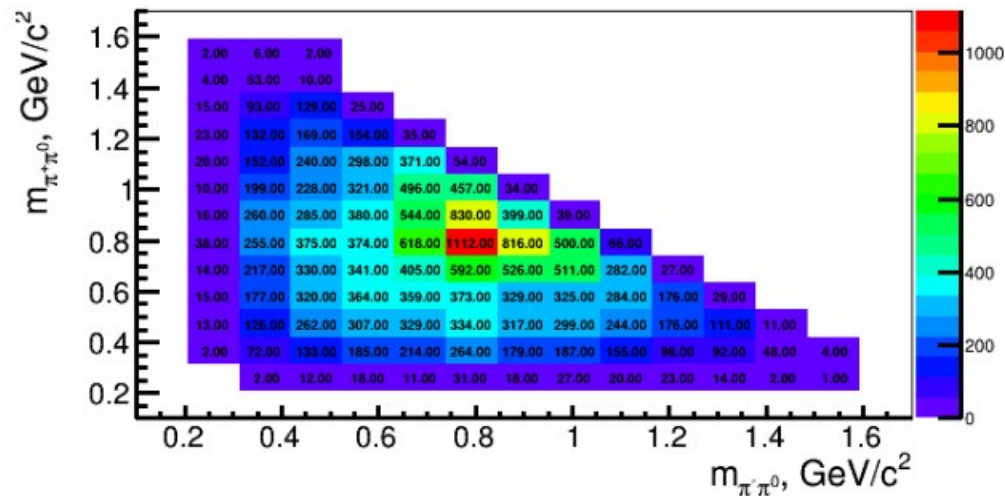
($ee \rightarrow \pi^+\pi^-\pi^0$)

1.6-1.7 GeV

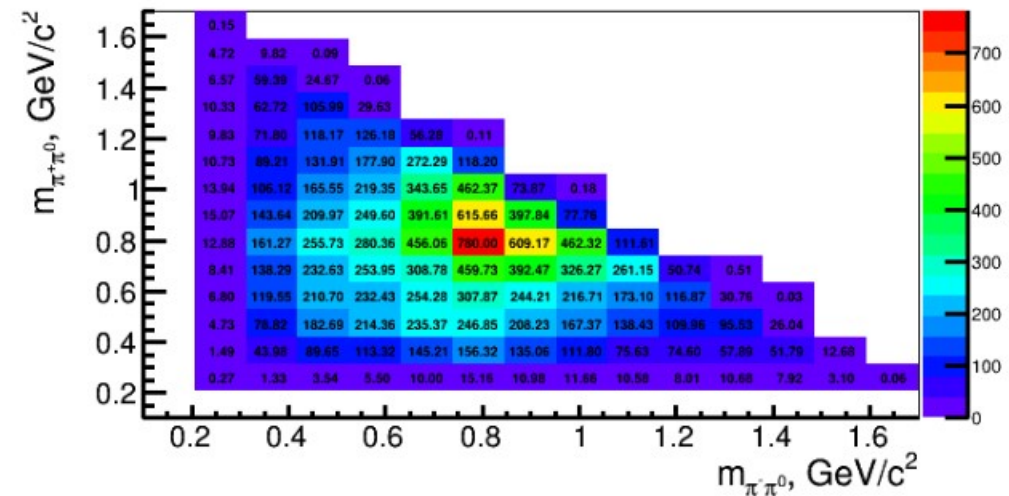
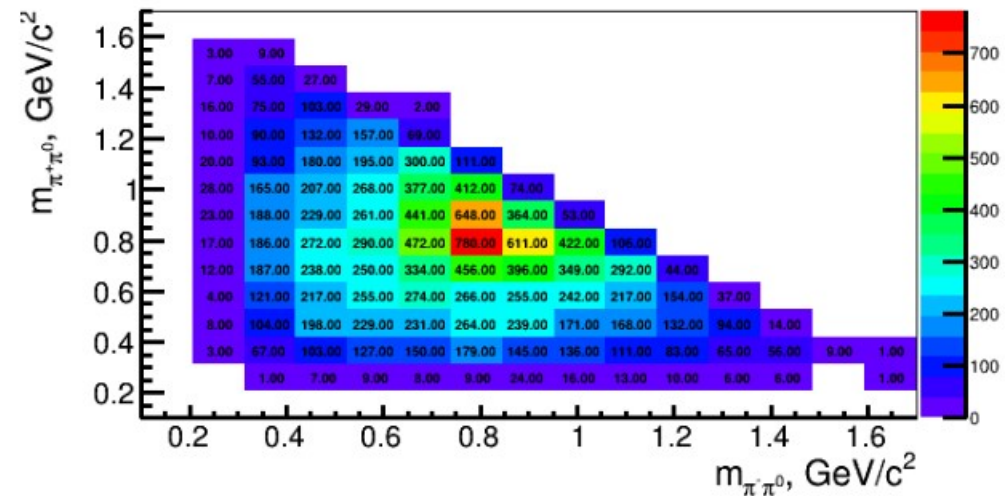


($ee \rightarrow \pi^+\pi^-\pi^0$)

1.8-1.87654 GeV

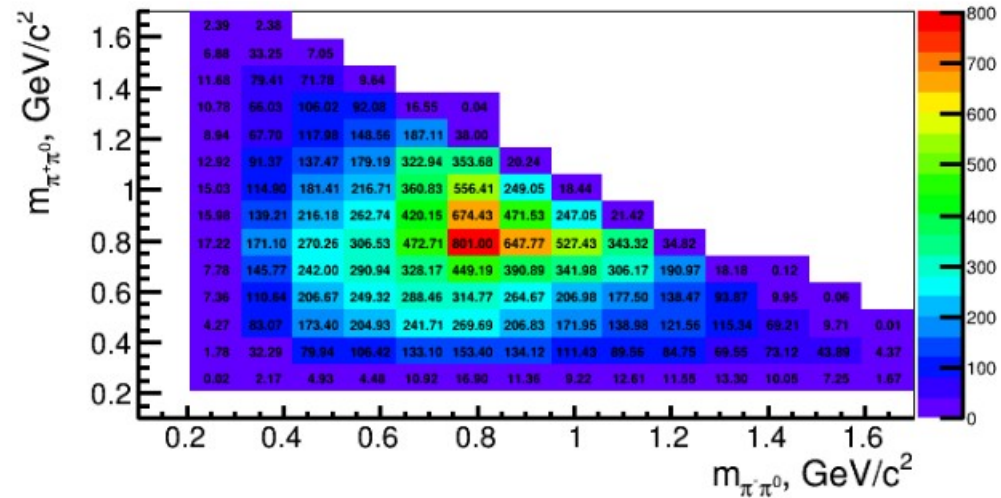
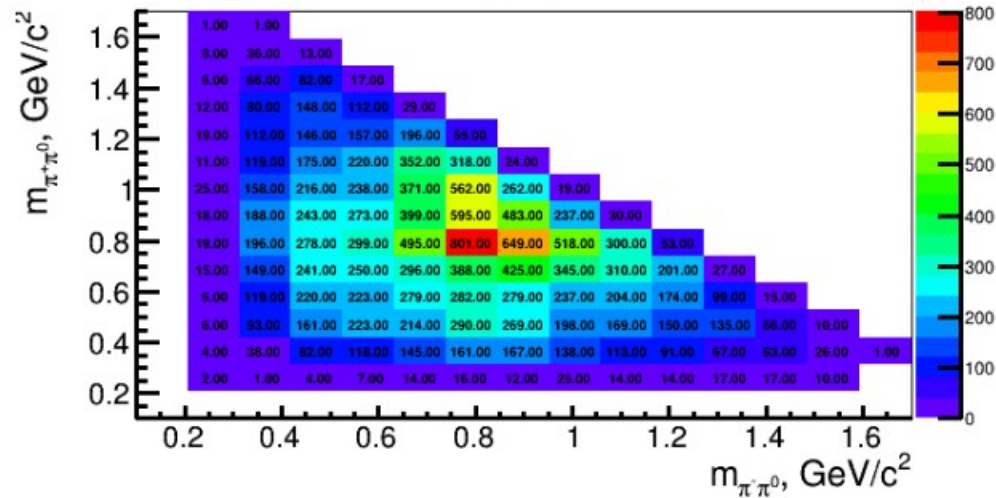


1.87654-1.9 GeV

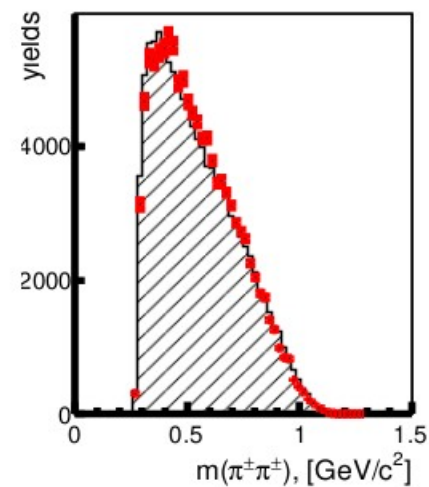
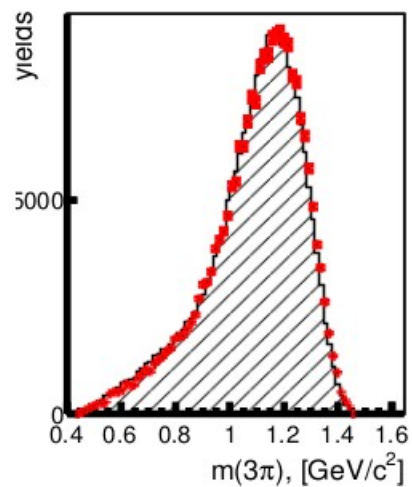
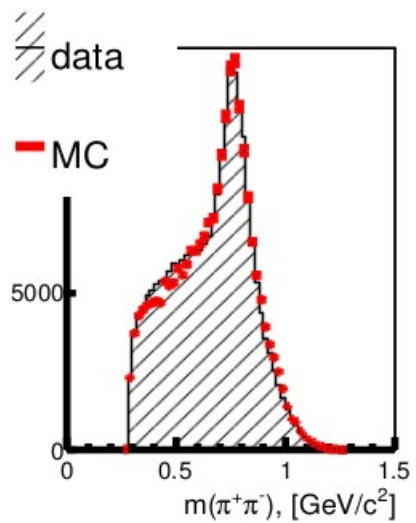
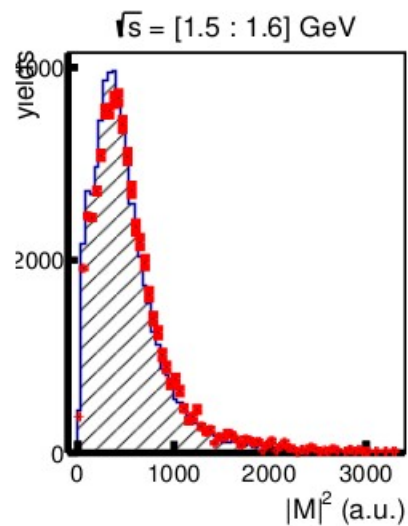
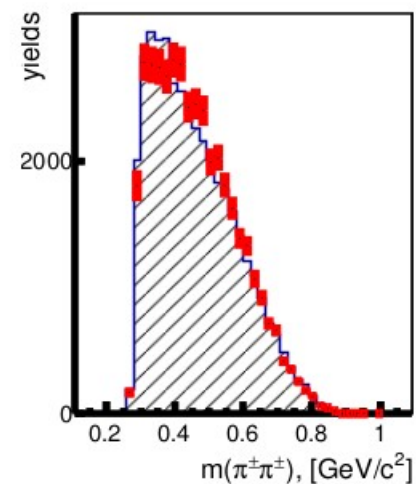
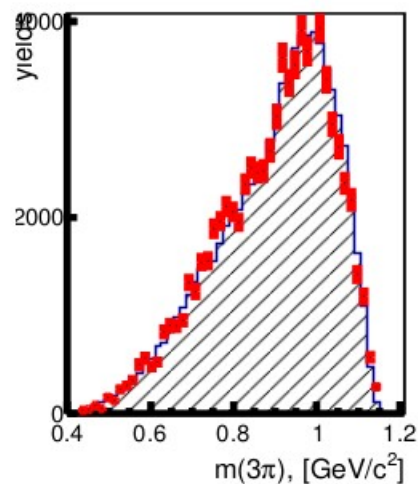
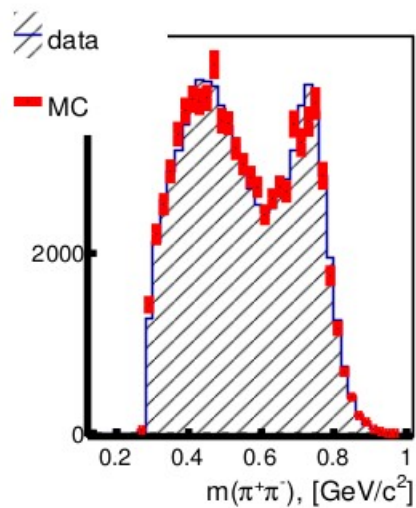
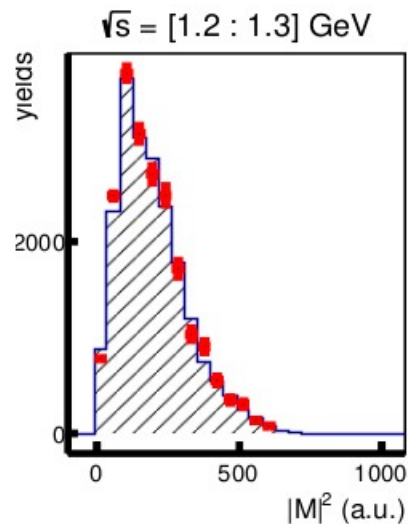


$(ee \rightarrow \pi^+\pi^-\pi^0)$

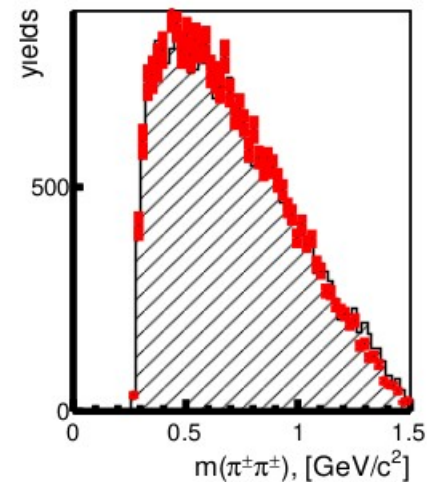
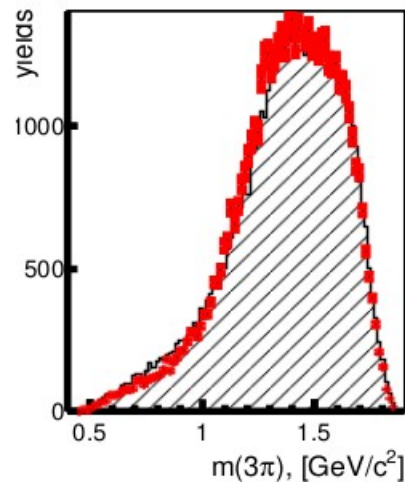
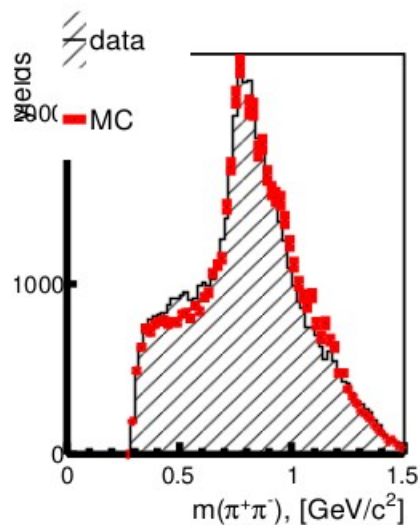
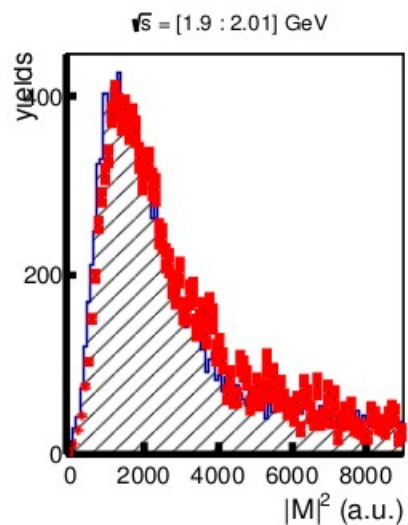
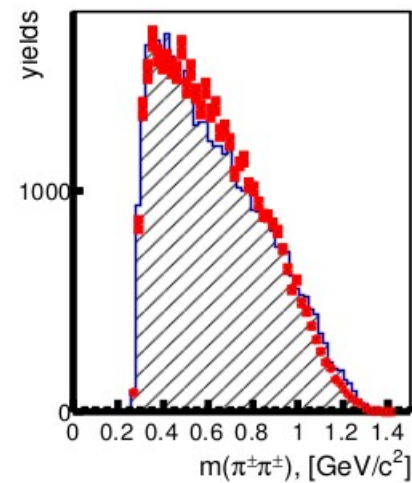
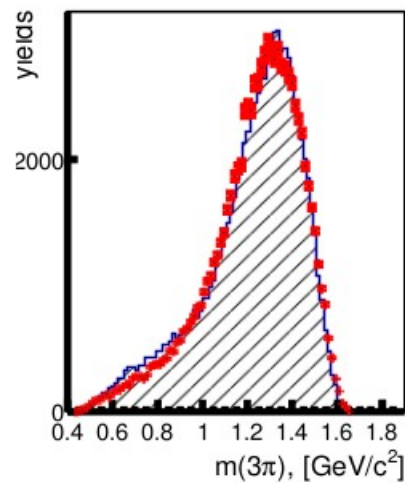
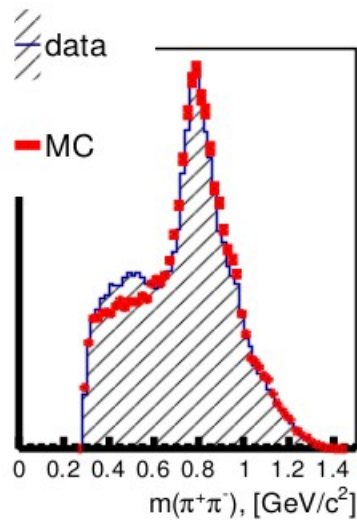
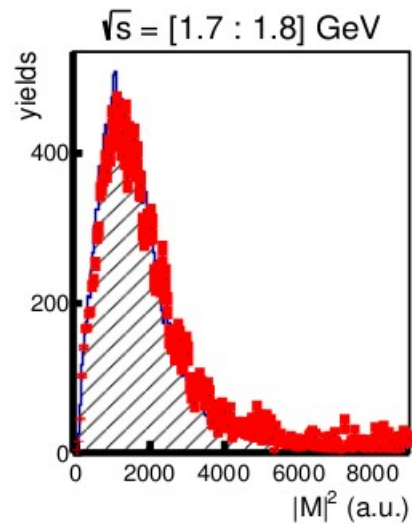
1.9-2.01 GeV



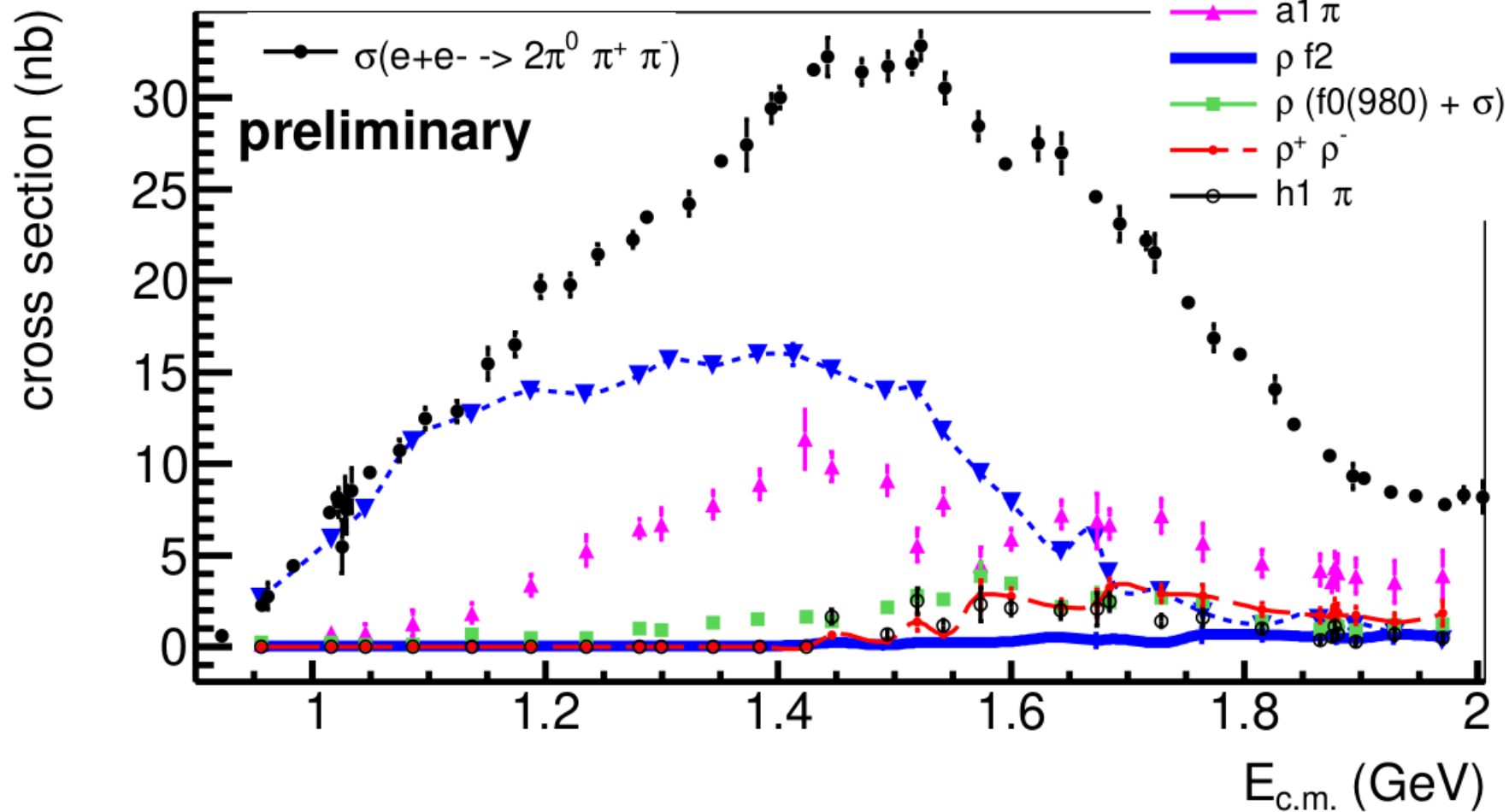
$(ee \rightarrow 2\pi^+2\pi^-)$



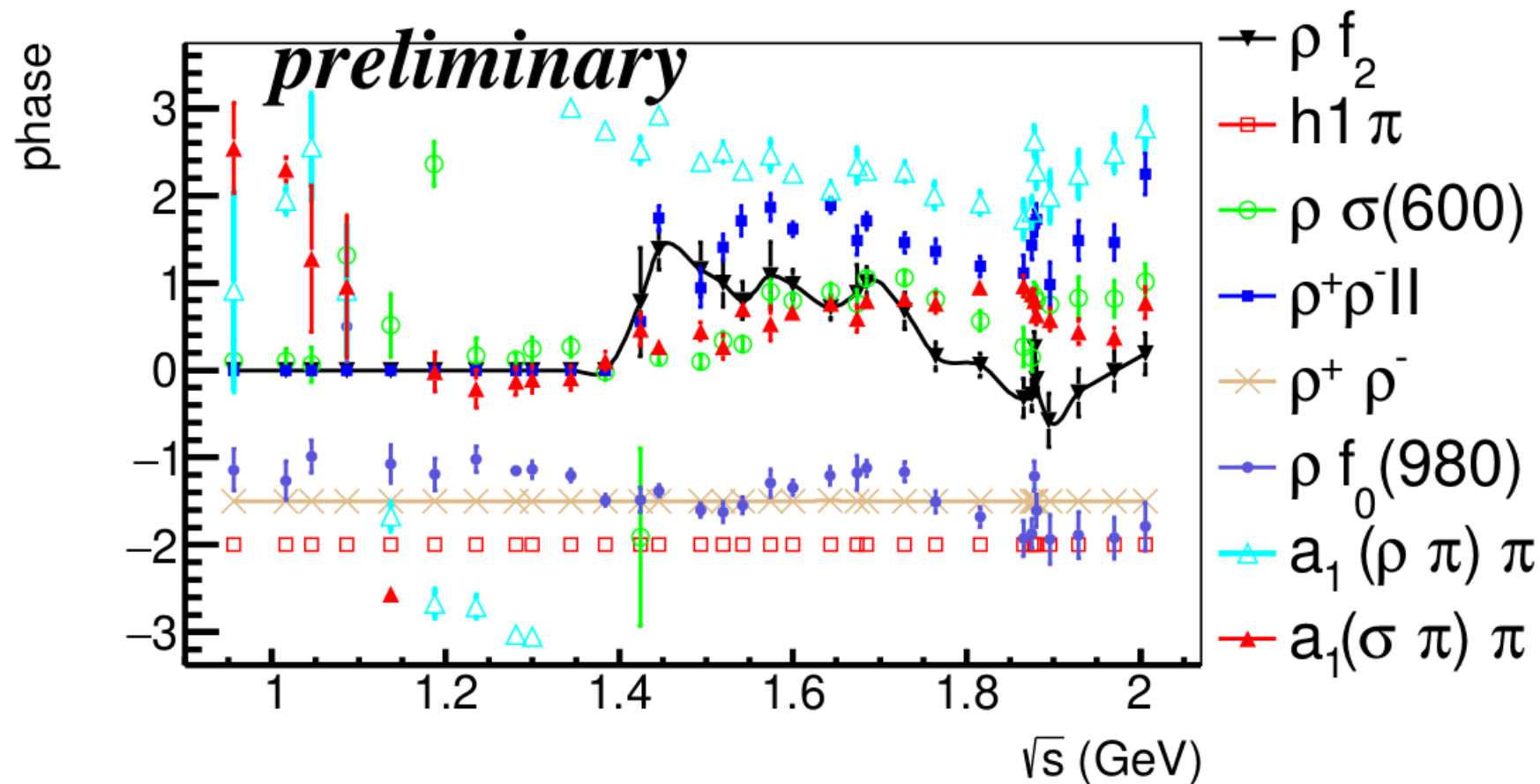
$(ee \rightarrow 2\pi^+2\pi^-)$

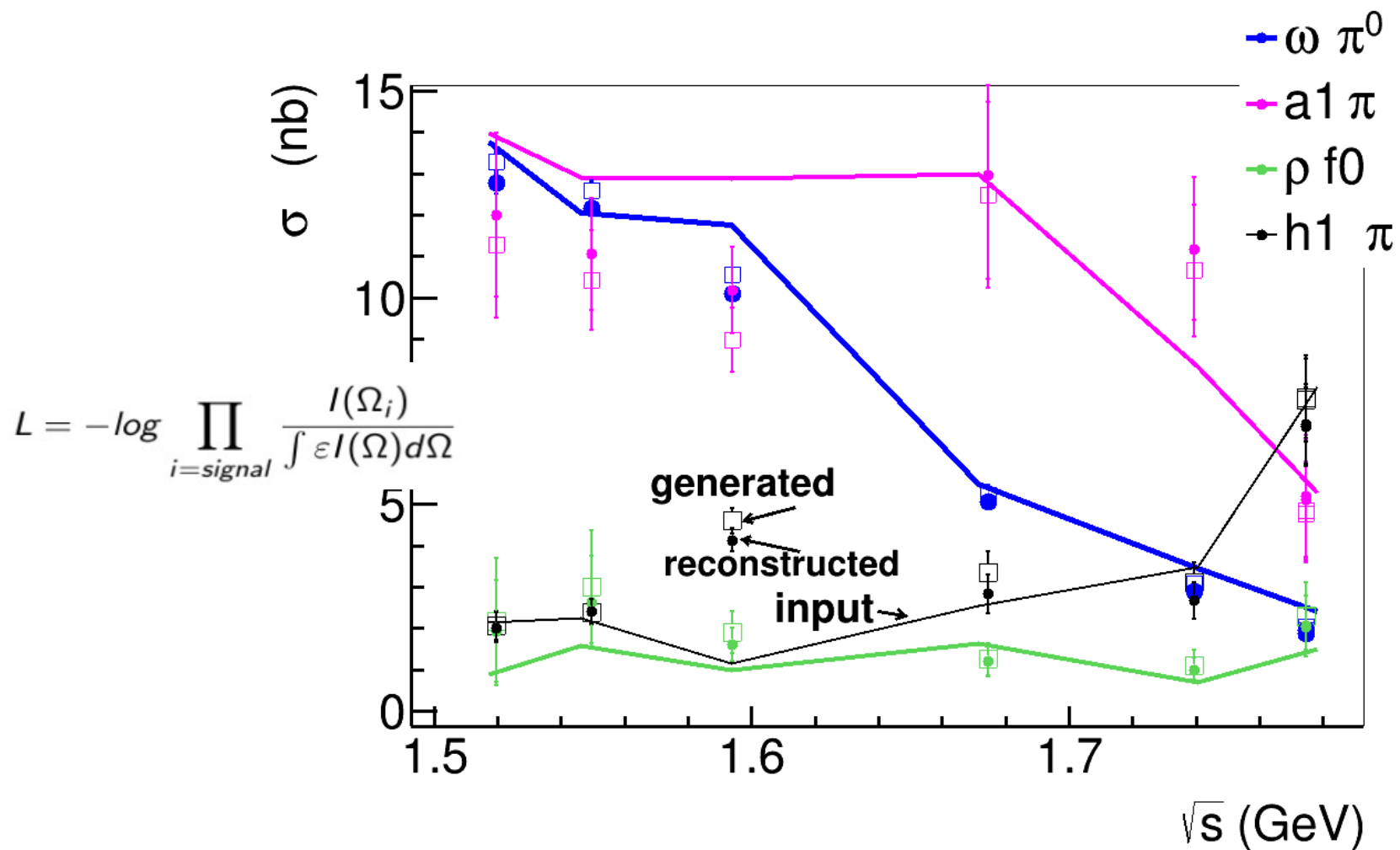


$$R_i(s) = \frac{\int |V_i \cdot \mathbf{H}_{i\perp}(\Omega)|^2 d\Phi}{\int |\sum_{\beta} V_{\beta} \mathbf{H}_{\beta\perp}(\Omega)|^2 d\Phi} \cdot \sigma_{2\pi^0 \pi^+ \pi^-}(s)$$



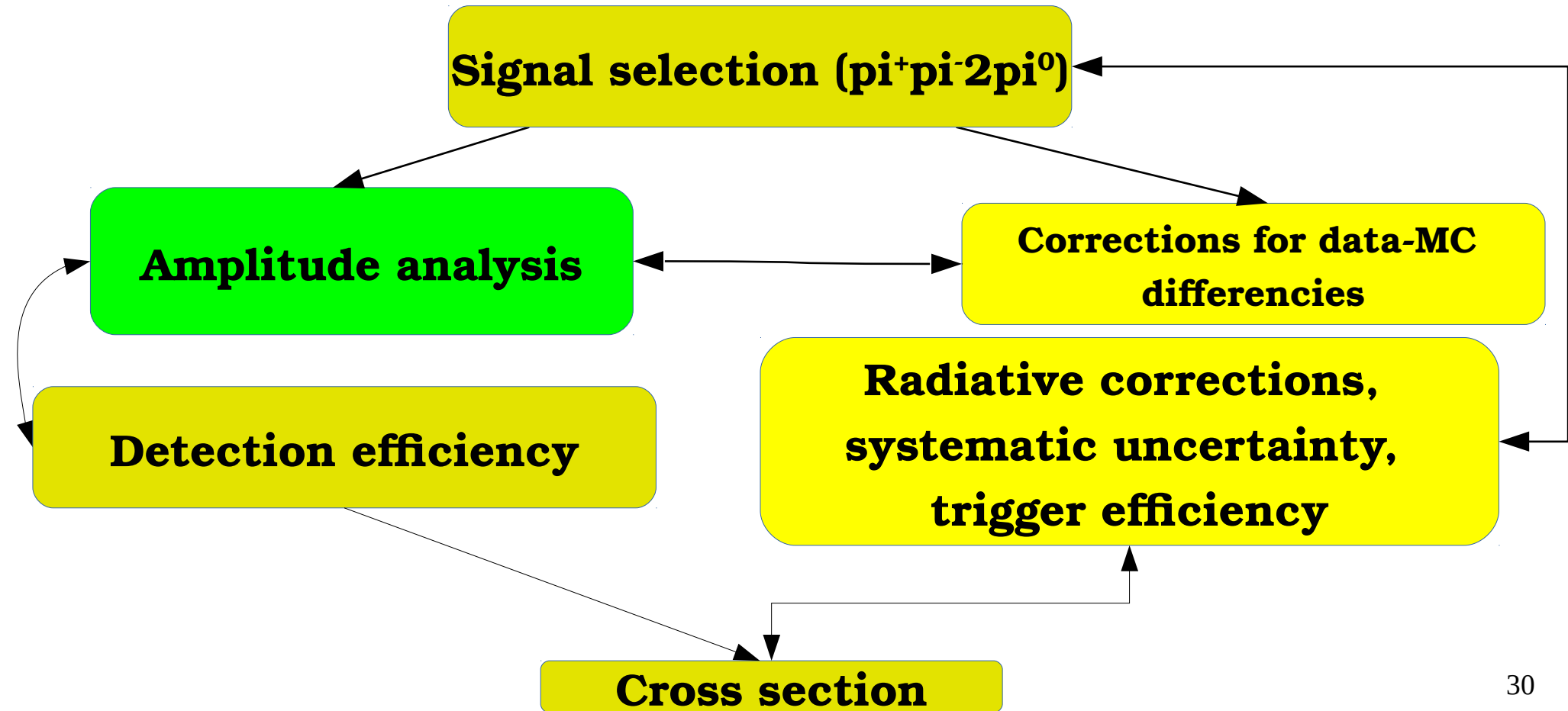
Значения фаз различных компонент матричного элемента





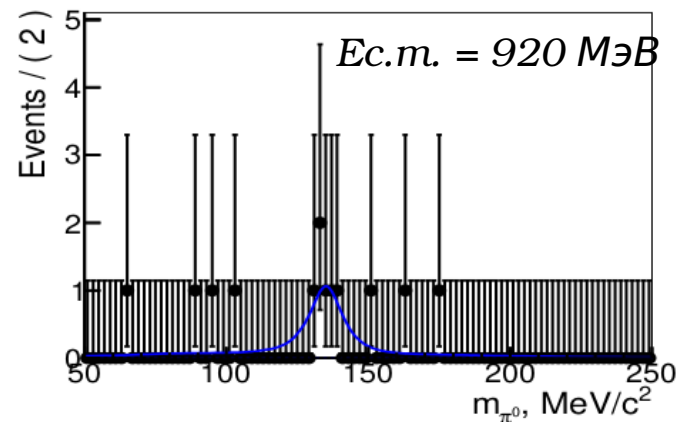
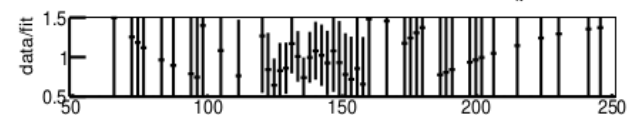
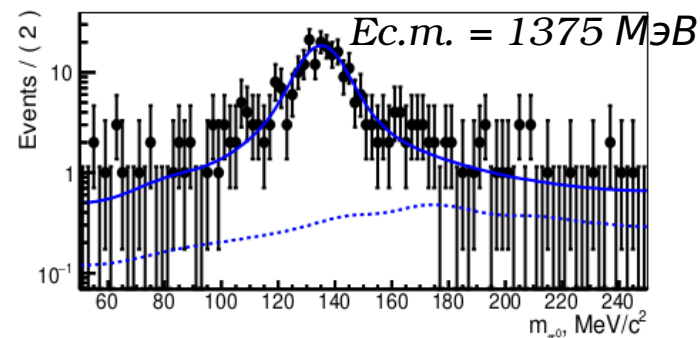
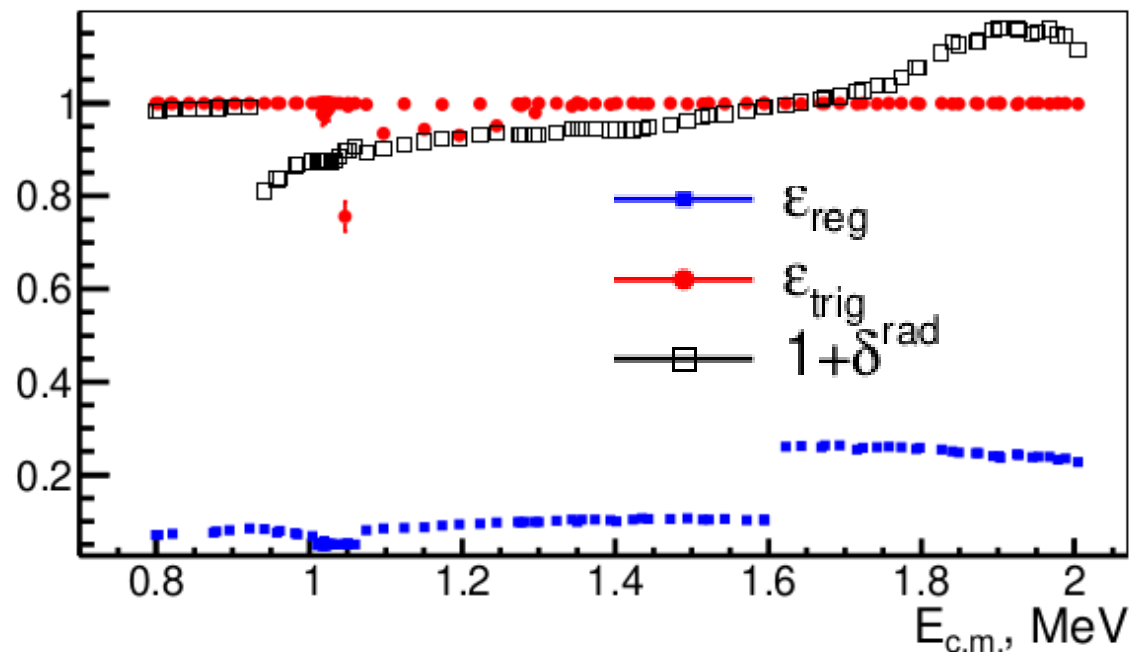
- Ошибка методики ~ 3-7%

Общая стратегия измерения сечения

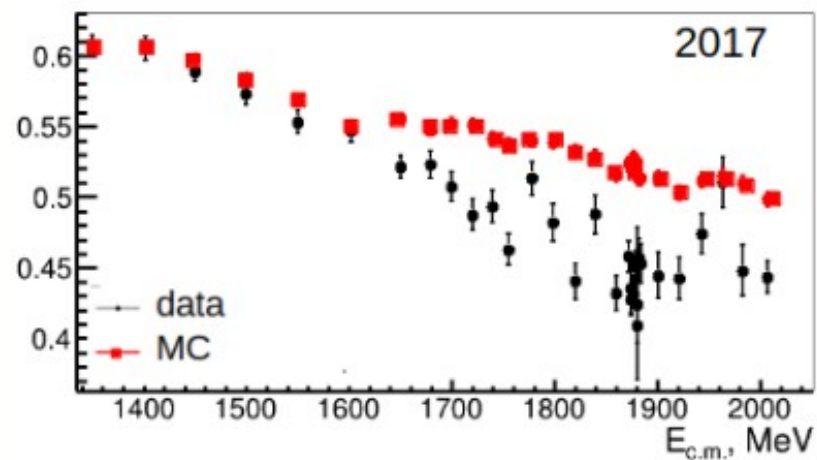
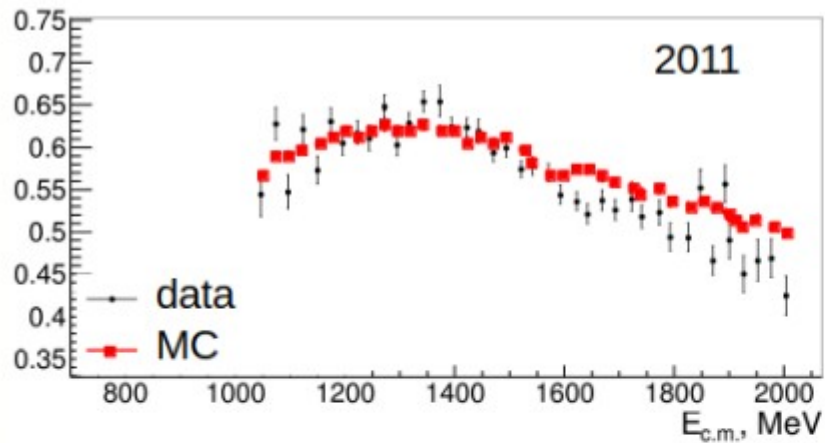


$$\sigma^{Born} = \frac{N_{exp}}{\epsilon_{reg}\epsilon_{trig}L(1 + \delta^{rad.})}$$

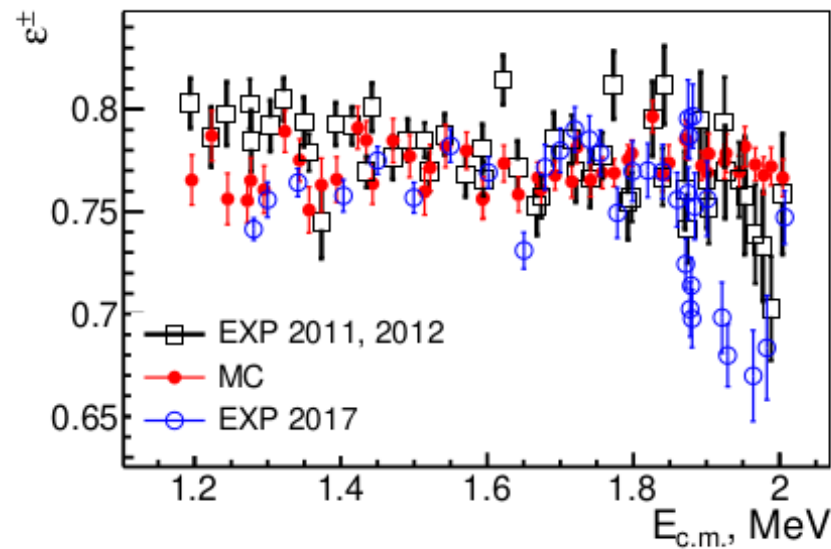
$$1 + \delta^{rad}(s) = \frac{\int F(x,s)\sigma^{Born}(s(1-x))dx}{\sigma^{Born}(s)}$$



Пример аппроксимации спектра инвариантной массы 3-го и 4-го фотонов



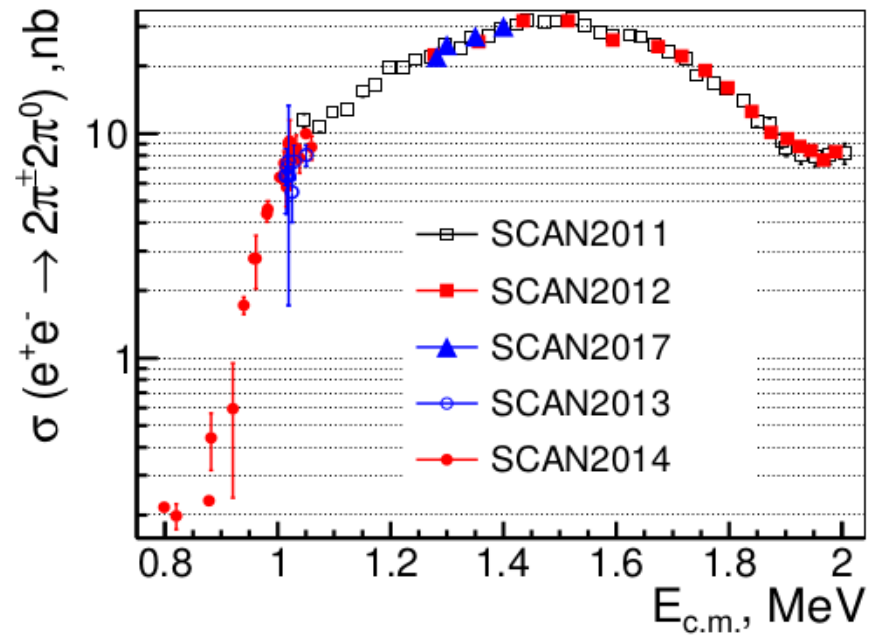
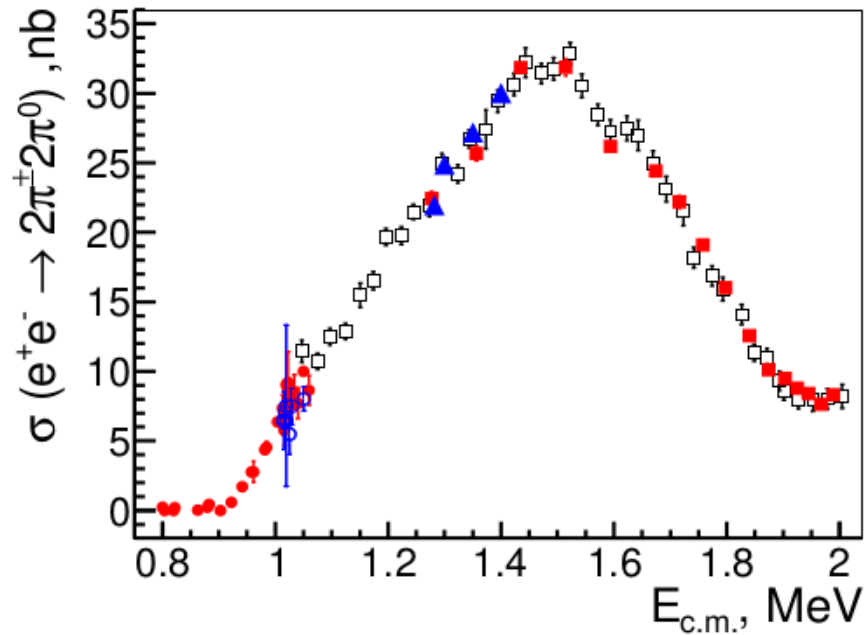
*Эффективность реконструкции
нейтральных пионов*



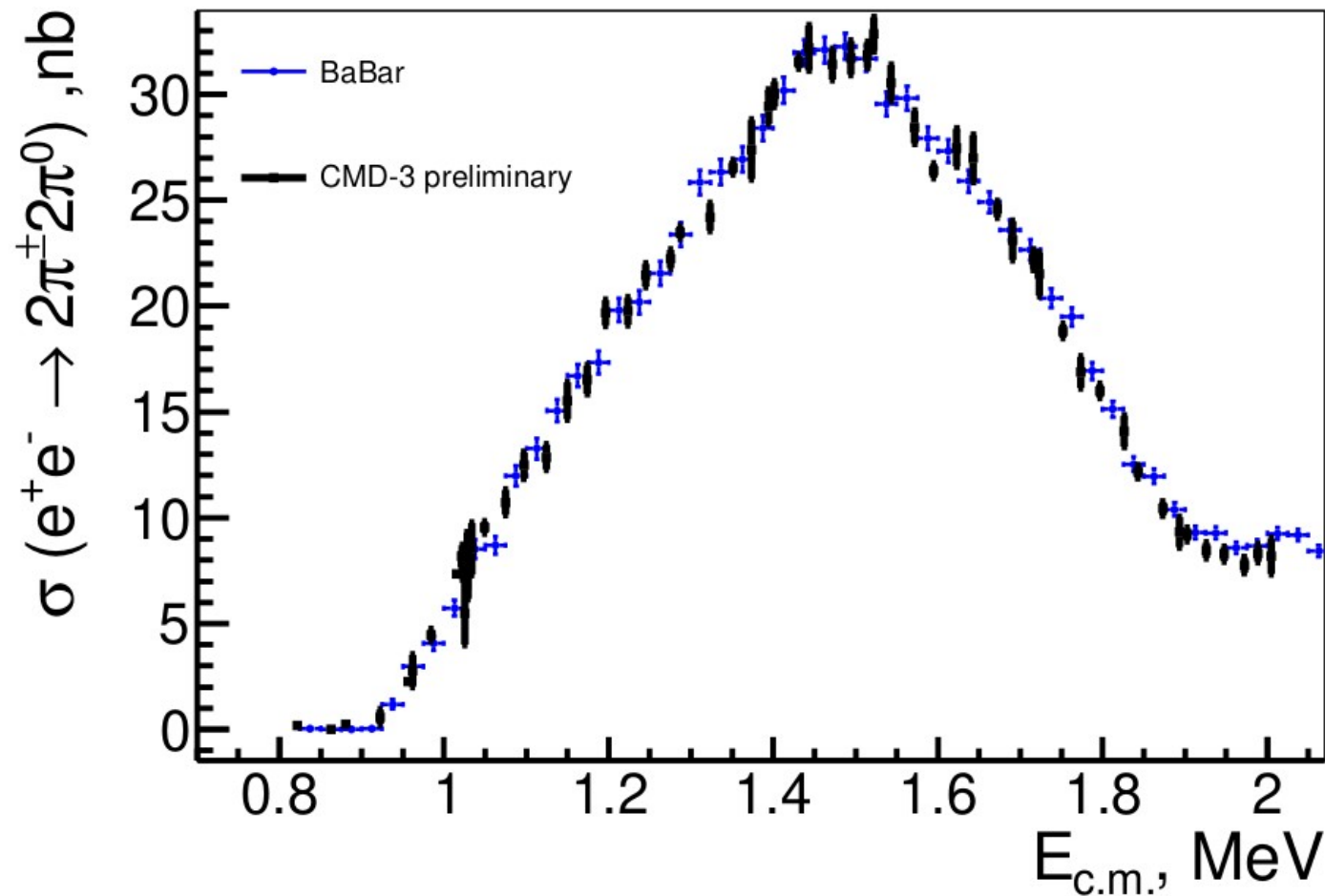
*Эффективность реконструкции
заряженных пионов*

Источник ошибки	$E_{c.m.} < 1.06$ ГэВ (%)	$1.06 < E_{c.m.} < 1.6$ ГэВ (%)	$1.6 < E_{c.m.}$ ГэВ (%)
Вычитание фона	3	1	4
Модельная ошибка	0.5	0.8	0.8
Условия отбора	4	4	2
Реконструкция заряженных пионов	2	2	2
Реконструкция нейтральных пионов	8	8	8
Эффективность триггера	0.3	0.3	0.3
Радиационная поправка	1.5	0.9	0.9
Светимость	1.0	1.0	1.0
Total	10	9	10

Источники систематической ошибки в сечении процесса $e^+e^- \rightarrow 2\pi^0\pi^+\pi^-$



Зависимость полученных значений сечений $e^+e^- \rightarrow 2\pi^0\pi^+\pi^-$ энергии для разных сезонов набора данных.

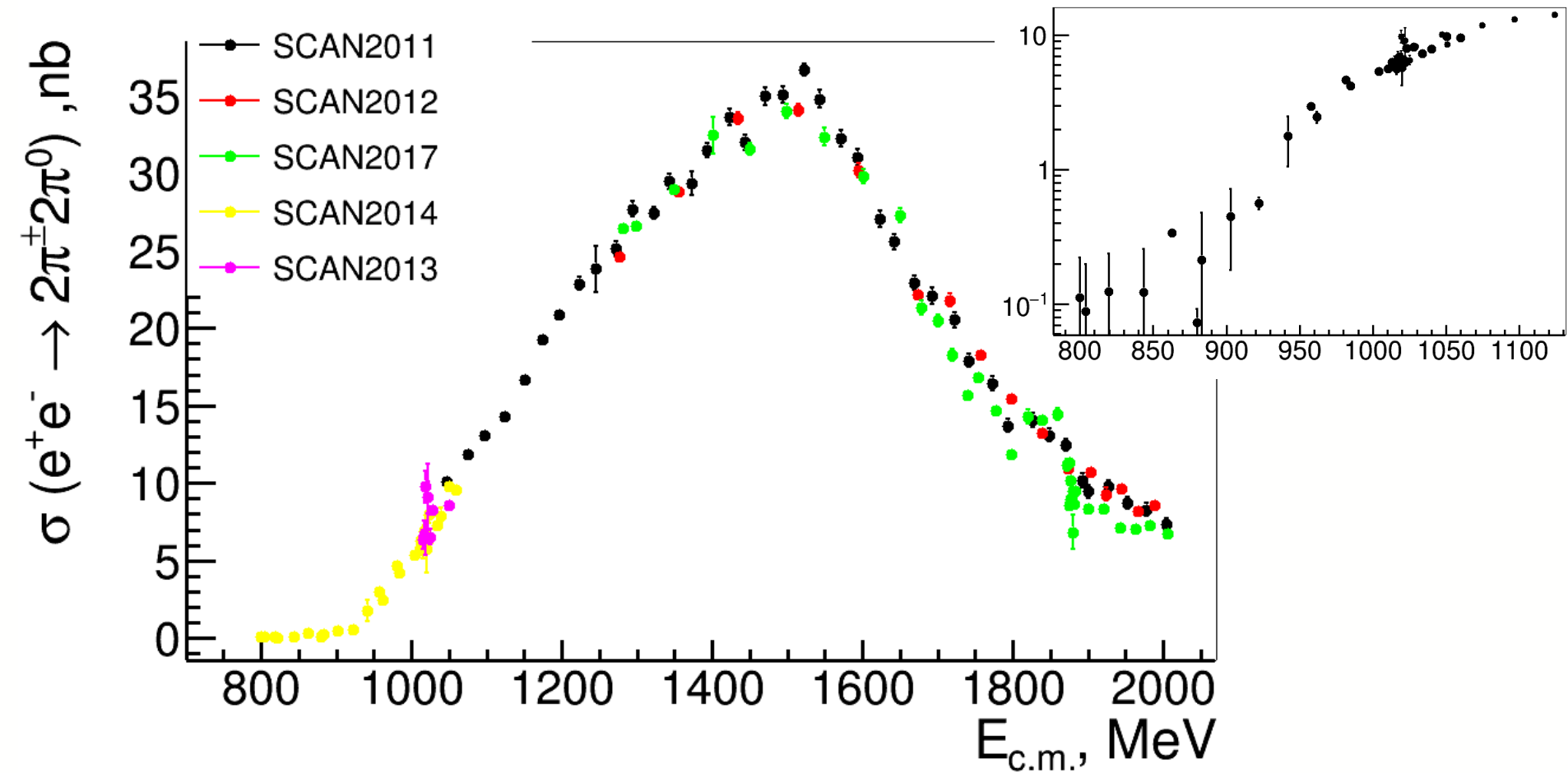


Сравнение полученных усредненных значений сечений $e^+e^- \rightarrow 2\pi^0\pi^+\pi^-$ от энергии с результатом на детекторе BaBar. 35

Заключение

- Выполнено предварительное изучение промежуточной динамики процесса $e^+e^- \rightarrow 4\pi$ в диапазоне энергий $\sqrt{s} = [0.95-2.01]$ ГэВ
- Подтверждена доминантность амплитуд $\omega\pi$ и $a_1\pi$ при $\sqrt{s} < 2$ ГэВ
- Канал $\omega\pi$ является доминирующим в процессе $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$ при $\sqrt{s} < 1.5$ ГэВ
- Определены вклады амплитуд $\rho\sigma$, ρf_0 , $\rho^- \rho^+$, ρf_2 , $h_1\pi$
- Измерено сечение процесса $e^+e^- \rightarrow 2\pi^0\pi^+\pi^-$ с систематической точностью около 10% в области энергий $E = 0.8 \div 2$ ГэВ.

back-up

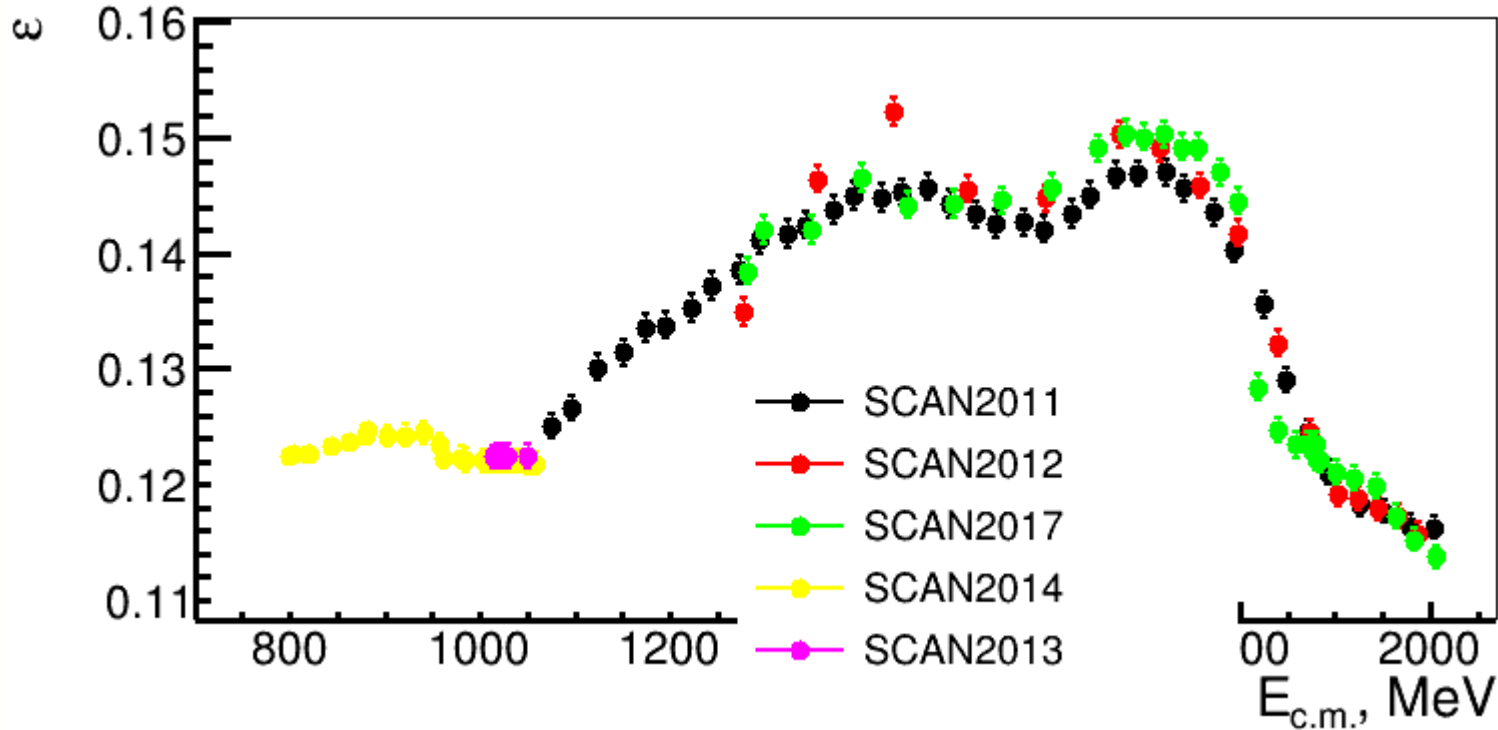


The cross-section of $e^+e^- \rightarrow 2\pi^0\pi^+\pi^-$ vs $E_{\text{c.m.}}$ for different scans

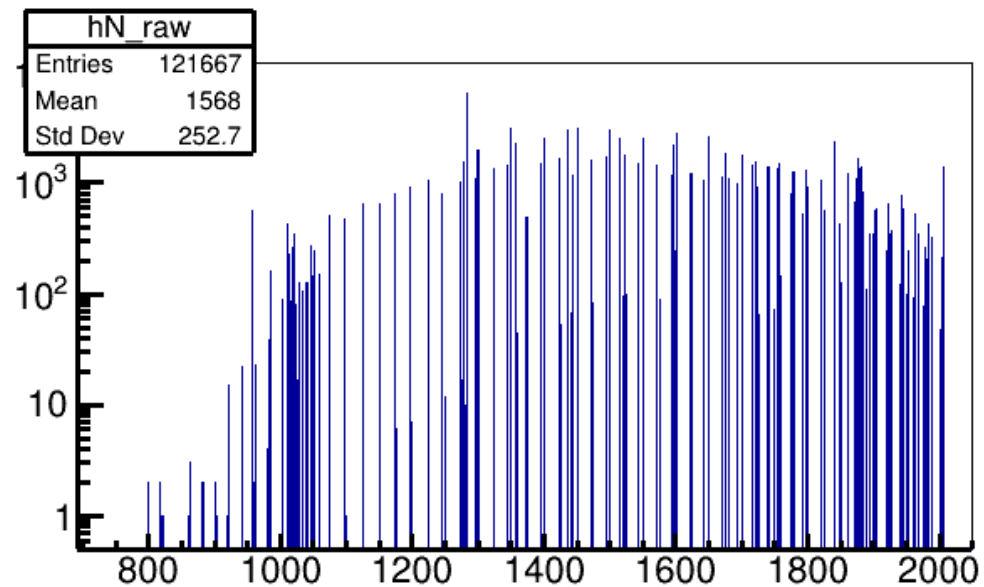
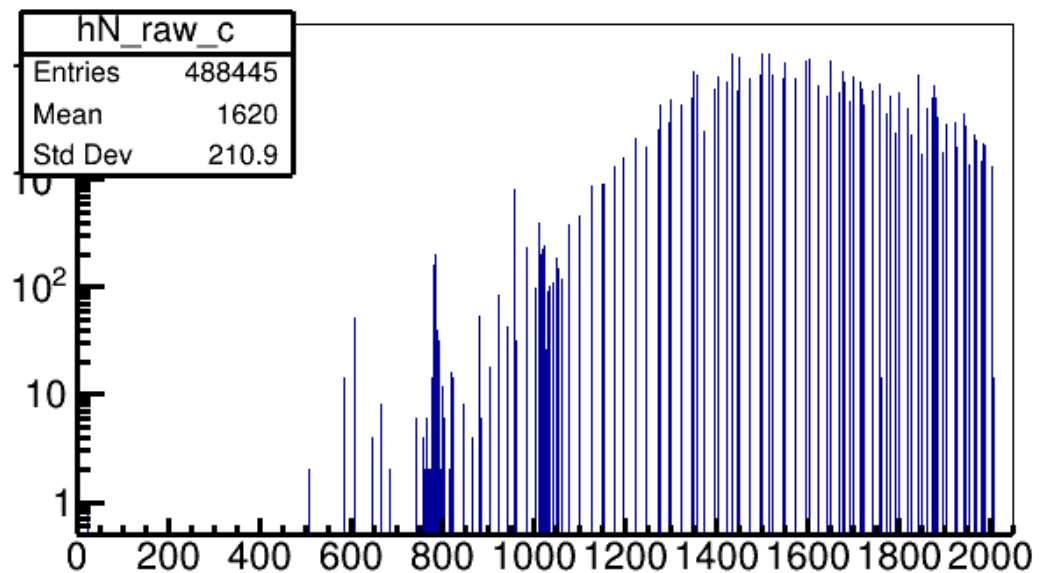
Soon we will publish the cross section of the cross section

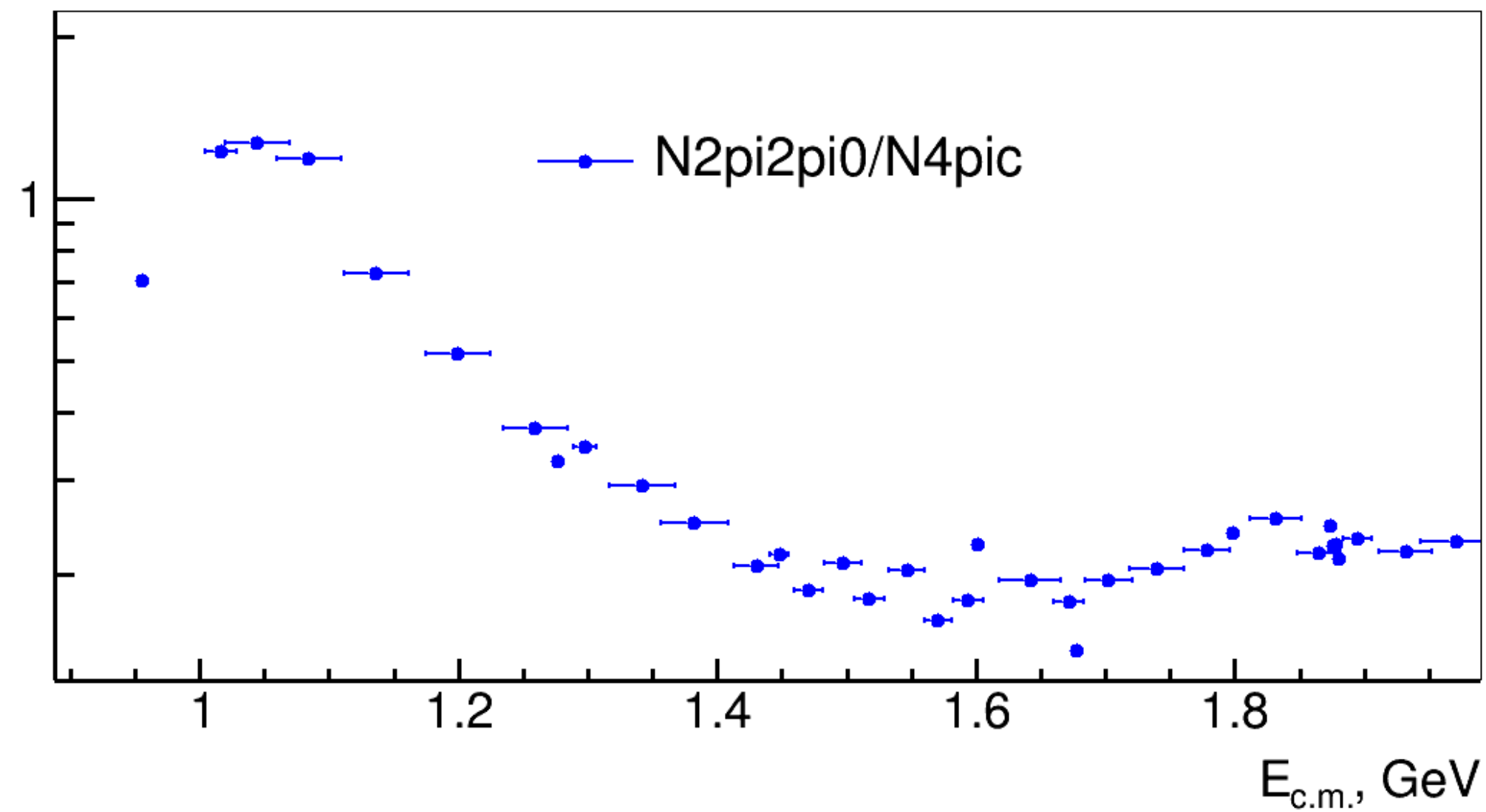
$$\varepsilon_{4\pi} = \frac{\sum_{det}^{ph.sp.} (|M(p_{\pi^0}, p_{\pi^0}, p_{\pi^+}, p_{\pi^-})|^2)}{\sum_{gen}^{ph.sp.} (|M(p_{\pi^0}, p_{\pi^0}, p_{\pi^+}, p_{\pi^-})|^2)}$$

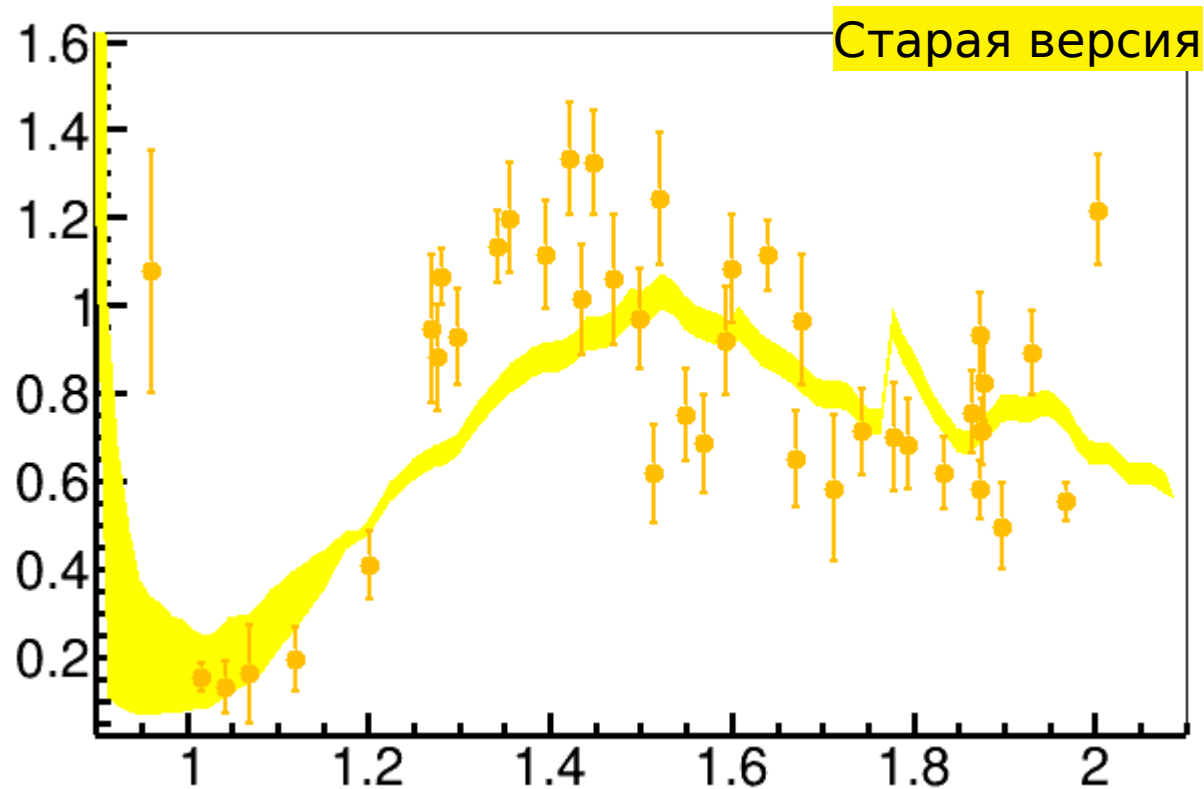
The max energy of simulated ISR is 0.2 GeV.



Detection efficiency vs. c.m. energy.



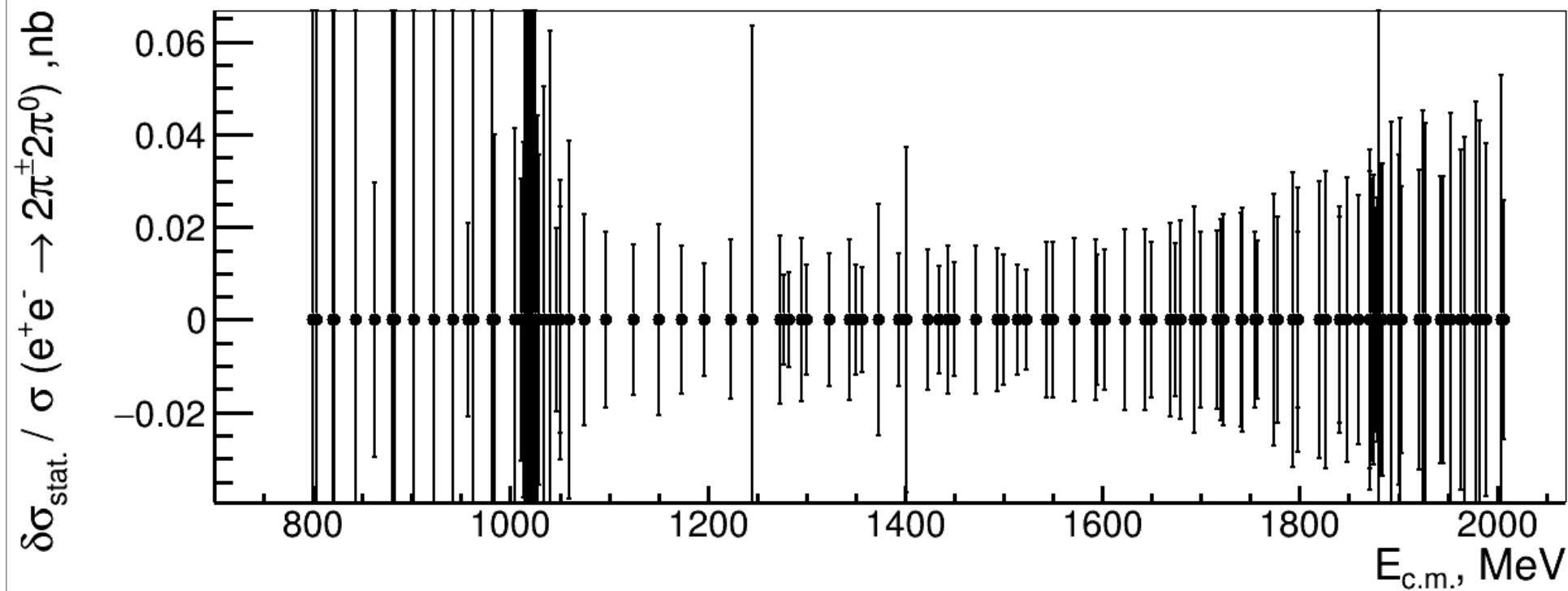




Отношение сечений $e^+e^- \rightarrow 2\pi^+2\pi^-$ к $e^+e^- \rightarrow \pi^+\pi^-2\pi^0$.

Закрашенная область — эксперимент (из др. работ).

Точки с ошибками — результат амплитудного анализа (из данной работы).



Statistical precision is about 2-3% in average