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Detecting Majorana nature of neutrinos in muon and tau decay

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The Majorana nature of neutrinos can be detected by the precise measurement of muon decay. This possibility comes from the presence of charged Higgs boson interaction for Majorana neutrinos. We study the effects of the neutrino Yukawa interaction via charged Higgs bosons in muon decay processes such as $\mu \to e\nu\overline{\nu}$ and $\mu \to e\gamma$. The Higgs triplet model with small vacuum expectation value is of special importance whose neutrino Yukawa coupling can affect significantly muon decays. External neutrino lines in the Feynman diagrams of $\mu \to e\nu\overline{\nu}$ can be crossed because of its Majorana nature. This fact provides the interference contribution between the W boson exchange diagram and that of charged Higgs boson, which may be detectable in near future experiments.

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We have obtained many definite information on neutrino over the last decade, showing manifestly new physics beyond the standard model (SM). There are still undetermined parameters θ_{13}, absolute mass values and leptonic CP phase in neutrino sector. Determinations of mass parameters are not the end of the story. The smallness of neutrino masses implies new origin of mass scale which may be related to the Majorana nature of neutrinos. The question of whether neutrino is Majorana or Dirac particles is one of the most important issues in particle physics.

The direct detection of Majorana nature of neutrino has been tried in neutrinoless double beta decay [1, 2, 3]. New type of experiments using the interference of neutrinos emitted from high Rydberg atom is also on going [4]. These attempts use the rare decay processes. On the other hand, the direct detection of the Majorana nature by precise measurement of muon decay $\mu \to e\nu\overline{\nu}$ was also studied in the model with V+A currents [5, 6]. In the seesaw framework[7], these effects are characterized by the mixing between neutrinos and anti-neutrino which is determined by a ratio m_D/M_R where m_D is the Dirac mass and M_R is the Majorana mass for neutrinos. The presence of the V+A current can affect the muon decay rate, but the deviation distinguishing Majorana from Dirac neutrino are suppressed by $\mathcal{O}\left((m_D/M_R)^2\right)$. Even if we assume the scale of right-handed neutrinos to be TeV region, i.e., $M_R \sim \mathcal{O}(\text{TeV})$ with $m_\nu \sim \mathcal{O}(\text{eV})$ [8, 9], the deviation is of $\mathcal{O}\left(10^{-12}\right)$ at most. However, present experimental accuracies of muon decay is $10^{-3}\%$ level [10]. So the detection is still out of the scope in near future experiments in their framework.

In this letter, we show that Majorana nature of neutrinos may be detectable by precise measurements of muon decay and the other decay processes. This is realized by the minimal extension of SM, including only the $SU(2)_L$ scalar triplet additionally (Higgs triplet model; HTM [11]). A detectability of the Majorana nature of neutrinos is discussed by using the interference effect of muon decay amplitude between W-boson exchange diagrams and that of charged Higgs boson in the HTM. The deviation of the muon decay rate from the SM prediction can reach several times $10^{-4}\%$. It can be detectable at further precise measurements of muon decay. Our method also gives the possibility to measure the effective neutrino mass for tau leptons in its leptonic decay.

Before studying the case for the HTM, we discuss the effects of the charged Higgs from more general standpoint, which is necessary to assure that the deviation is indeed due to the interference of Majorana neutrinos. That is, we argue the effects due to the large neutrino Yukawa coupling in the two-Higgs-doublet model (THDM) [12] first, and then in the HTM. These effects would be measured at future low energy experiments for relatively small vacuum expectation values of extra Higgs bosons ($v_\nu, v_\Delta \ll v$). It is shown that the deviation of $\Gamma(\mu \to e\nu\overline{\nu})$ from SM can become detectable level only in the HTM because of the Majorana nature of neutrinos.

In the SM, the weak interaction Lagrangian is written by

$$\mathcal{L}_{W^\pm} = -\frac{g}{\sqrt{2}} U_{\ell i}\, \overline{\ell_L} \gamma^\alpha \nu_i W^-_\alpha + \text{H.c.} \tag{1}$$

where L, ν are lepton doublet and neutrino fields in each mass diagonal bases, and $U_{\ell i}$ is the neutrino mixing matrix. The muon decay rate is calculated as

$$\Gamma^{\text{SM}}_{\mu\to e\nu\overline{\nu}} = \frac{G_F^2 m_\mu^5}{192\pi^3}\, f\left(\frac{m_e^2}{m_\mu^2}\right)(1+R.C.) \times \left(1+\frac{3m_\mu^2}{5m_W^2}\right). \tag{2}$$

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Here G_F is defined by

$$G_F \equiv \frac{g^2}{4\sqrt{2}M_W^2} \tag{3}$$

with the universal weak coupling constant g. The function is $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$, and $R.C.$ represents radiative corrections which is given by [13]

$$\begin{aligned} R.C. =& \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\left[1 + \frac{\alpha}{\pi}\left(\frac{2}{3}\ln\frac{m_\mu}{m_e} - 3.7\right)\right. \\ &+ \left.\left(\frac{\alpha}{\pi}\right)^2\left(\frac{4}{9}\ln^2\frac{m_\mu}{m_e} - 2.0\ln\frac{m_\mu}{m_e} + C\right)\right]. \end{aligned} \tag{4}$$

As we will show soon, the signal of Majorana nature can be $O(10^{-6})$, and we have written in Eq.(2) up to this precision order.

Let us begin with the THDM, where we consider an additional Higgs doublet which interacts with Dirac neutrinos. All other fermions only couples to the SM Higgs doublet. There are three neutral Higgs bosons and a pair of charged Higgs boson $H^\pm$. In order to generate small neutrino masses, the vacuum expectation value (v_ν) of the extra Higgs doublet can be taken as small $v_\nu/v \ll 1$. Such small vacuum expectation value predicts a very light scalar which is experimentally allowed because of the highly suppressed couplings with gauge bosons and charged fermions. Its detailed phenomenology is discussed in [14]. Neutrino masses are obtained by the diagonalization of $m_\nu = y_\nu v_\nu/\sqrt{2}$. In this setup, the neutrino Yukawa coupling can be taken to be large. In the THDM, the muon decay rate is corrected by the contribution of charged Higgs boson through the charged lepton Yukawa interaction [15]. This effect is suppressed by a factor v_ν/v. We here consider the extent of the large neutrino Yukawa coupling. Neutrino Yukawa interaction with charged Higgs bosons are given by

$$\mathcal{L}_{H^\pm} = -\frac{\sqrt{2}m_{\nu i}}{v_\nu} U_{\ell i}\,\overline{\ell_L}\nu_i H^- + \text{H.c.} \tag{5}$$

where ℓ and ν are charged lepton and neutrino fields in mass diagonal basis of charged leptons, $m_{\nu i}$ represents neutrino masses, and $U_{\ell i}(i = 1\text{–}3)$ is the neutrino mixing matrix. The effect of the neutrino Yukawa interaction in muon decay is calculated as

$$\delta\Gamma^{\rm THDM}_{\mu\to e\nu\overline{\nu}} = \frac{G_F^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right)\left(\frac{v}{v_\nu}\right)^4 \frac{\langle m_\nu^2\rangle_\mu \langle m_\nu^2\rangle_e}{4m_{H^\pm}^4}, \tag{6}$$

where $\langle m_\nu^2\rangle_\ell = \sum_i {m_\nu}_i^2 |U_{\ell i}|^2$. In the above formula, we neglect neutrino masses but keep its Yukawa interaction, i.e., $m_{\nu i}/m_\ell \ll m_{\nu i}/v_\nu (\ell = e, \mu)$.

The muon lifetime has been precisely measured $\tau_\mu = (2.197019 \pm 0.000021) \times 10^{-6}$s [10]. Once charged Higgs boson is observed, and its mass is determined at the LHC, the deviation of the muon decay rate from the SM prediction can constrain v_ν. The search for charged Higgs bosons has been examined at LEP via its pair production. It gives the lower limit on the mass of charged Higgs boson $m_{H^\pm} \gtrsim 79.3$ GeV[16]. Requiring the small deviation less than 10^{-3}%, the lower limit of v_ν would be obtained as

$$v_\nu \gtrsim 2\text{eV}\left(\frac{m_\nu}{0.05\text{eV}}\right)\left(\frac{100\text{GeV}}{m_H^\pm}\right). \tag{7}$$

The large neutrino Yukawa coupling induces rare muon decay $\mu \to e\gamma$. Its decay branching fraction is calculated as

$$\text{Br}(\mu \to e\gamma)^{\rm THDM} = \frac{\alpha_{\rm EM}}{24\pi}\left(\frac{v}{v_\nu}\right)^4 \frac{\left|{m_\nu}_j^2 U_{ej}U_{\mu j}\right|^2}{m_{H^\pm}^4}. \tag{8}$$

Taking into account the experimental upper limit $\text{Br}(\mu \to e\gamma)^{\rm exp} < 1.2 \times 10^{-11}$ [17], we obtain the slightly stronger lower bound on v_ν,

$$v_\nu \gtrsim 6\text{eV}\left(\frac{m_\nu}{0.05\text{eV}}\right)\left(\frac{100\text{GeV}}{m_H^\pm}\right). \tag{9}$$

Applying the bound in Eq. (9), the possible deviation of $\Gamma(\mu \to e\nu\overline{\nu})$ is only several times 10^{-8} which will not be measured in near future.

Next we discuss the effect of Majorana neutrinos on muon decay in the framework of the HTM. We introduce an $SU(2)_L$ triplet Higgs boson Δ in addition to the SM. The origin of neutrino masses can be adapted by the interactions term,

$$\mathcal{L}_{\rm HTM} = \overline{L^c} h_M\, i\tau_2 \Delta L + \text{H.c.} \tag{10}$$

Here neutrinos are required to be Majorana particles. The matrix h_M is coupling strength and $\tau_i(i = 1\text{–}3)$ denote the Pauli matrices. The triplet Higgs boson field with hypercharge $Y = 2$ can be parameterized by

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \frac{v_\Delta}{\sqrt{2}} + \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}, \tag{11}$$

where v_Δ is the vacuum expectation value of triplet Higgs boson. Mass eigenvalues of neutrinos are determined by diagonalization of $m_\nu = \sqrt{2}h_M v_\Delta$. There is a tree level contribution to rho parameter from the triplet vacuum expectation value as $\rho \approx 1 - 2v_\Delta^2/v^2$. LEP precision result gives an upper limit $v_\Delta \lesssim 5$ GeV. There is no stringent bound from $b \to s\gamma$ on the charged Higgs boson mass because the triplet Higgs boson does not couple to quarks.

The Yukawa interaction of the singly and the doubly charged Higgs boson is written by

$$\begin{aligned} \mathcal{L}_\Delta =& -\frac{m_{\nu i}}{v_\Delta} U_{\ell i}\,\overline{\ell_L} N_i^c \Delta^- \\ &- \frac{m_{\nu i}}{\sqrt{2}v_\Delta} U_{\ell i}U^*_{\ell' i}\,\overline{\ell_L}{\ell'}_{L i}^{c}\Delta^{--} + \text{H.c.}, \end{aligned} \tag{12}$$

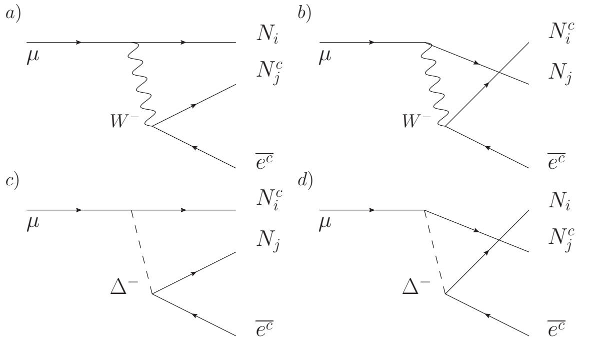

FIG. 1: The Feynman diagrams for the muon decay $\mu^- \to e^- N\overline{N}$.

where N_i represent Majorana neutrinos which satisfy conditions $N_i = N_i^c = C\overline{N_i}^T$. Therefore Majorana fields can contract not only with fermions but also with anti-fermions. In FIG. 1, we depict the Feynman diagrams for the muon decay in the HTM. The contribution to the muon decay from the singly charged Higgs boson is calculated as

$$\delta\Gamma^{\rm HTM}_{\mu\to eN\overline{N}} = \frac{G_F^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right) \times \left[2\left(\frac{v}{v_\Delta}\right)^2 \frac{|\langle m_\nu\rangle_{\mu e}|^2}{m_{\Delta^\pm}^2} + \left(\frac{v}{v_\Delta}\right)^4 \frac{\langle m_\nu^2\rangle_\mu \langle m_\nu^2\rangle_e}{m_{\Delta^\pm}^4}\right], \quad (13)$$

where the effective mass of neutrino is defined as $\langle m_\nu\rangle_{\mu e} = \sum_j m_{\nu j} U_{\mu j}^* U_{ej}$. The first term in Eq. (13) comes from the interference between Fig.(1a) and (1d). The presence of interference term is a consequence of the Majorana nature of neutrinos. Again we note that in the above formula we neglect neutrino masses but keep its Yukawa coupling. The effective neutrino masses should be understood by the ratio of the Yukawa coupling and the triplet vacuum expectation value.

Let us estimate the magnitude of these effects. The most stringent constraint on the triplet Yukawa coupling comes from $\mu \to ee\bar{e}$ through the tree level contribution due to the doubly charged Higgs boson [18]. The branching ratio for this decay is given by

$${\rm Br}(\mu \to ee\bar{e})^{\rm HTM} = \frac{1}{8}\left(\frac{v}{v_\Delta}\right)^4 \frac{|\langle m_\nu\rangle_{\mu e}\langle m_\nu\rangle_{ee}|^2}{m_{\Delta^{\pm\pm}}^4}. \quad (14)$$

The experimental bound ${\rm Br}(\mu \to ee\bar{e})^{\rm exp} < 1.0 \times 10^{-12}$ [19] gives the upper limit for the neutrino Yukawa coupling, which is translated into the lower bound on v_Δ

$$v_\Delta \gtrsim 70{\rm eV}\left(\frac{m_\nu}{0.05{\rm eV}}\right)\left(\frac{100{\rm GeV}}{m_{\Delta^{\pm\pm}}}\right)^2. \quad (15)$$

In order to avoid the large one loop contribution to rho parameter from the triplet Higgs bosons, we take their masses to be degenerate [20]. Under these conditions, the decay branching ratio of $\mu \to e\gamma$ is suppressed enough. The deviation from the SM muon decay rate can reach to several times 10^{-4}%. The difference between the effect and the current experimental accuracy is only a few factor. It might be accessible by further precise measurement of the muon decay. Unfortunately, muon decay is used as a precision measurement of Fermi coupling constant (G_F), and this deviation is renormalized in G_F [21]. So we need an independent determination of G_F for final decision of Majorana nature.

We comment on the detectability of the effective neutrino masses for tau leptons. The decay formulae for tau decay $\tau \to \ell N\overline{N}, (\ell = e, \mu)$ are easily obtained by substituting $\mu \to \tau$ and $e \to \ell$ in the preceding formula. Experimental accuracies of such tau decays are not so good, $\mathcal{O}(10^{-2})$ [22]. However, the upper bounds for the tau associated neutrino Yukawa couplings in the HTM from the lepton flavor violating decays $\tau \to \ell\ell'\ell''$ are also loose. In fact, upper bounds for these branching fractions are order 10^{-4} smaller than that of $\mu \to ee\bar{e}$. Therefore, we can expect larger deviation from the SM predictions on this process. The tau leptonic decays may be useful to distinguish the Dirac and Majorana nature of neutrinos even if the effect of neutrino Yukawa coupling is renormalized.

We have discussed detectability of Majorana nature in muon decay $\mu \to e\nu\overline{\nu}$. The effect of the large neutrino Yukawa coupling on muon decays are studied in the THDM and the HTM. In the THDM with small vacuum expectation value of extra Higgs boson, the size of neutrino Yukawa coupling is restricted by the upper limit of $\mu \to e\gamma$. Under the constraint, we have evaluated the possible deviation of decay rate for $\mu \to e\nu\overline{\nu}$. Its effect is not so large which is out of the scope in near future experiments. In the HTM, the larger values of neutrino Yukawa couplings are strongly bounded by non observation of $\mu \to ee\bar{e}$. Despite of stringent limits, we have found that muon decay rate can deviate to accessible level. This possibility comes from the additional indifference contribution between the W-boson and the charged Higgs boson mediation diagrams through the Majorana nature of neutrinos. We have shown that the effective masses for tau leptons can also be observed in tau leptonic decays.

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