Intermediate dynamics of four-pion production in e^+e^- annihilation and tau decay

Evgeny Kozyrev for the CMD-3 collaboration

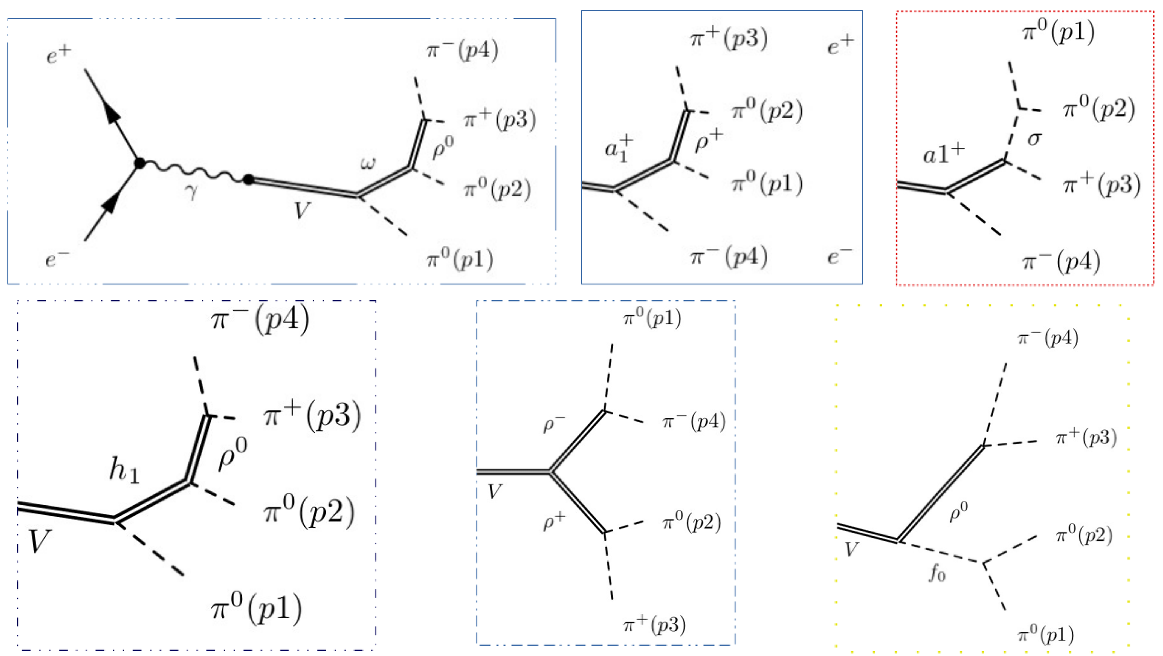
e.a.kozyrev@inp.nsk.su

Budker Institute Of Nuclear Physics, Russia

The processes $e^+e^- \to 2\pi^0\pi^+\pi^-$ and $e^+e^- \to 2\pi^+2\pi^-$ dominate the hadronic cross section at low center-of-mass energies $E_{c.m.} < 2$ GeV. A large data sample of their events collectedby the CMD-3 experiment at the VEPP-2000 e^+e^- collider allows foran Amplitude Analysis (AA) at $0.95 < E_{c.m.} < 2.007$ GeV. This study uses about 43 pb^{-1} of an integrated luminosity accumulated in four scans at 85 energy points.

Introduction

This work is aimed to probe the structure of the hadronic current of the $e^+e^- \to 4\pi$:



In the assumption of the dominance of the two-body decays we consider the following list of the intermediate states:

- $\omega[J^{PC} = 1^{--}]$ $\pi^0[0^{-+}]$ (only $\to 2\pi^{\pm}2\pi^0$)
- $a_1(1200)[1^+]$ $\pi[0^-]$
- $\rho[1^{--}]$ $f_0(980)[0^{++}]$
- $\rho[1^{--}]$ $\sigma(500)[0^{++}]$
- $\rho[1^{--}]$ $f_2(1270)[2^{++}]$
- $\rho^+[1^-]$ $\rho^-[1^-]$ (only $\to 2\pi^{\pm}2\pi^0$)
- $a_2(1320)[2^{++}]$ $\pi[0^-]$
- $h_1(1170)[1^{+-}]$ $\pi^0[0^{-+}]$ (only $\to 2\pi^{\pm}2\pi^0$)
- $\pi(1300)^{\pm}[0^-]\pi^{\mp}[0^-]$

Data set

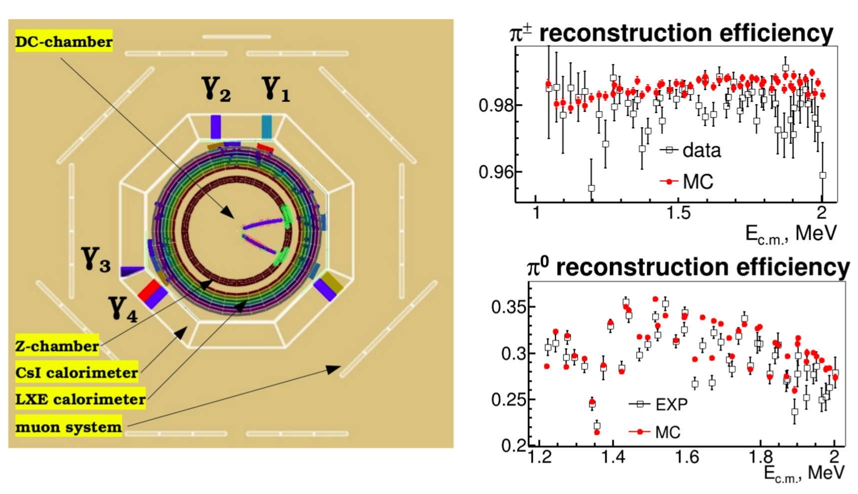
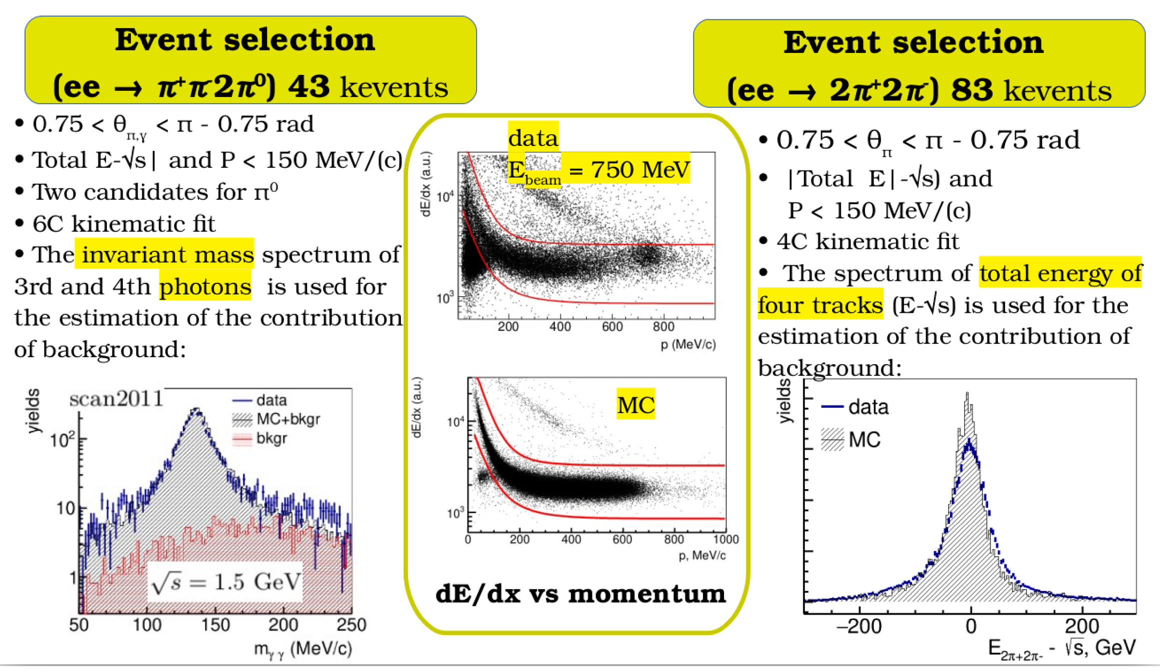


Figure 1: Transverse view of the CMD-3 detector and a typical event of $e^+e^- \to 2\pi^0\pi^+\pi^-$

The Cryogenic Magnetic Detector (CMD-3)[1] is installed in one of the two interaction regions of the VEPP-2000 e^+e^- collider [2] in **Novosibirsk**.



Amplitude analysis

AA is performed by minimization of the unbined Likelihood function for a particular model

$$L = -\ln \prod_{i}^{e^+e^- \to 2\pi^0\pi^+\pi^-} \frac{|\sum_\alpha \mathbf{V}_\alpha A^0_\alpha(\Omega_i)|^2}{\frac{1}{N^{gen}_{MC}} \sum_k^{rec.\ ph.\ space\ MC} |\sum_\alpha \mathbf{V}_\alpha A^0_\alpha(\Omega_k)|^2}$$

$$- \ln \prod_{j}^{e^+e^- \to 2\pi^+2\pi^-} \frac{|\sum_\alpha \mathbf{V}_\alpha A^{\pm}_\alpha(\Omega_j)|^2}{\frac{1}{N^{gen}_{MC}} \sum_k^{rec.\ ph.\ space\ MC} |\sum_\alpha \mathbf{V}_\alpha A^{\pm}_\alpha(\Omega_k)|^2},$$

where i and j run over all selected events. The sum α runs over all intermediate states, $\mathbf{V}_\alpha$ - a complex model parameter. The quantities $\mathbf{V}_\alpha A^{\pm}_\alpha\big(p_1(\pi^+), p_2(\pi^-), p_3(\pi^+), p_4(\pi^-)\big)$ and $\mathbf{V}_\alpha A^0_\alpha\big(p_1(\pi^0), p_2(\pi^0), p_3(\pi^+), p_4(\pi^-)\big)$ are the specific components α of the total matrix elements at particular points in phase space for the $e^+e^- \to 2\pi^+2\pi^-$ and $e^+e^- \to 2\pi^0\pi^+\pi^-$ channels, respectively. The sum in the denominator runs over all MC events, flatly generated in phase space and passed all selection criteria above. The effects of detection resolution and initial state radiation are under control by the AA of the toy MC samples.

An example of an amplitude: for decay $\rho' \to \rho f_2(1270)$

$$A(\rho' \rho f_2) = \mathbf{V}_{\rho' \rho f_2} \cdot \rho'_\alpha \rho^*_\beta \cdot \omega^*_{\alpha\beta}, \cdot \delta^{ab} \phi^a_{\rho'} \phi^{*b}_{\rho},$$

where $\omega_{\alpha\beta}$ - the polarization tensor of f_2-meson; a,b - isospin indexes.

The parametrization of the different components of the matrix element is the same as used in Ref. [3].

C invariance: $A^0_\alpha(p_1, p_2, p_3, p_4) = -A^0_\alpha(p_1, p_2, p_4, p_3)$,

Bose symmetry: $A^0_\alpha(p_1, p_2, p_3, p_4) = A^0_\alpha(p_2, p_1, p_3, p_4)$

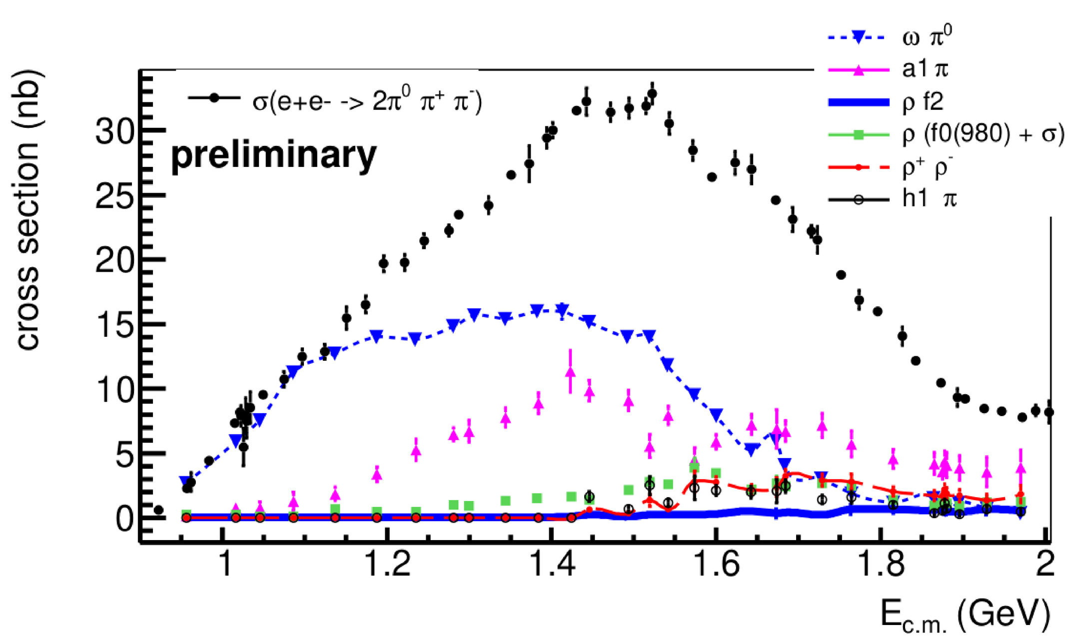
Gauge invariance: $q^\mu A_\mu(p_1, p_2, p_3, p_4) = 0$, where $q = p_{e^-} + p_{e^+}$.

In the isospin limit: $A^{\pm}_\alpha\big(p_1(\pi^+), p_2(\pi^-), p_3(\pi^+), p_4(\pi^-)\big) = A^0_\alpha\big(p_1, p_2, p_3, p_4\big) + A^0_\alpha\big(p_3, p_2, p_1, p_4\big) + A^0_\alpha\big(p_1, p_4, p_3, p_2\big) + A^0_\alpha\big(p_3, p_4, p_1, p_2\big)$. Masses and central values of resonance widths are fixed according to PDG.

A fraction f_X of an individual component of the matrix element is calculated as

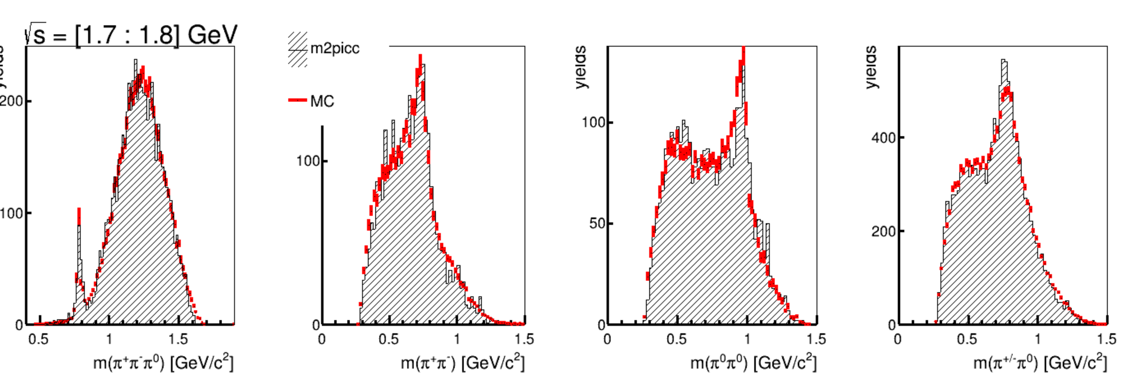
$$f_X^{\pm/0} = \frac{\int |\mathbf{V}_X A_X^{\pm/0}(\Omega)|^2 d\Omega}{\int |\sum_\alpha \mathbf{V}_\alpha A_\alpha^{\pm/0}(\Omega)|^2 d\Omega}. \quad (1)$$

$f^0_X \cdot \sigma(e^+e^- \to 2\pi^0\pi^+\pi^-)$:

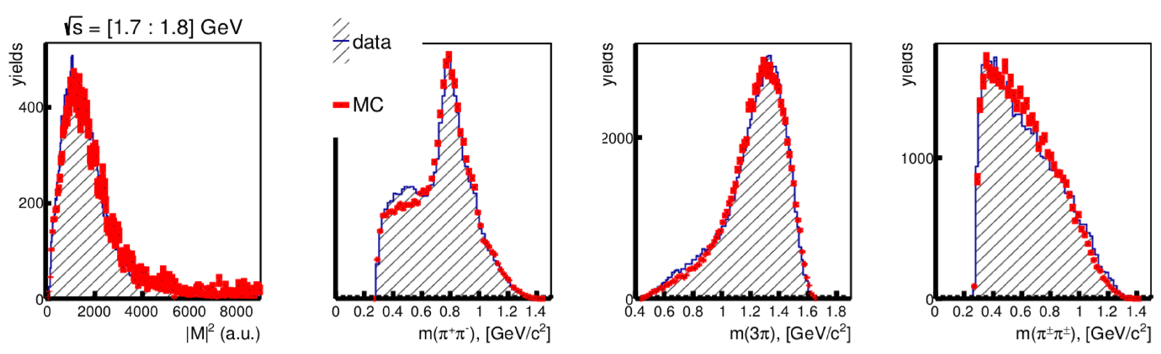


DATA-MC comparison

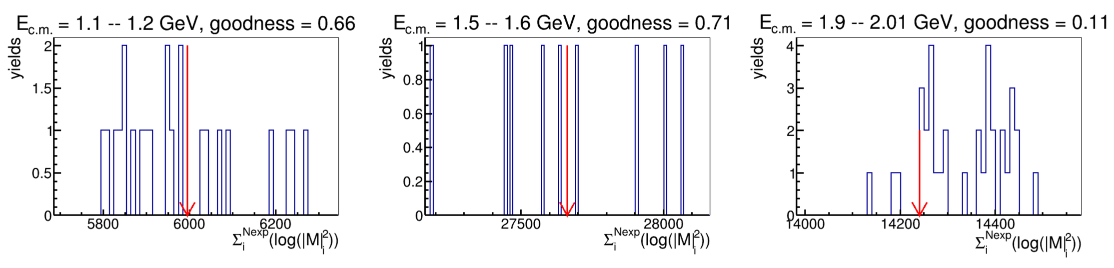
Comparison of inv. mass spectra for $e^+e^- \to 2\pi^0\pi^+\pi^-$:



Comparison of inv. mass spectra for $e^+e^- \to 2\pi^+2\pi^-$:



We generate 10-30 toy Monte Carlo samples at each energy point. Each sample is generated with parameters obtained in the fit to the data and the number of events in each sample is equal to that observed in the data. The goodness-of-fit is estimated as a fraction of samples where $\sum_i^{N_{exp}}(\log|M_i|^2)$ is less than in the data, where $M = |\sum_\alpha \mathbf{V}_\alpha A_\alpha(\Omega_k)|^2$ - is the total matrix element of the $e^+e^- \to 4\pi$.



$\frac{\Gamma(\tau^- \to \nu_\tau X) \cdot B(X \to 4\pi)}{\Gamma(\tau^- \to \nu_\tau 4\pi)}$

The obtained amplitudes $A^{\pm/0}(\Omega)$ and CVC hypothesis [4] allow to predict the matrix elements of the charged vector current relevant for τ decay. Fractions $f_X^{\pm/0}$ can be used for the calculation of τ decays to intermediate states:

$$R_X = \frac{\Gamma(\tau^- \to \nu_\tau X) \cdot B(X \to 4\pi)}{\Gamma(\tau^- \to \nu_\tau 4\pi)} =$$

$$= \frac{\int_0^{m_\tau^2} dQ^2 V(Q^2)\Big(f_X^{\pm}(Q^2) \cdot \sigma(e^+e^- \to 2\pi^+2\pi^-)(Q^2) + f_X^0(Q^2) \cdot \sigma(e^+e^- \to 2\pi^0\pi^+\pi^-)(Q^2)\Big)}{\int_0^{m_\tau^2} dQ^2 V(Q^2)\Big(\sigma(e^+e^- \to 2\pi^+2\pi^-)(Q^2) + \sigma(e^+e^- \to 2\pi^0\pi^+\pi^-)(Q^2)\Big)}$$

$$V(Q^2) = Q^2 \cdot (m_\tau^2 - Q^2)^2(m_\tau^2 + 2Q^2)$$

X	R_X
$\omega\pi^-$	0.41 ± 0.01
$a_1(1260)\pi$	0.197 ± 0.008
$\rho^-[\sigma(500) + f_0(980)]$	0.044 ± 0.001
$\rho^- f_2(1270)$	0.00076 ± 0.00008
$\rho^-\rho^0$	0.0045 ± 0.0005
$h_1(1170)\pi^0$	0.006 ± 0.001

where only statistical uncertainty of f_X us taken into account. The predictions can be checked in future detailed studies of tau decays.

Conclusion

- The preliminary study of internal dynamics is performed for the process $e^+e^- \to 4\pi$ at $E_{c.m.} < 2$ GeV;
- The dominance of states $\omega\pi^0$ and $a_1\pi$ is observed;
- The estimation of the contributions of $\rho(\sigma(500) + f_0(980))$, $\rho^+\rho^-$, $\rho f_2(1270)$ and $h_1(1170)\pi^0$ is shown.

This study was supported by the RFBR (grant No. 18-32-01020).

References

[1] B. I. Khazin *et al.*, Nucl. Phys. B (Proc. Suppl.) **376**, 181 (2008).

[2] Yu. M. Shatunov *et al.*, in Proceedings of the 7th European Particle Accelerator Conference, Vienna, 2000, p. 439.

[3] R.R. Akhmetshin *et al.* (CMD-2 Collaboration), Physics Letters B **466** 392–402 (1999).

[4] Tsai Y.S., Physical Review D, **4**, 2821 (1971).