Measurement of Michel parameters in τ decays at Belle

D. Epifanov (BINP)

BAM, August 30th, 2016

Outline:

1. Introduction
2. Study of systematic effects
3. Summary & Plan
4. Current status

Introduction

In the SM charged weak interaction is described by the exchange of $W^{\pm}$ with a pure vector coupling to only left-handed fermions ("V-A" Lorentz structure). Deviations from "V-A" indicate New Physics. $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ ($\ell = e, \mu$) decays provide clean laboratory to probe electroweak couplings.
The most general, Lorentz invariant four-lepton interaction matrix element:

$$\mathcal{M} = \frac{4G}{\sqrt{2}} \sum_{\substack{N=S,V,T \\ i,j=L,R}} g_{ij}^N \left[\bar{u}_i(l^-)\Gamma^N v_n(\bar{\nu}_l)\right] \left[\bar{u}_m(\nu_\tau)\Gamma_N u_j(\tau^-)\right],$$

$$\Gamma^S = 1,\ \Gamma^V = \gamma^\mu,\ \Gamma^T = \frac{i}{2\sqrt{2}}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

Ten couplings g_{ij}^N, in the SM the only non-zero constant is $g_{LL}^V = 1$

Four bilinear combinations of g_{ij}^N, which are called as Michel parameters (MP): ρ, η, ξ and δ appear in the energy spectrum of the outgoing lepton:

$$\frac{d\Gamma(\tau^\mp)}{d\Omega dx} = \frac{4G_F^2 M_\tau E_{\max}^4}{(2\pi)^4}\sqrt{x^2 - x_0^2}\Bigg(x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)$$

$$\mp\frac{1}{3}P_\tau cos\theta_\ell \xi\sqrt{x^2 - x_0^2}\left[1 - x + \frac{2}{3}\delta\left(4x - 4 + \sqrt{1 - x_0^2}\right)\right]\Bigg),\ x = \frac{E_\ell}{E_{\max}},\ x_0 = \frac{m_\ell}{E_{\max}}$$

In the SM: $\rho = \frac{3}{4}$, $\eta = 0$, $\xi = 1$, $\delta = \frac{3}{4}$

Unbinned maximum likelihood fit

Total PDF

$$\mathcal{P}(x) = \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\Big((1-\sum_i \lambda_i)\frac{S(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} S(x)dx} + \lambda_{3\pi}\frac{\tilde{B}_{3\pi}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\tilde{B}_{3\pi}(x)dx} + \lambda_{\pi}\frac{\tilde{B}_{\pi}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\tilde{B}_{\pi}(x)dx} + \lambda_{\rho}\frac{\tilde{B}_{\rho}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\tilde{B}_{\rho}(x)dx} +$$

$$+(1-\sum_i \lambda_i)\frac{N^{\rm sel}_{\rm rest}(x)}{N^{\rm sel}_{\rm sig}(x)} S_{\rm SM}(x)\Big)$$

$$\tilde{B}_{3\pi}(x) = \int 2(1-\varepsilon_{\pi^0}(y))\varepsilon_{\rm add}(y)B_{3\pi}(x,y)dy,\ \tilde{B}_{\pi}(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)}B_{\pi}(x)$$

$$\tilde{B}_{\rho}(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)}\int (1-\varepsilon_{\pi^0}(y))\varepsilon_{\rm add}(y)B_{\rho}(x,y)dy,\ \overline{\varepsilon(x)} = \epsilon^{\rm trg}_{\rm corr}(x)\epsilon^{\ell {\rm ID}}_{\rm corr}(x)\varepsilon(x)$$

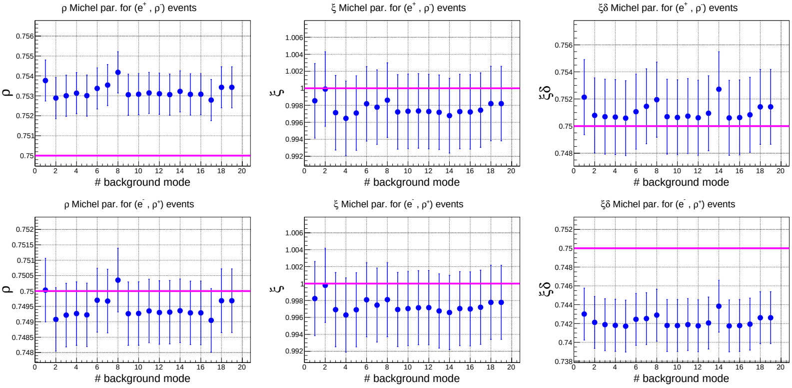
- $x = (p_\ell,\ \Omega_\ell,\ p_\rho,\ \Omega_\rho,\ m^2_{\pi\pi},\ \tilde{\Omega}_\pi);\ y = (p_{\pi^0},\ \Omega_{\pi^0})$;
- $S(x)$ - theoretical density of signal ($\ell^\mp,\ \pi^\pm\pi^0$) events;
- $B_{3\pi}(x,y)$ - theoretical density of background ($\ell^\mp,\ \pi^\pm 2\pi^0$) events;
- $B_\pi(x)$ - theoretical density of background ($\pi^\mp,\ \pi^\pm\pi^0$) events;
- $B_\rho(x)$ - theoretical density of background ($\pi^\mp\pi^0,\ \pi^\pm\pi^0$) events;
- $\varepsilon(x)$ - detection efficiency for signal events (**common multiplier**);
- $N^{\rm sel}_{\rm rest}(x)/N^{\rm sel}_{\rm sig}(x)$ - number of the selected (remaining/signal) MC events in the multidimensional cell around "x". Admixture of the remaining background is $(1 \div 2)\%$.
- λ_i - i-th background fraction (from MC)
- $\varepsilon_{\pi^0}(y)$ - π^0 detection efficiency (tabulated from MC);
- $\varepsilon_{\rm add}(y) = \varepsilon^{3\pi}_{\rm add}(y)/\varepsilon^{\rm sig}_{\rm add}$ - ratio of the $E^{\rm LAB}_{\gamma{\rm rest}}$ cut efficiencies (tabulated from MC);
- $\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)/\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)$ is tabulated from MC;
- $\epsilon^{\rm trg}_{\rm corr}(x),\ \epsilon^{\ell{\rm ID}}_{\rm corr}(x)$ - EXP/MC efficiency corrections.

- We established the contents of the remaining background, all modes with the absolute admixtures $\geq$ 0.1% were considered (19 modes for the $(e^{\pm},\ \rho^{\mp})$ and 25 modes for the $(\mu^{\pm},\ \rho^{\mp})$ events)
- Switching ON/OFF modes one by one in the fitted MC sample we found 2 modes with the largest impact on Michel parameters
- $(\pi^{\pm},\ \pi^{\mp}\pi^{0})$ (with the absolute admixture of about 0.15%) in $(e^{\pm},\ \pi^{\mp}\pi^{0})$ events
- $(\pi^{\pm}\pi^{0},\ \pi^{\mp}\pi^{0})$ (with the absolute admixture of about 0.5%) in $(\mu^{\pm},\ \pi^{\mp}\pi^{0})$ events
- They are described analytically in the total PDF, the accuracy of the description of the remaining backgound is sufficient now.

Remaining background for the $(e^{\pm}, \rho^{\mp})$ events

#	(e, h) mode	$E^{LAB}_{\gamma\ rest} < 0.1$ GeV	$E^{LAB}_{\gamma\ rest} < 0.3$ GeV
1	(e, other)	11.5%	13.2%
2	(e, π)	8.3%	7.6%
3	$(e, 3\pi)$	2.4%	2.1%
4	$(e, \pi K_S)$	4.9%	7.0%
5	$(e, \pi K_L)$	1.5%	1.3%
6	$(e, K\pi^0)$	11.6%	8.5%
7	$(e, 3\pi\pi^0)$	9.9%	8.5%
8	$(e, \pi 3\pi^0)$	8.8%	15.9%
9	$(e, \pi K_S K_L)$	0.7%	1.0%
10	$(e, K\pi^0 K_S)$	0.3%	0.3%
11	$(e, K\pi^0 K_L)$	1.4%	1.2%
12	$(e, K2\pi^0)$	0.4%	0.5%
13	$(e, \pi\pi^0 K_S)$	3.5%	3.2%
14	$(e, \pi\pi^0 K_L)$	22.1%	17.1%
15	$(e, \pi 4\pi^0)$	0.2%	0.4%
16	$(e, \pi\omega\pi^0)$	0.2%	0.3%
17	$(\pi, \pi\pi^0)$	6.9%	4.9%
18	$(\pi, \pi 2\pi^0)$	0.5%	0.7%
19	$(\pi\pi^0, \pi\pi^0)$	2.0%	2.8%
	sum	97.0%	96.7%
	rest	3.0%	3.3%

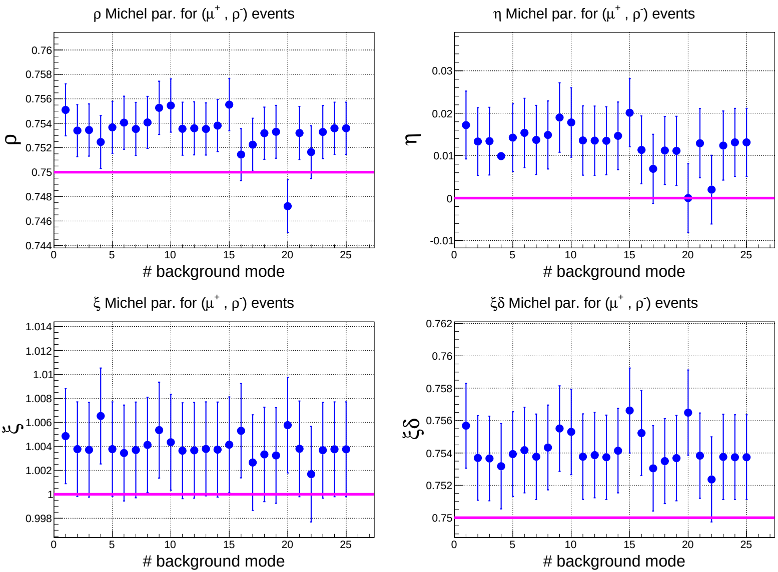
$(e,\ \rho)$ MC fit with analytical description of $(\pi,\ \rho)$

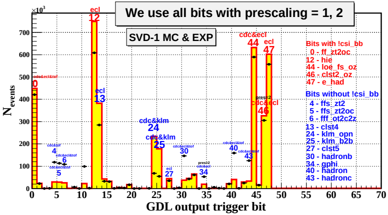


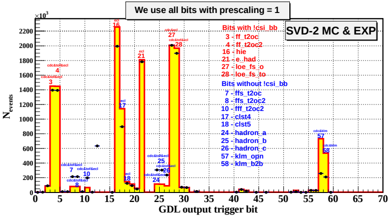
Remaining background for the $(\mu^{\pm},\ \rho^{\mp})$ events

#	$(\mu,\ h)$ mode	$E_{\gamma\ rest}^{LAB} < 0.1$ GeV	$E_{\gamma\ rest}^{LAB} < 0.3$ GeV
1	$(\mu,\ \text{other})$	9.1%	9.3%
2	$(\mu,\ e)$	0.8%	0.6%
3	$(\mu,\ \mu)$	0.6%	0.4%
4	$(\mu,\ \pi)$	7.6%	6.3%
5	$(\mu,\ 3\pi)$	1.7%	1.5%
6	$(\mu,\ \pi K_S)$	3.9%	4.9%
7	$(\mu,\ \pi K_L)$	1.4%	1.1%
8	$(\mu,\ K\pi^0)$	8.9%	6.2%
9	$(\mu,\ 3\pi\pi^0)$	8.8%	7.0%
10	$(\mu,\ \pi 3\pi^0)$	6.6%	10.6%
11	$(\mu,\ \pi K_S K_L)$	0.5%	0.7%
12	$(\mu,\ KK_L\pi^0)$	1.0%	0.7%
13	$(\mu,\ K2\pi^0)$	0.3%	0.3%
14	$(\mu,\ \pi K_S\pi^0)$	2.2%	2.0%
15	$(\mu,\ \pi K_L\pi^0)$	15.5%	11.5%
16	$(\pi,\ \pi 2\pi^0)$	4.4%	5.5%
17	$(\pi\pi^0,\ \pi\pi^0)$	10.8%	15.9%
18	$(\pi\pi^0,\ \pi 2\pi^0)$	0.9%	1.8%
19	$(3\pi,\ \pi\pi^0)$	0.5%	0.4%
20	$(\pi 2\pi^0,\ \pi\pi^0)$	0.6%	1.1%
21	$(K,\ \pi\pi^0)$	7.2%	4.9%
22	$(K,\ \pi 2\pi^0)$	0.5%	0.6%
23	$(\pi K_L,\ \pi\pi^0)$	1.4%	1.1%
24	$(K\pi^0,\ \pi\pi^0)$	0.4%	0.5%
25	$(KK_L,\ \pi\pi^0)$	0.4%	0.3%
	sum	95.8%	95.3%
	rest	4.2%	4.7%

$(\mu,\ \rho)$ MC fit with analytical description of $(\rho,\ \rho)$







$$\mathcal{R}_{trg} = \frac{\varepsilon^{EXP}_{trg}}{\varepsilon^{MC}_{trg}} = \frac{\left(\frac{N_{N\&Z}}{N_N}\right)^{EXP} + \left(\frac{N_{N\&Z}}{N_Z}\right)^{EXP} - \left(\frac{N_{N\&Z}}{N_N}\frac{N_{N\&Z}}{N_Z}\right)^{EXP}}{\left(\frac{N_{N\&Z}}{N_N}\right)^{MC} + \left(\frac{N_{N\&Z}}{N_Z}\right)^{MC} - \left(\frac{N_{N\&Z}}{N_N}\frac{N_{N\&Z}}{N_Z}\right)^{MC}}$$

SVD1 GDL output bits assignment:

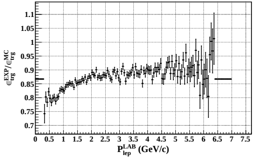
$$\text{Charged trigger}: \ Z = \mathbf{4} \text{ or } \mathbf{24} \text{ or } \mathbf{25}, \quad \text{Neutral triger}: \ N = \mathbf{13} \text{ or } \mathbf{27}$$

SVD2 GDL output bits assignment:

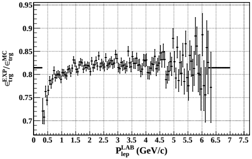
$$\text{Charged trigger}: \ Z = \mathbf{57} \text{ or } \mathbf{58}, \quad \text{Neutral triger}: \ N = \mathbf{17} \text{ or } \mathbf{18}$$

EXP data fit with $\mathcal{R}_{\rm trg}(P_\ell)$

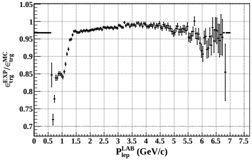
SVD1 (e, ρ)



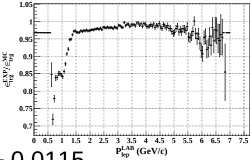
SVD2 (e, ρ)



SVD1 (μ, ρ)



SVD2 (μ, ρ)



$$\rho = 0.7586 \pm 0.0013 \pm 0.0115$$

$$\eta = -0.0276 \pm 0.0062 \pm 0.0219$$

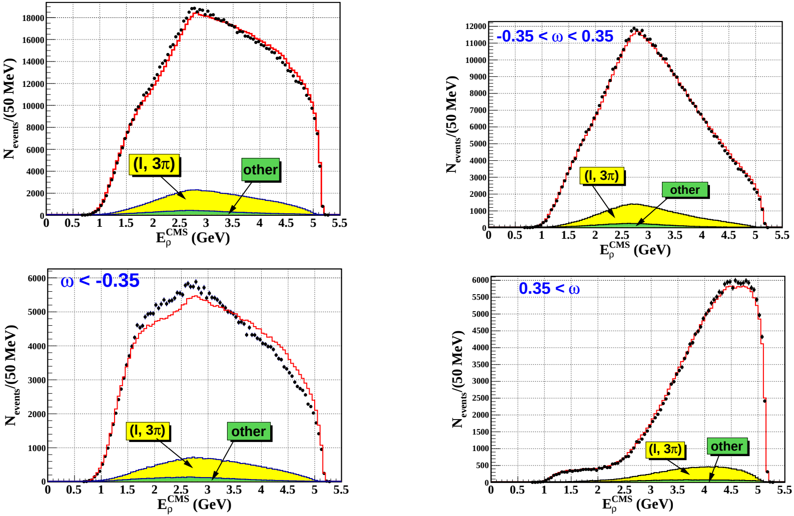
$$\xi = 0.9973 \pm 0.0039 \pm 0.0479$$

$$\xi\delta = 0.7520 \pm 0.0025 \pm 0.0478$$

The second uncertainty is systematic one. It was obtained as a maximal difference of Michel parameter over 4 configurations and over 3 extra gamma energy cuts (very conservative estimation).

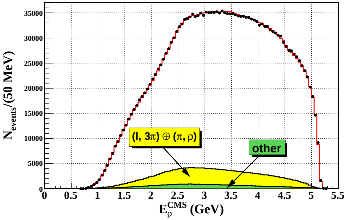
We think that the systematic uncertainty can be substantially reduced.

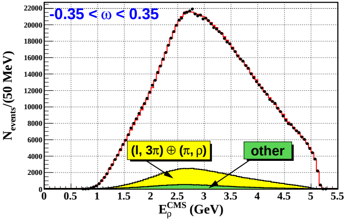
Fit of EXP (e, ρ) events

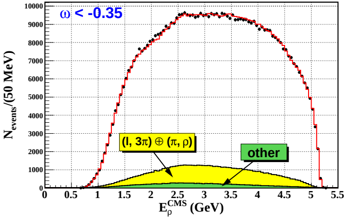


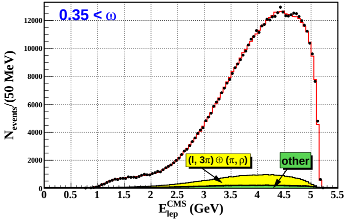
We observe notable disagreement between experimental and optimal ρ energy spectra for ONLY (e, ρ) events (especially for events with $\omega < -0.35$). This is certainly an effect of the trigger efficiency correction.

Fit of EXP $(\mu,\ \rho)$ events



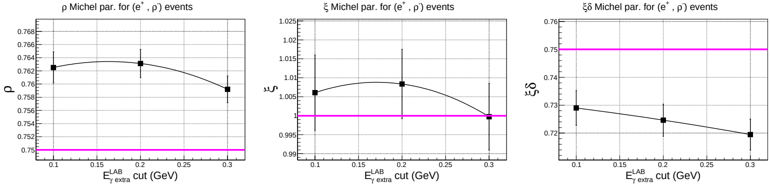




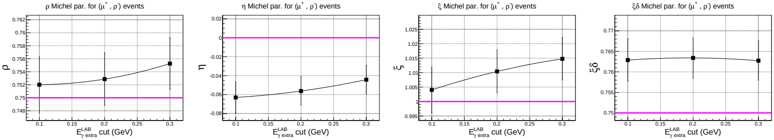


$$\mathcal{R}_{\rm trg} = \frac{\mathcal{R}_{\rm 2D}(P_\ell, \cos\psi_{\ell\rho})\mathcal{R}_{\rm 2D}(P_\ell, P_\rho)}{\mathcal{R}_{\rm 1D}(P_\ell)}$$

$(e^+;\ \rho^-)$



$(\mu^+;\ \rho^-)$

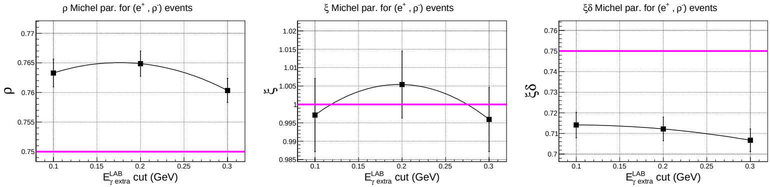


Notable impact on the ξ ((2.5 $\div$ 3)%), ρ (up to 1%), η (up to 2%) and $\xi\delta$ was observed.

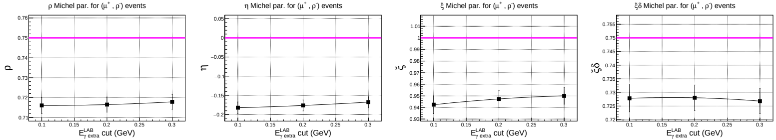
Fit with the $(2D)^7/(1D)^5$ scheme

$$\mathcal{R}_{\rm trg} = \frac{\mathcal{R}_{\rm 2D}(P_\ell,\cos\psi_{\ell\rho})\mathcal{R}_{\rm 2D}(P_\ell,P_\rho)\mathcal{R}_{\rm 2D}(P_\ell,\cos\theta_\rho)\mathcal{R}_{\rm 2D}(P_\rho,\cos\tilde\theta_\pi)\mathcal{R}_{\rm 2D}(P_\rho,m^2_{\pi\pi})}{\mathcal{R}^2_{\rm 1D}(P_\ell)\mathcal{R}^2_{\rm 1D}(P_\rho)} \times$$
$$\times \frac{\mathcal{R}_{\rm 2D}(\cos\psi_{\ell\rho},\varphi_\ell-\varphi_\rho)\mathcal{R}_{\rm 2D}(\varphi_\rho,\tilde\varphi_\pi)}{\mathcal{R}_{\rm 1D}(\cos\psi_{\ell\rho})}$$

$(e^+;\ \rho^-)$

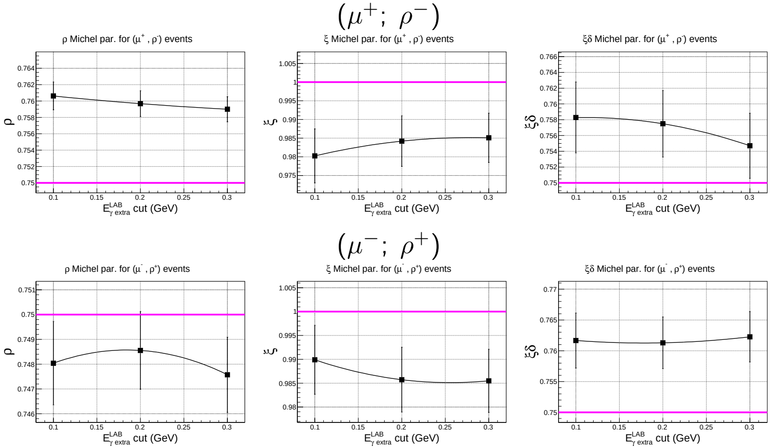


$(\mu^+;\ \rho^-)$



$(e^\pm;\ \rho^\mp)$ events: notable impact on the $\xi\delta$ ($(3\div4)\%$) was observed.
$(\mu^\pm;\ \rho^\mp)$ events: very large impact on the η parameter (large deviation of the EXP and optimal spectra at small momenta)

Fit with the $(2D)^7/(1D)^5$ scheme, $\eta = 0$-fixed



Agreement between different charge configurations is $\lesssim 1\%$

Michel parameters agree with the SM expectation within $(1.0 \div 1.5)\%$

The second way to tabulate trigger efficiency

In reality, the GDL output trigger bits are constructed from a few simple parameters, characterizing physical event, i.e. the **neutral trigger** depends on the **number of isolated clusters (icn)** in ECL and the **total energy deposition in ECL (E_{tot})**. **Charged trigger** ((cdc_open OR cdc_bb)& klm_br) depends on the transversal lepton ($p_{\perp\ell}$) and pion ($p_{\perp\pi}$) momenta and track acollinearity ($\Delta\varphi_{\ell\pi}$) in the $R-\varphi$ plane. **It is enough to tabulate charged and neutral trigger efficiencies as a functions of these, natural, variables:** $\epsilon_N(icn, E_{tot})$, $\epsilon_Z(p_{\perp\ell}, p_{\perp\pi}, \Delta\varphi_{\ell\pi})$.

It is easy to show that in this approach there are no additional multiplicative factors like in the previous method.

To increase an accuracy of the approximation of ϵ_Z, it is written as ((2D×2D)/1D):

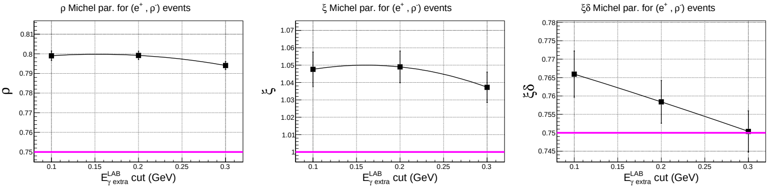
$$\epsilon_Z = \frac{\sum_i \epsilon_Z(p_{\perp\ell}, p_{\perp\pi}(i), \Delta\varphi_{\ell\pi}) \sum_j \epsilon_Z(p_{\perp\ell}, p_{\perp\pi}, \Delta\varphi_{\ell\pi}(j))}{\sum_{i,j} \epsilon_Z(p_{\perp\ell}, p_{\perp\pi}(i), \Delta\varphi_{\ell\pi}(j))}$$

So, the procedure is: tabulate $\epsilon_N^{MC}(icn, E_{tot})$, $\epsilon_Z^{MC}(p_{\perp\ell}, p_{\perp\pi}, \Delta\varphi_{\ell\pi})$, $\epsilon_N^{EXP}(icn, E_{tot})$, $\epsilon_Z^{EXP}(p_{\perp\ell}, p_{\perp\pi}, \Delta\varphi_{\ell\pi})$ (instead of icn we use number of extra photons in the event), and then for each experimental event in the fit calculate the trigger efficiency correction according to the formula:

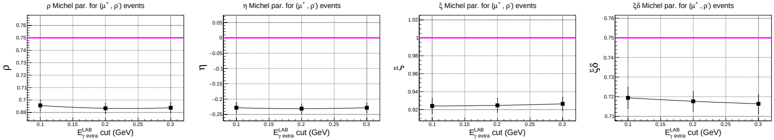
$$\mathcal{R}_{trg} = \frac{\epsilon_Z^{EXP} + \epsilon_N^{EXP} - \epsilon_Z^{EXP}\epsilon_N^{EXP}}{\epsilon_Z^{MC} + \epsilon_N^{MC} - \epsilon_Z^{MC}\epsilon_N^{MC}}$$

Fit of EXP data with the new scheme

$(e^+;\ \rho^-)$



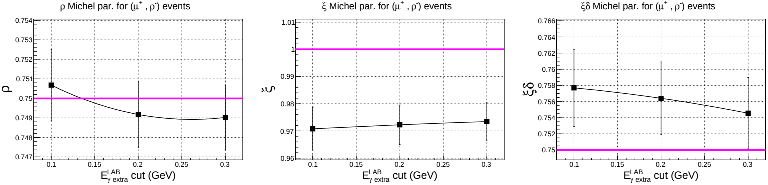
$(\mu^+;\ \rho^-)$



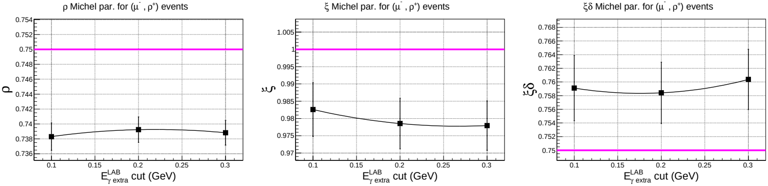
$(e^\pm;\ \rho^\mp)$ events: large impact on all Michel parameters.
$(\mu^\pm;\ \rho^\mp)$ events: very large impact on the η parameter (large deviation of the EXP and optimal spectra at small momenta)

Fit of EXP data with the new scheme, $\eta = 0$-fixed

$(\mu^+;\ \rho^-)$



$(\mu^-;\ \rho^+)$



Agreement between different charge configurations is $\lesssim$ 1%
Michel parameters agree with the SM expectation within $(1.0 \div 3.0)$%

Summary & Plan

- We reached acceptable description of the backgrounds in the PDF.
- We are working on the EXP/MC trigger efficiency correction, $\mathcal{R}_{\rm trg}$.
- In the scheme where all 2D-correlations are accounted in the $\mathcal{R}_{\rm trg}$ ($(2D)^7/(1D)^5$) large impact on the η parameter was observed (induces bias in the other Michel parameters through correlations), region of small lepton momenta shows the largest difference between experimental and optimal distributions. **It might indicate that at this point we already encounter with the effect of the track reconstruction EXP/MC efficiency correction, which is expected to be large at small lepton/cherged pion momenta.**
- As the mentioned schemes suffer from intrinsic inaccuracies (effect of the additional multiplicative factors, distorting true shape of the $\mathcal{R}_{\rm trg}$), we developed another scheme, where the trigger efficiencies are tabulated as a functions of natural variables. Fit of experimental data in this scheme showed the same problems, as in the previous $(2D)^7/(1D)^5$ scheme. We are still performing various tests with the new scheme.
- We plan to add the remaining corrections, track reconstruction efficiency correction and π^0 reconstruction efficiency correction, to observe the full picture and establish the appropriate scheme for the trigger efficiency correction.

Current status

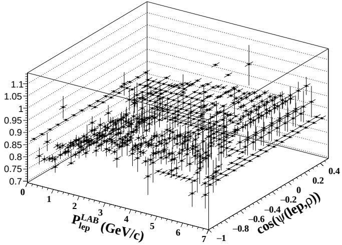
- **Belle note N1351**:
  http://belle.kek.jp/secured/belle_note/gn1351/michel_note_v1.2.pdf
  Version v1.4 is ready (will be issued after we solve the problem with $\mathcal{R}_{\rm trg}$).
- **Belle-CONF-1402**: http://arxiv.org/abs/1409.4969
  Central values of Michel parameters are not published in the Belle-CONF-1402, only statistical sensitivities and expected systematic uncertainties are given.
- Two talks at the international conferences: **Tau-2014** and **17th Lomonosov Conf. 2015**.
  No contributions were sent to the proceedings of the conferences (don't want to publish only sensitivities).
- Plan to make report at the coming **Tau-2016** conf.
- We are studying systematic effects related to EXP/MC corrections.

Backups

$\mathcal{R}_{\rm trg}(P_\ell, \cos\psi_{\ell\rho})$

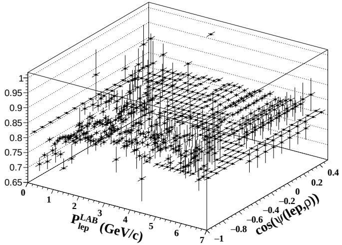
SVD1 (e, ρ)

Graph2D



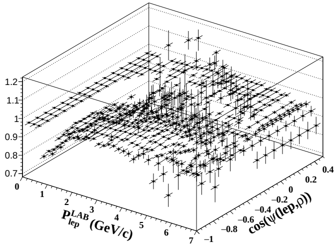
SVD2 (e, ρ)

Graph2D



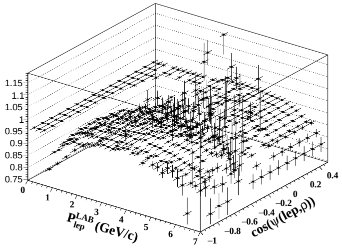
SVD1 (μ, ρ)

Graph2D



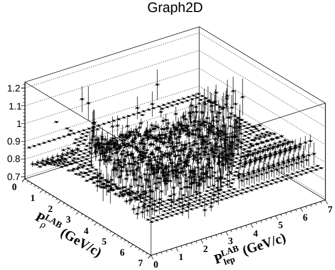
SVD2 (μ, ρ)

Graph2D

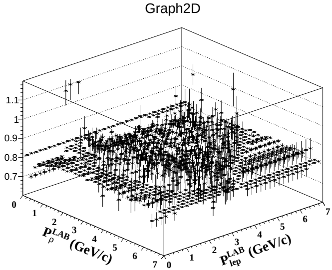


$\mathcal{R}_{\text{trg}}(P_\ell, P_\rho)$

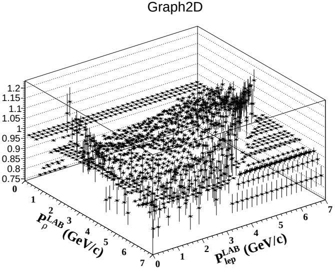
SVD1 (e, ρ)



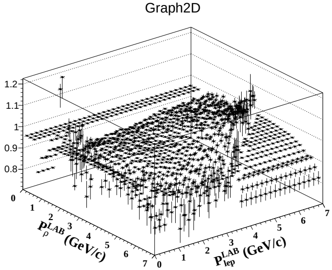
SVD2 (e, ρ)



SVD1 (μ, ρ)



SVD2 (μ, ρ)



Trigger efficiency as a function of kinematical par.

$$\mathrm{N_Z} = \mathrm{N_0} \int \mathrm{P(x,y)}\varepsilon\mathrm{(x,y)}\epsilon_\mathrm{Z}\mathrm{(x,y)dxdy}$$
$$\mathrm{N_N} = \mathrm{N_0} \int \mathrm{P(x,y)}\varepsilon\mathrm{(x,y)}\epsilon_\mathrm{N}\mathrm{(x,y)dxdy}$$
$$\mathrm{N_{N\&Z}} = \mathrm{N_0} \int \mathrm{P(x,y)}\varepsilon\mathrm{(x,y)}\epsilon_\mathrm{N}\mathrm{(x,y)}\epsilon_\mathrm{Z}\mathrm{(x,y)dxdy},$$

where $P(x,y)$ - initial PDF determined by the differential cross section, $\varepsilon(x,y)$ - detection efficiency, $\epsilon_i(x,y)$ ($i = Z,\ N,\ N\&Z$) - i-trigger efficiency. Suppose we want to tabulate the Z-trigger efficiency as a functions of "x" or "y" from the data:

$$\epsilon_Z^{tab}(x) = \frac{N_{N\&Z}(x)}{N_N(x)} = \frac{\int P_N(x,y)\epsilon_Z(x,y)dy}{\int P_N(x,y)dy},$$

where $P_N(x,y) = P(x,y)\varepsilon(x,y)\epsilon_N(x,y)$. If the $\epsilon_Z(x,y)$ can be factorized as: $\epsilon_Z(x,y) = \epsilon_Z^{(1)}(x) \cdot \epsilon_Z^{(2)}(y)$ we obtain:

$$\epsilon_Z^{tab}(x) = \frac{\int P_N(x,y)\epsilon_Z^{(1)}(x)\epsilon_Z^{(2)}(y)dy}{\int P_N(x,y)dy} = f_Z^{(2)}(x)\epsilon_Z^{(1)}(x),\ f_Z^{(2)}(x) = \frac{\int P_N(x,y)\epsilon_Z^{(2)}(y)dy}{\int P_N(x,y)dy}$$
$$\epsilon_Z^{tab}(y) = \frac{\int P_N(x,y)\epsilon_Z^{(1)}(x)\epsilon_Z^{(2)}(y)dx}{\int P_N(x,y)dx} = f_Z^{(1)}(y)\epsilon_Z^{(2)}(y),\ f_Z^{(1)}(y) = \frac{\int P_N(x,y)\epsilon_Z^{(1)}(x)dx}{\int P_N(x,y)dx}$$

As a result the total **tabulated** Z-trigger efficiency:

$$\epsilon_Z^{tab}(x,y) = \epsilon_Z^{tab}(x)\epsilon_Z^{tab}(y) = f_Z^{(2)}(x)f_Z^{(1)}(y)\epsilon_Z(x,y)$$

Similar situation we have for the N-trigger:

$$\epsilon_N^{tab}(x) = \frac{N_{N\&Z}(x)}{N_Z(x)} = \frac{\int P_Z(x,y)\epsilon_N(x,y)dy}{\int P_Z(x,y)dy},$$

where $P_Z(x,y) = P(x,y)\varepsilon(x,y)\epsilon_Z(x,y)$. If the $\epsilon_N(x,y)$ can be factorized as: $\epsilon_N(x,y) = \epsilon_N^{(1)}(x) \cdot \epsilon_N^{(2)}(y)$ we obtain:

$$\epsilon_N^{tab}(x) = \frac{\int P_Z(x,y)\epsilon_N^{(1)}(x)\epsilon_N^{(2)}(y)dy}{\int P_Z(x,y)dy} = f_N^{(2)}(x)\epsilon_N^{(1)}(x),\ f_N^{(2)}(x) = \frac{\int P_Z(x,y)\epsilon_N^{(2)}(y)dy}{\int P_Z(x,y)dy}$$

$$\epsilon_N^{tab}(y) = \frac{\int P_Z(x,y)\epsilon_N^{(1)}(x)\epsilon_N^{(2)}(y)dx}{\int P_Z(x,y)dx} = f_N^{(1)}(y)\epsilon_N^{(2)}(y),\ f_N^{(1)}(y) = \frac{\int P_Z(x,y)\epsilon_N^{(1)}(x)dx}{\int P_Z(x,y)dx}$$

As a result the total **tabulated** N-trigger efficiency:

$$\epsilon_N^{tab}(x,y) = \epsilon_N^{tab}(x)\epsilon_N^{tab}(y) = f_N^{(2)}(x)f_N^{(1)}(y)\epsilon_N(x,y)$$

We don't earn additional factors only if we tabulate Z- and N-trigger efficiencies in the total phase space (9D). If we tabulate efficiency in the reduced space, additional multiplicative factors appear, and they distort the true shape of the tabulated efficiency:

$$\mathcal{R}_{trg}^{tab} = \frac{\epsilon_Z^{EXP,tab}+\epsilon_N^{EXP,tab}-\epsilon_Z^{EXP,tab}\epsilon_N^{EXP,tab}}{\epsilon_Z^{MC,tab}+\epsilon_N^{MC,tab}-\epsilon_Z^{MC,tab}\epsilon_N^{MC,tab}} = \frac{f_Z^{EXP}\epsilon_Z^{EXP}+f_N^{EXP}\epsilon_N^{EXP}-f_Z^{EXP}f_N^{EXP}\epsilon_Z^{EXP}\epsilon_N^{EXP}}{f_Z^{MC}\epsilon_Z^{MC}+f_N^{MC}\epsilon_N^{MC}-f_Z^{MC}f_N^{MC}\epsilon_Z^{MC}\epsilon_N^{MC}}$$